

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

--	--	--	--	--

--	--	--	--	--

## Pearson Edexcel International GCSE

**Tuesday 31 October 2023**

Morning (Time: 2 hours)

Paper  
reference

**4PM1/01**

### Further Pure Mathematics PAPER 1



**Calculators may be used.**

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.  
Anything you write on the formulae page will gain **NO** credit.

### Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

P73584A

©2023 Pearson Education Ltd.  
Z:1/1/1/



  
Pearson

## International GCSE in Further Pure Mathematics Formulae sheet

### Mensuration

Surface area of sphere =  $4\pi r^2$

Curved surface area of cone =  $\pi r \times$  slant height

Volume of sphere =  $\frac{4}{3}\pi r^3$

### Series

#### Arithmetic series

Sum to  $n$  terms,  $S_n = \frac{n}{2}[2a + (n - 1)d]$

#### Geometric series

Sum to  $n$  terms,  $S_n = \frac{a(1 - r^n)}{(1 - r)}$

Sum to infinity,  $S_\infty = \frac{a}{1 - r}$   $|r| < 1$

#### Binomial series

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$$

### Calculus

#### Quotient rule (differentiation)

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

### Trigonometry

#### Cosine rule

In triangle  $ABC$ :  $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

### Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**BLANK PAGE**





**Question 1 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area with horizontal dotted lines for writing.

**(Total for Question 1 is 4 marks)**





**Question 2 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Handwriting practice area with 28 horizontal dotted lines.

**(Total for Question 2 is 7 marks)**







**Question 3 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area with horizontal dotted lines for writing.

**(Total for Question 3 is 8 marks)**



4 The point  $A$  with coordinates  $(12, 14)$  and the point  $B$  with coordinates  $(q, 2)$  where  $q$  is a constant, lie on the straight line with equation  $3y - 2x - p = 0$  where  $p$  is a constant.

(a) Find the value of  $p$  and the value of  $q$

(3)

The line  $L$  is perpendicular to  $AB$  and passes through the point  $X$ , which lies on  $AB$  such that  $AX : XB = 1 : 2$

(b) Find an equation for  $L$  in the form  $ax + by + c = 0$  where  $a$ ,  $b$  and  $c$  are integers to be found.

(6)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



**Question 4 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Handwriting practice area with 25 horizontal dotted lines.



**Question 4 continued**

Area with horizontal dotted lines for writing.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 4 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area with horizontal dotted lines for writing.

**(Total for Question 4 is 9 marks)**



5

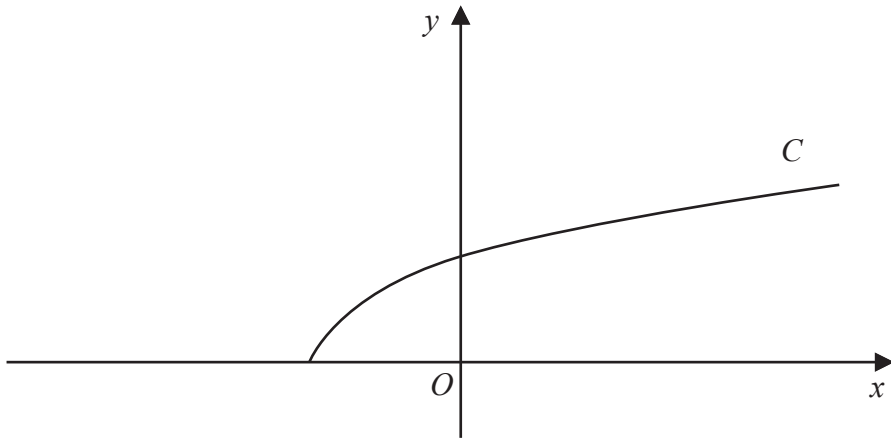


Figure 2

Figure 2 shows the graph of part of the curve  $C$  with equation  $y = \sqrt{2x + 6}$ . The finite region enclosed by the curve  $C$  and the straight line with equation  $3y - x = 3$  is rotated through  $360^\circ$  about the  $x$ -axis.

Use algebraic integration to find the exact volume of the solid generated. Give your answer in terms of  $\pi$

(8)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**Question 5 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area with horizontal dotted lines for writing.

**(Total for Question 5 is 8 marks)**



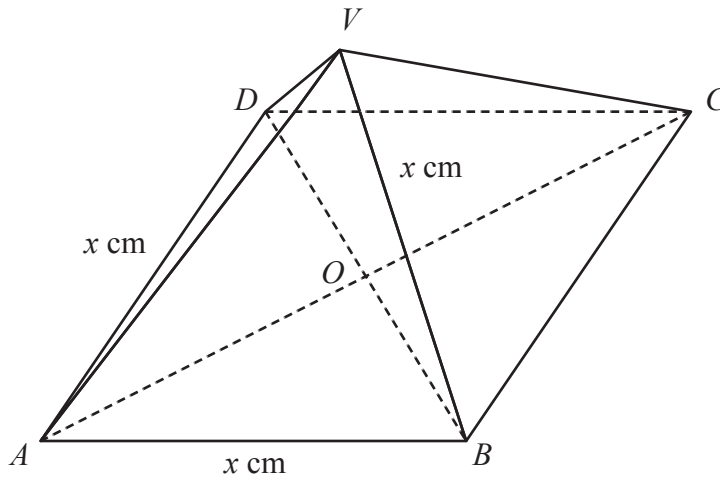


Diagram **NOT**  
accurately drawn

**Figure 3**

Figure 3 shows a right pyramid with a horizontal square base.

$$AB = BC = CD = DA = x \text{ cm}$$

$$AV = BV = CV = DV = x \text{ cm}$$

$O$  is the point of intersection of the diagonals of the base.  
The vertex  $V$  of the pyramid is vertically above  $O$

(a) Show that  $VO = \frac{\sqrt{2}}{2}x \text{ cm}$  (3)

(b) Find, in degrees, the size of the angle  $AVC$  (2)

(c) Find, in degrees to one decimal place, the size of the angle between the plane  $VAB$  and the plane  $VDC$  (3)

The volume of the pyramid is  $200 \text{ cm}^3$

Given that the volume of a pyramid =  $\frac{1}{3} \times \text{base area} \times \text{height}$

(d) Find to 3 significant figures, the value of  $x$  (3)

.....

.....

.....

.....

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA





**Question 6 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area with horizontal dotted lines for writing.



**Question 6 continued**

Area with horizontal dotted lines for writing.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 6 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area with horizontal dotted lines for writing.

**(Total for Question 6 is 11 marks)**



7 A geometric series  $G$  with common ratio  $r$ , has first term 16 and third term  $\frac{2704}{625}$

(a) Find the two possible values of  $r$  (2)

Given that  $r > 0$

(b) find the sum to infinity of  $G$  (2)

The sum to  $n$  terms of  $G$  is greater than 33

(c) Find, using logarithms, the least possible value of  $n$   
Show your working clearly. (5)

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---



**Question 7 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area with horizontal dotted lines for writing.

**(Total for Question 7 is 9 marks)**



8

$$y = \frac{2e^{3x+1}}{5x^2}$$

(a) Find  $\frac{dy}{dx}$

Give your answer in the form  $\frac{Ae^{3x+1}(Bx - A)}{Cx^3}$  where  $A$ ,  $B$  and  $C$  are prime numbers to be found.

(5)

The value of  $x$  increases by 2%

(b) Use your answer to part (a) to find an estimate, in terms of  $x$ , for the percentage change in  $y$

Give your answer in the form  $(Px - Q)$  where  $P$  and  $Q$  are integers.

(3)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 8 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Handwriting practice area with 25 horizontal dotted lines.



**Question 8 continued**

Area with horizontal dotted lines for writing.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA





**Question 8 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area with horizontal dotted lines for writing.

**(Total for Question 8 is 8 marks)**



- 9 (a) Expand  $(1-8x^2)^{\frac{1}{2}}$  in ascending powers of  $x$ , up to and including the term in  $x^6$  giving each coefficient as an integer. (3)

$$g(x) = \frac{a+bx}{\sqrt{1-8x^2}} \quad \text{where } a \text{ and } b \text{ are prime numbers}$$

Given that the fourth and fifth terms, in ascending powers of  $x$ , in the series expansion of  $g(x)$  are  $20x^3$  and  $48x^4$  respectively,

- (b) find the value of  $a$  and the value of  $b$  (4)

Using the first five terms, in ascending powers of  $x$ , in the series expansion of  $g(x)$

- (c) obtain an estimate, to 4 significant figures, of  $\int_0^{0.2} g(x) dx$  (4)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area with horizontal dotted lines for writing.



**Question 9 continued**

Area with horizontal dotted lines for writing.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 9 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area with horizontal dotted lines for writing.

**(Total for Question 9 is 11 marks)**



10 (a) Using formulae on page 2, show that

(i)  $\sin 2A = 2 \sin A \cos A$

(ii)  $\cos 2A = 2 \cos^2 A - 1$

(3)

$$f(\theta) = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

(b) Show that  $f(\theta) = \sin 2\theta$

(4)

(c) Solve, in radians to 3 significant figures, for  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ , the equation

$$5 \tan \left( x + \frac{\pi}{6} \right) = \left[ 1 + \tan^2 \left( x + \frac{\pi}{6} \right) \right] \left[ 1 - 2 \cos^2 \left( x + \frac{\pi}{6} \right) \right]$$

(6)

(d) Using calculus, find the exact value of

$$\int_0^{\frac{\pi}{2}} \left( \frac{4 \tan \theta}{1 + \tan^2 \theta} - \cos 5\theta + 2 \right) d\theta$$

(4)

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---



**Question 10 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Handwriting practice area with 25 horizontal dotted lines.



P 7 3 5 8 4 A 0 3 1 3 6

**Question 10 continued**

Area with horizontal dotted lines for writing.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA





**Question 10 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area with horizontal dotted lines for writing.

**(Total for Question 10 is 17 marks)**



P 7 3 5 8 4 A 0 3 3 3 6

11 Solve the simultaneous equations

$$2\log_4 x = \log_3 3y^2$$

$$\log_2 x^3 + 8\log_9 y = 13$$

Show your working clearly.

(8)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 11 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Handwriting practice area with 25 horizontal dotted lines.



