Mark Scheme (Results)

Summer 2023

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 01R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks
    - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
    - cao – correct answer only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case

  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=....$

$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0$, $q\neq 0$ leading to $x=....$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n\rightarrow x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n\rightarrow x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either**
quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values
or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

4PM1 Paper 1R
Mark Scheme

Question	Scheme	Marks
1	$\left(4-\sqrt{2}\right)\left(a+b\sqrt{2}\right)=10+\sqrt{2}\Rightarrow a+b\sqrt{2}=\dfrac{10+\sqrt{2}}{4-\sqrt{2}}$	B1
	$\dfrac{\left(10+\sqrt{2}\right)}{\left(4-\sqrt{2}\right)}\times\dfrac{\left(4+\sqrt{2}\right)}{\left(4+\sqrt{2}\right)}=\dfrac{42+14\sqrt{2}}{14}=3+\sqrt{2}=a+b\sqrt{2}$	M1
	$\Rightarrow a=3,\quad b=1$	A1A1 [4]
	ALT	
	$\left(4-\sqrt{2}\right)\left(a+b\sqrt{2}\right)=4a+4b\sqrt{2}-a\sqrt{2}-2b$ $\Rightarrow 4a-2b+\sqrt{2}\left(4b-a\right)=10+\sqrt{2}$	B1
	$4a-2b=10$ $-a+\ 4b=1$	M1
	$\Rightarrow a=3,\quad b=1$	A1A1 [4]
		Total 4 marks

Mark	Notes
B1	For forming the correct equation $[a+b\sqrt{2}=]\frac{10+\sqrt{2}}{4-\sqrt{2}}$ Allow as an expression with $a+b\sqrt{2}$ omitted.
M1	For multiplying the denominator and the numerator by $4+\sqrt{2}$ The calculation must be shown. Minimum working is: $\frac{(10+\sqrt{2})}{(4-\sqrt{2})}\times\frac{(4+\sqrt{2})}{(4+\sqrt{2})}=\frac{42+14\sqrt{2}}{14}$
A1	Either a = 3 OR b = 1 Allow embedded.
A1	Both a = 3 AND b = 1 Allow embedded.
ALT	
B1	For forming a simplified expression in a and b for the area. Minimum required is expression of the form $4a+4b\sqrt{2}-a\sqrt{2}-2b$
M1	For solving the simultaneous equations in a and b Condone arithmetic errors, but process for solving must be correct.
A1	Either a = 3 OR b = 1 Allow embedded.
A1	Both a = 3 AND b = 1 Allow embedded.

Question	Scheme	Marks

2(a)	$6\times\left(-\frac{1}{2}\right)^2-\left(-\frac{1}{2}\right)q=5\Rightarrow\frac{3}{2}+\frac{q}{2}=5\Rightarrow q=7$ $y=px+9\Rightarrow 7=p\times-\frac{1}{2}+9\Rightarrow p=4$	M1A1 M1A1 [4]
	ALT	
	$6x^2-x(px+9)=5\ \Rightarrow\frac{3}{2}-\frac{p}{4}+\frac{9}{2}-5=0\ \Rightarrow\ p=4$ $y=px+9\ \Rightarrow\ q=4\times-\frac{1}{2}+9\ \Rightarrow q=7$	[M1A1 M1A1]
	ALT 2	
	$6x^2-x(px+9)-5=0$ and $(2x+1)(ax+b)=0\Rightarrow 2ax^2+x(2b+a)+b=0$ $6-p=2a$ and $b=-5$ $-9=2b+a\quad\Rightarrow\quad -9=2\times-5+a\Rightarrow a=1$ $\therefore 6-p=2\Rightarrow p=4$ $q=4\times-\frac{1}{2}+9=7$	[M1A1 M1A1]
(b)	$6x^2-x(4x+9)=5\Rightarrow 6x^2-4x^2-9x-5=0\Rightarrow 2x^2-9x-5=0$ $2x^2-9x-5=0\Rightarrow(2x+1)(x-5)=0\Rightarrow x=5$ $y=4\times5+9=29\Rightarrow(5,\ 29)$	M1 M1A1 A1 [4]
		Total 8 marks

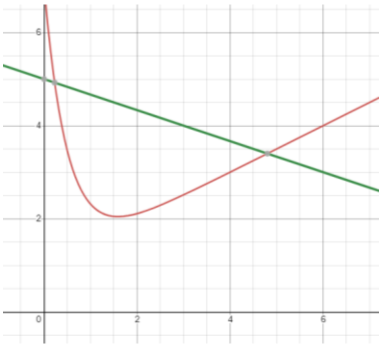
Part	Mark	Notes
(a)	M1	Substitutes $\left(-\frac{1}{2}\right)$ into the quadratic to find the value of q (or y)
	A1	For $q = 7$
	M1	For using the linear equation with y = their q and $x = -\frac{1}{2}$ to find the value of p
	A1	For $p = 4$
	ALT	
	M1	Substitutes the linear expression for y and $\left(-\frac{1}{2}\right)$ into the quadratic to find the value of p
	A1	For $p = 4$
	M1	For using the linear equation with their p and $x = -\frac{1}{2}$ to find the value of q
	A1	For $q = 7$
	ALT 2	
	M1A1	For the complete method which must be shown: • Substituting the linear expression for y into the quadratic • Writing $(2x + 1)(ax + b) = 0$ and expanding • Comparing coefficients for form equations in a, b and p • Solving simultaneous equations to find a, b and p
	M1	For using the linear equation with their p and $x = -\frac{1}{2}$ to find the value of q
	A1	For $q = 7$
(b)	M1	For substituting the linear equation (with their value of p) into the quadratic equation to form a 3TQ in x
	M1	For solving their 3TQ, must be a 3TQ. Independent of the first M. See general guidance on what constitutes an attempt to solve a 3TQ.
	A1	For $x = 5$
	A1	For the other solution to the equations. Does not have to be given as coordinates.
	M1M1A1 may be awarded for correct work in (a) if ALT2 was used, then award the final A mark as per scheme.	

Question	Scheme	Marks
3(a)	$BC^2 = 10^2 + 8^2 - 2 \times 10 \times 8 \times \cos 1\,00^{\circ}$ $BC = 13.848\ldots \approx 13.8$	M1 A1 [2]
(b)	(i) $\sin ABC = \dfrac{8\sin 100^{\circ}}{13.848\ldots} \Rightarrow ABC = 34.6752\ldots^{\circ} \approx 34.7^{\circ}$ (ii) $\angle ACB = 180^{\circ} - 100^{\circ} - 34.6752\ldots^{\circ} = 45.324\ldots^{\circ} \approx 45.3^{\circ}$	M1A1 B1FT [3]
	(i)(ii) $\cos ACB = \frac{8^2 + 13.848\ldots^2 - 10^2}{2 \times 8 \times 13.848\ldots} \Rightarrow ACB = 45.330\ldots$ $ABC = 180° - 100° - 45.330\ldots = 34.6699\ldots$	[M1A1 B1FT]
(c)	$\angle MBC = 34.675 \div 2 = 17.337\ldots^{\circ}$ and $\angle BMC = 180^{\circ} - 17.337^{\circ} = 117.33\ldots^{\circ}$ $MC = \dfrac{13.848\sin 17.337^{\circ}}{\sin 117.33^{\circ}} = 4.6293\ldots$ Area of BMC = $\dfrac{1}{2} \times 4.629 \times 13.848 \times \sin 45.324^{\circ} = 22.79\ldots \approx 22.8\,(\text{cm}^2)$	B1FT M1 M1A1 [4]
	ALT	
	$ABM = 34.675 \div 2 = 17.337\ldots°$ and $AMB = 180 - 100 - 17.33\ldots = 62.66\ldots$ $BM = \frac{10\sin 100}{\sin 62.66\ldots} = 11.086\ldots$ Area of BMC = $\frac{1}{2} \times 13.848\ldots \times 11.086\ldots \times \sin 17.337\ldots = 22.87\ldots$ $\approx 22.9\,(\text{cm}^2)$	[B1FT M1 M1A1]
		Total 9 marks

Part	Mark	Notes
(a)	M1	For applying the cosine rule correctly to obtain BC^2
	A1	For awrt 13.8 (cm)
(b)(i)	M1	For applying the sine rule correctly to find angle ABC Accept any appropriate trigonometry. $\frac{\sin ABC}{8} = \frac{\sin 100}{13.8}$ leading to $ABC = 34.8$ is M1A0
	A1	For awrt 34.7°
(ii)	B1FT	For awrt 45.3° FT $180 - 100 - \mathit{their}\ 34.3752\ldots$ $0 < \mathit{their}\ 34.3752\ldots < 80$
ALT		
(b)(i)(ii)	M1	For applying cosine rule correctly to find angle ACB

		Accept any appropriate trigonometry.
	A1	For awrt 45.3º (angle *ACB*)
	B1FT	FT $180 - 100 - \mathit{their}\ 45.330$ ... (angle *ABC*)
(c)	B1ft	For both angles *MBC* and *BMC*
	M1	For applying the sine rule using their angles and *BC* to find length *MC* Allow alternative correct methods to find *MC*.
	M1	For using Area = $\frac{1}{2}ab\sin C$ using their length and angles. $\mathit{Their}\ MC \neq 4$
	A1	Awrt 22.8 or awrt 22.9
ALT		
(c)	B1ft	For both angles *ABM* and *AMB*
	M1	For applying sine rule using their angles to find length *BM*. Allow alternative correct methods to find *BM*.
	M1	For using Area = $\frac{1}{2}ab\sin C$ using their length and angles. $\mathit{Their}\ MC \neq 4$ if used.
	A1	Awrt 22.8 or awrt 22.9

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>4(a)</td><td>
<table>
<tr><td>x</td><td>0</td><td>1</td><td>1.5</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>y</td><td>7</td><td>2.3</td><td>2.0</td><td>2.1</td><td>2.5</td><td>3</td><td>3.5</td><td>4</td></tr>
</table>
</td><td>B2
[2]</td></tr>
<tr><td>(b)</td><td>


</td><td>B2ft
[2]</td></tr>
<tr><td>(c)</td><td>
$2x+\ln\left(24-5x\right)-\ln 36=0 \Rightarrow \ln\left(\frac{24-5x}{36}\right)=-2x$
$\Rightarrow \frac{24-5x}{36}=\mathrm{e}^{-2x} \Rightarrow \frac{2}{3}-\frac{5x}{36}=\mathrm{e}^{-2x}$
$\Rightarrow 4-\frac{5x}{6}=6\mathrm{e}^{-2x}$
$\Rightarrow 5-\frac{x}{3}=\frac{x}{2}+6\mathrm{e}^{-2x}+1 \Rightarrow y=5-\frac{x}{3}$
$\Rightarrow x=0.2,\quad 4.8$


</td><td>M1
M1A1
M1A1FT
[5]</td></tr>
<tr><td colspan="3" align="right">Total 9 marks</td></tr>
</table>

Part	Mark	Notes
(a)	B1	At least two correct values rounded correctly.
	B1	All values correct, rounded correctly.
(b)	B1ft	Their points plotted within half of one square.
	B1ft	Their points joined up in a smooth curve. Note: do not allow ruled sections.
(c)	M1	Combines the logs correctly and raises both sides to the power of base e
	M1	Rearranges the equation to obtain the equation of the curve on one side and the equation of the straight line on the other.
	A1	Achieves a correct equation of the straight line. Note: equivalent forms are acceptable.
	M1	Draws their line provided it is of the form $y = k \pm \frac{x}{3}$ when simplified.
	A1FT	Provided M1M1A1M1 awarded. For both correct values of *x* to 1 decimal place, their values **must** follow through from their graph. 0.2, 0.3, 0.4 and 4.8 Condone coordinates. Note: correct values seen with no working scores M0M0A0M0A0

Question	Scheme	Marks
5(a)	$\mathrm{f}(1)=2\times1^3+a\times1^2-14\times1+b=0 \quad \Rightarrow \quad a+b=12$ $\mathrm{f}(4)=2\times4^3+a\times4^2-14\times4+b=39\Rightarrow16a+b=-33$ $\Rightarrow15a=-45\Rightarrow a=-3*$ $\Rightarrow b=15$	M1 M1 M1A1 cso B1 [5]
	ALT – polynomial division	
	$(2x^3+ax^2\ -14x+b)\div(x-1)=2x^2+(a+2)x-14+a+2$ and comparison of final step of division with b to obtain an equation **or** $(2x^3+ax^2\ -14x+b)\div(x-4)=2x^2+(a+8)x+(18+4a)$ r 39 and comparison of final step of division with obtaining remainder 39 to obtain an equation $(2x^3+ax^2\ -14x+b)\div(x-1)=2x^2+(a+2)x-14+a+2$ and comparison of final step of division with b to obtain an equation **and** $(2x^3+ax^2\ -14x+b)\div(x-4)=2x^2+(a+8)x+(18+4a)$ r 39 and comparison of final step of division with obtaining remainder 39 to obtain an equation $b=12-a$ $b+4(18+4a)=39$ $a\ =\ -3*$ $b=15$	[M1 M1 M1 A1cso B1]
(b)	$\begin{array}{r} 2x^2-x-15 \\ (x-1)\overline{)2x^3-3x^2-14x+15} \end{array}$ $\mathrm{f}(x)=(x-1)(2x+5)(x-3)$	M1A1 dM1A1 [4]
(c)	$p=-\frac{5}{2} \quad q=1 \quad r=3$	B1ft B1ft B1ft [3]

(d)	$A=\left\lvert\int_1^3\left(2x^3-3x^2-14x+15\right)\,\mathrm{d}x\right\rvert+\int_{-\frac{5}{2}}^{1}\left(2x^3-3x^2-14x+15\right)\,\mathrm{d}x$	M1
	$A=\left\lvert\left[\frac{2x^4}{4}-\frac{3x^3}{3}-\frac{14x^2}{2}+15x\right]_1^3\right\rvert+\left[\frac{2x^4}{4}-\frac{3x^3}{3}-\frac{14x^2}{2}+15x\right]_{-\frac{5}{2}}^{1}$	M1*
	$A=\left\lvert\left[\left(\frac{3^4}{2}-3^3-7\times3^2+15\times3\right)-\left(\frac{1^4}{2}-1^3-7\times1^2+15\times1\right)\right]\right\rvert$ $+\left[\left(\frac{1^4}{2}-1^3-7\times1^2+15\times1\right)-\left(\frac{\left(-\frac{5}{2}\right)^4}{2}-\left(-\frac{5}{2}\right)^3-7\times\left(-\frac{5}{2}\right)^2+15\times\left(-\frac{5}{2}\right)\right)\right]$	dM1
	$A=12+\frac{1715}{32}=\frac{2099}{32}$ or 65.59375	A1 [4]
		Total 16 marks

Part	Mark	Notes
(a)	M1	For use of f (1) = 0 **or** use of f (4) = 39
	M1	For use of f (1) = 0 **and** use of f (4) = 39
	M1	For solving the simultaneous equations in a and b Must see at least one line of working before a or b obtained. Must see an equation in just one variable.
	A1cso	For showing $a = -3$* Must see expression only in terms of a
	B1	For the correct value of b = 15 seen. Note: this is an A mark in epen.
	ALT – polynomial division	
	M1	Correct method for forming one equation by polynomial division together with comparison of the final step of the division with the required result (no remainder for division by $(x - 1)$ and remainder 39 for division by $(x - 4)$)
	M1	Correct method for forming **both** equations by polynomial division together with comparison of the final step of the division with the required result (no remainder for division by $(x - 1)$ and remainder 39 for division by $(x - 4)$)
	M1	For solving the simultaneous equations in a and b Must see at least one line of working before a or b obtained.
	A1cso	For showing $a = -3$* Must see expression only in terms of a ie $ka = -3k$ before $a = -3$
	B1	For the correct value of b = 15 seen. Note: this is an A mark in epen.
	Assuming $a = -3$ and solving to find b = 15 scores maximum M1M0M0A0A1	
(b)	**Condone (b) completed in (c)**	
	M1	For carrying out polynomial division/comparing coefficients to find the quadratic factor. Must reach $2x^2 \pm \cdots$. Allow statement of quadratic factor if polynomial division by $(x - 1)$ used in (a).
	A1	For the correct quadratic factor.
	dM1	For factorising their 3TQ See general guidance for what constitutes an attempt to factorise a 3TQ.
	A1	For the correct factorisation of f (x) on one line. Condone omission of $\mathrm{f}(x) =$
	Note: complete factorisation seen with no method scores M0A0M0A0.	
(c)	B1ft	For one correct value for any of p, r or q FT their linear factors in (b). Values need not be identified as p, r, q, but if they are labelled these should be $p < q < r$.
	B1ft	For two correct values. FT their linear factors in (b). Values need not be identified as p, r, q, but if they are labelled these should be $p < q < r$.
	B1ft	For all three correct values. FT their linear factors in (b), but **1 must be one of the values**. Values need not be identified as p, r, q, but if they are labelled these should be $p < q < r$.

(d)	M1	For a correct statement for the area of the shaded regions with their limits correctly used. Minimum required is: $\int_p^q (2x^3 - 3x^2 - 14x + 15)\mathrm{d}x - \int_q^r (2x^3 - 3x^2 - 14x + 15)\mathrm{d}x$ with $p < q < r$ shown as numerical limits or $\int_p^q (2x^3 - 3x^2 - 14x + 15)\mathrm{d}x + \left\lvert \int_q^r (2x^3 - 3x^2 - 14x + 15)\mathrm{d}x \right\rvert$ with $p < q < r$ shown as numerical limits
	M1*	For an attempt to integrate their expression. Minimum required is: • two powers of terms to increase by 1 • no powers of terms to decrease Condone $+c$
	dM1	For substituting their limits correctly to evaluate the integrated expression. Must show the substitution if their integral or their limits are incorrect. Correct exact answer following correct integral with correct limits implies this mark. Condone $+c$ as long as this is cancelled out during the substitution.
	A1	For the correct area. Exact area required.

Question	Scheme	Marks
6(a)	$\tan 30^{\circ} = \frac{r}{h} \Rightarrow r = \frac{h}{\sqrt{3}}$	B1
	$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h \Rightarrow V = \frac{1}{9}\pi h^3 *$	M1A1 cso [3]
(b)	$\frac{\mathrm{d}V}{\mathrm{d}t} = -4$	B1
	$A = \pi r^2 \Rightarrow A = \pi\left(\frac{h}{\sqrt{3}}\right)^2 = \frac{\pi h^2}{3}$	B1
	$\frac{\mathrm{d}A}{\mathrm{d}h} = \frac{2\pi h}{3} \qquad \frac{\mathrm{d}V}{\mathrm{d}h} = \frac{3\pi h^2}{9} = \frac{\pi h^2}{3}$	M1M1A1
	$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}h} \times \frac{\mathrm{d}V}{\mathrm{d}t} \times \frac{1}{\frac{\mathrm{d}V}{\mathrm{d}h}}$	M1
	$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{2\pi h}{3} \times (-4) \times \frac{3}{\pi h^2} = -\frac{8}{h} \Rightarrow \frac{\mathrm{d}A}{\mathrm{d}t} = -\frac{8}{24} = -\frac{1}{3} \quad \left(\mathrm{cm}^2/\mathrm{s}\right)$	dM1A1 [8]
	ALT – working in terms of *r* rather than *h*	
	$\frac{\mathrm{d}V}{\mathrm{d}t} = -4$	[B1
	$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi r^2\left(\sqrt{3}r\right) = \frac{\sqrt{3}}{3}\pi r^3$	B1
	$A = \pi r^2 \Rightarrow \frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r \qquad V = \frac{\sqrt{3}}{3}\pi r^3 \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}r} = \sqrt{3}\pi r^2$	M1M1A1
	$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}r} \times \frac{1}{\frac{\mathrm{d}V}{\mathrm{d}r}} \times \frac{\mathrm{d}V}{\mathrm{d}t}$	M1
	$\frac{\mathrm{d}A}{\mathrm{d}t} = 2\pi r \times \frac{1}{\sqrt{3}\pi r^2} \times -4 = -\frac{8}{\sqrt{3}r} \Rightarrow \frac{\mathrm{d}A}{\mathrm{d}t} = -\frac{8}{\sqrt{3}\times\frac{24}{\sqrt{3}}} = -\frac{1}{3}$	dM1A1]
		Total 11 marks

Part	Mark	Notes
(a)	B1	For $r=\frac{h}{\sqrt{3}}$ or $r^2=\frac{1}{3}h^2$ $r=\frac{h}{\sqrt{3}}$ may be implied by substitution into the correct formula for volume of a cone.
	M1	For using the correct volume of a cone and substituting in their expression for r or r^2 to find an expression for the volume in terms of h
	A1	For the correct expression with no errors seen. Must have $V=$
(b)	B1	For stating $\frac{\mathrm{d}V}{\mathrm{d}t}=-4$ (allow +) May be embedded in working.
	B1	For finding the correct expression for the area of a circle in terms of h Note: if differentiation is attempted before writing in terms of h then award 1st M1 for $\frac{\mathrm{d}A}{\mathrm{d}h}=\frac{\mathrm{d}A}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}h}$ with attempts at $\frac{\mathrm{d}A}{\mathrm{d}r},\frac{\mathrm{d}r}{\mathrm{d}h}$ found and substituted in and attempt to substitute for r in terms of h. B1 once $\frac{\mathrm{d}A}{\mathrm{d}h}$ found completely in terms of h
	M1	For attempting to differentiate their expression for A in terms of h See general guidance for what constitutes an attempt to differentiate. A must have the form $A=kh^2$ (dimensionally correct for area).
	M1	For attempting to differentiate the given V See general guidance for what constitutes an attempt to differentiate.
	A1	Both derivatives correct. $\frac{\mathrm{d}A}{\mathrm{d}h}$ and $\frac{\mathrm{d}V}{\mathrm{d}h}$
	M1	For a correct expression of chain rule to find $\frac{\mathrm{d}A}{\mathrm{d}t}$
	dM1	For substituting their derivatives into a correct chain rule. Dep on previous three M marks.
	A1	For the correct $\frac{\mathrm{d}A}{\mathrm{d}t}=-\frac{1}{3}$ $(\mathrm{cm}^2/\mathrm{s})$ Allow $-0.\dot{3}$ but not -0.3. ISW once exact answer found.
	ALT	
	B1	For stating $\frac{\mathrm{d}V}{\mathrm{d}t}=-4$ (allow +) May be embedded in working.
	B1	For finding the correct expression for the volume in terms of r Must be working with correct volume expression as given in (a).
	M1	For attempting to differentiate expression for A in terms of r See general guidance for what constitutes an attempt to differentiate. A must have the form $A=kr^2$ (dimensionally correct for area).
	M1	For attempting to differentiate expression for V in terms of r See general guidance for what constitutes an attempt to differentiate.

	A1	Both derivatives correct. $\frac{\mathrm{d}A}{\mathrm{d}r}$ and $\frac{\mathrm{d}V}{\mathrm{d}r}$
	M1	For a correct expression of chain rule to find $\frac{\mathrm{d}A}{\mathrm{d}t}$
	dM1	For substituting their derivatives into a correct chain rule. Dep on previous three M marks.
	A1	For the correct $\frac{\mathrm{d}A}{\mathrm{d}t} = -\frac{1}{3}$ $\left(\mathrm{cm}^2 / \mathrm{s}\right)$ Allow $-0.\dot{3}$ but not -0.3. ISW once exact answer found.

Question	Scheme	Marks
7(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=2mx+\frac{1}{2}\times 64x^{-\frac{1}{2}}$ $32x^{-\frac{1}{2}}+2mx=0\Rightarrow 32\times 4^{-\frac{1}{2}}+2m\times 4=0\Rightarrow 16+8m=0\Rightarrow m=-2$ $n=39+64\sqrt{4}-2\times 4^2=135$	M1A1 dM1A1 dM1A1 [6]
(b)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{1}{2}\times 32x^{-\frac{3}{2}}-4=-16x^{-\frac{3}{2}}-4$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-6<0\Rightarrow$ maximum	M1 A1FT [2]
		Total 8 marks

Part	Mark	Notes
(a)	M1	For an attempt to differentiate the given expression. Minimum required is **two** of: • $mx^2 \to kmx,\quad k\neq 0$ • $64\sqrt{x}\to nx^{-\frac{1}{2}},\ n\neq 0$ • $32\to 0$
	A1	For a correct derivative in terms of *m*
	dM1	For setting their $\frac{\mathrm{d}y}{\mathrm{d}x}=0$, substitute $x=4$ and attempting to solve to find the value of *m* Dep on 1st M mark.
	A1	For $m=-2$
	dM1	For using their value of *m* to find a value for *n* $n=4^2\times m+167$ Dep on 1st M mark.
	A1	For the correct value of *n*
(b)	M1	For finding the second derivative. Minimum required is $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=km+px^{-\frac{3}{2}}$ with *k*, *p* not 0. Allow for substituting x values on either side of $x=4$ into *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ provided $\frac{\mathrm{d}y}{\mathrm{d}x}=kmx+nx^{-\frac{1}{2}}$ with *k*, *n* not 0.
	A1FT	For substituting their value of *m* and forming a correct conclusion. Minimum required is substitution of their value of *m* together with $x=4$ into their second derivative, evaluation and correct conclusion. Allow for evaluating of $\frac{\mathrm{d}y}{\mathrm{d}x}$ on either side of $x=4$ and forming a correct conclusion. $x=3.9, \frac{\mathrm{d}y}{\mathrm{d}x}=0.6038\ldots \qquad x=4.1, \frac{\mathrm{d}y}{\mathrm{d}x}=-0.596\ldots$

Question	Scheme	Marks
8(a)	$U_5=\left(\frac{25}{4}\right)\left(\frac{3}{5}\right)^5=\frac{243}{500}$ oe	B1 [1]
(b)	$U_1=\left(\frac{25}{4}\right)\left(\frac{3}{5}\right)^1=\frac{15}{4}$ oe $S_n=\left(\frac{15}{4}\right)\left(\frac{3}{5}\right)^0+\left(\frac{15}{4}\right)\left(\frac{3}{5}\right)^1+\ldots\ldots+\left(\frac{15}{4}\right)\left(\frac{3}{5}\right)^{n-1}$ $\Rightarrow S_n=\sum_{r=1}^{n}\left(\frac{15}{4}\right)\left(\frac{3}{5}\right)^{r-1}$ oe	B1 M1 A1 [3]
	ALT 1	
	$U_1=\left(\frac{25}{4}\right)\left(\frac{3}{5}\right)^1=\frac{15}{4}$ $r=\frac{3}{5}\qquad U_n=\frac{15}{4}\times\left(\frac{3}{5}\right)^{n-1}$ $S_n=\sum_{r=1}^{n}\frac{15}{4}\times\left(\frac{3}{5}\right)^{r-1}$	[B1 M1 A1]
	ALT 2	
	$S_n=\sum_{r=1}^{n}\frac{25}{4}\left(\frac{3}{5}\right)^{r}$ $=\sum_{r=1}^{n}\frac{25}{4}\times\frac{3}{5}\times\left(\frac{3}{5}\right)^{r-1}$ $=\sum_{r=1}^{n}\frac{15}{4}\times\left(\frac{3}{5}\right)^{r-1}$	[B1 M1 A1]
(c)	$S_\infty=\dfrac{\frac{15}{4}}{1-\frac{3}{5}}=\frac{75}{8}$ or $9.375\qquad S_n=\dfrac{\frac{15}{4}\left(1-\left[\frac{3}{5}\right]^n\right)}{1-\frac{3}{5}}$ $9.375-\dfrac{\frac{15}{4}\left(1-\left[\frac{3}{5}\right]^n\right)}{1-\frac{3}{5}}<0.045\Rightarrow\dfrac{\frac{15}{4}\left(1-\left[\frac{3}{5}\right]^n\right)}{1-\frac{3}{5}}>9.33$ $\dfrac{\frac{15}{4}\left(1-\left[\frac{3}{5}\right]^n\right)}{1-\frac{3}{5}}>9.33\Rightarrow1-\left[\frac{3}{5}\right]^n>0.9952\Rightarrow\left[\frac{3}{5}\right]^n<\frac{3}{625}$ $n\lg\left[\frac{3}{5}\right]<\lg\frac{3}{625}\Rightarrow n>10.45$ $n=11$	M1M1 dM1 ddM1A1 A1 [6]
		Total 10 marks

Part	Mark	Notes
Mark (a) and (b) together		
(a)	B1	For the correct value only. Decimal value is 0.486.
(b)	B1	Finds the value of the first term. Must be identified as U₁ / first term.
	M1	Forms the sum of the series with at least the first term and the *n*th term.
	A1	For the correct expression.
	ALT 1	
	B1	Finds the value of the first term. Must be identified as U₁ / first term.
	M1	For finding r and writing $U_n = \frac{15}{4}\times\left(\frac{3}{5}\right)^{n-1}$
	A1	For the correct expression.
	ALT 2	
	B1	For correct summation statement using U_n from the question
	M1	For showing process to change index from r to $r-1$ and find the value of $\frac{A}{B}$ Minimum working is $\frac{25}{4}\times\left(\frac{3}{5}\right)^{r} = \frac{25}{4}\times\frac{3}{5}\times\left(\frac{3}{5}\right)^{r-1}$. May be with n as index.
	A1	For the correct expression.
(c)	Trial and improvement approaches for (c) should be sent to review.	
	M1	Correct method to find the sum to infinity using the correct formula with their $a = \frac{15}{4}$ and $r = \frac{3}{5}$. Their a should be $\frac{A}{B}$ if found in (b). Condone if working with $a = \frac{25}{4}$, r must be correct.
	M1	Using the correct formula, forms the sum to n terms using their $a = \frac{15}{4}$ and $r = \frac{3}{5}$. Their a should be $\frac{A}{B}$ if found in (b). Condone if working with $a = \frac{25}{4}$, r must be correct.
	dM1	Forms the inequality in terms of n with their S_∞ and S_n and simplifies to $\left[\frac{3}{5}\right]^n <\text{'}\frac{3}{625}\text{'}$ Must have dealt with negative. Allow for working with equation if inequality sign reinstated later. Reinstatement can be implied by rounding up to the next integer value once n found. Dep M1M1
	ddM1	Uses logarithms correctly to attempt to find the value of n They must reverse the inequality for this mark. Allow for working with equation if inequality sign reinstated later. Reinstatement can be implied by rounding up to the next integer value once n found. Dep previous M mark. Allow use of log base $\frac{3}{5}$.
	A1	For correct n awrt 10.5 Can be implied by $n = 11$ if decimal not seen.
	A1	For $n = 11$

Question	Scheme	Marks
9(a)	$(1+2x)^{-\frac{1}{3}}=1+\left(-\frac{1}{3}\right)(2x)+\frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)(2x)^2}{2!}+\frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)(2x)^3}{3!}+\ldots$ $=1-\frac{2x}{3}+\frac{8x^2}{9}-\frac{112x^3}{81}$	M1 A1A1 [3]
(b)	$-\frac{1}{2}<x\leqslant\frac{1}{2}$ accept $-\frac{1}{2}<x<\frac{1}{2}$	B1 [1]
(c)	$\mathrm{f}(x)=(2+kx^2)\left(1-\frac{2x}{3}+\frac{8x^2}{9}-\frac{112x^3}{81}\right)$ $=2-\frac{4x}{3}+\frac{16}{9}x^2+kx^2-\frac{224}{81}x^3-\frac{2k}{3}x^3$	M1 M1A1 [3]
(d)	$-\frac{224}{81}-\frac{2k}{3}=-\frac{8}{3}\Rightarrow k=-\frac{4}{27}$	M1A1 [2]
(e)	$\int_{0.1}^{0.2}\left(2-\frac{4x}{3}+\frac{44x^2}{27}-\frac{8}{3}x^3\right)\mathrm{d}x=\left[2x-\frac{4x^2}{2\times3}+\frac{44x^3}{3\times27}-\frac{8x^4}{4\times3}\right]_{0.1}^{0.2}$ $=\left(2\times0.2-\frac{2\times0.2^2}{3}+\frac{44\times0.2^3}{81}-\frac{2}{3}\times0.2^4\right)-\left(2\times0.1-\frac{2\times0.1^2}{3}+\frac{44\times0.1^3}{81}-\frac{2}{3}\times0.1^4\right)$ $=0.1828$ NB: Calculator value is 0.18301744	B1FT M1A1ft M1 A1 [5]
		Total 14 marks

Part	Mark	Notes
(a)	M1	For an attempt at binomial expansion. - The first term is 1 - The denominators are correct. - The powers of *x* are correct.
	A1	For at least one algebraic term correct and simplified.
	A1	For a fully correct and simplified expansion.
(b)	B1	For the correct range. Accept $\|x\| < \frac{1}{2}$ Do not accept $-\frac{1}{2} \le x \le \frac{1}{2}$
(c)	M1	For showing the intent to multiply their expansion by $\left(2+kx^2\right)$
	M1	For expanding the two brackets up to terms in x^3. Ignore terms in higher powers.
	A1	For the correct expansion. This does not have to be simplified. Must be in ascending powers, but accept $\frac{16}{9}x^2$ and kx^2 in either order and accept $-\frac{224}{81}x^3$and $-\frac{2k}{3}x^3$ in either order. $\text{f}(x) = 2 - \frac{4x}{3} + x^2\left(\frac{16}{9} + k\right) + x^3\left(-\frac{224}{81} - \frac{2k}{3}\right)$ ISW errors in collection of coefficients once correct expansion in ascending powers seen.
(d)	M1	For setting their coefficient of x^3 equal to $-\frac{8}{3}$
	A1	For the correct value of *k*
(e)	B1FT	For substituting in their *k* into coefficient for x^2 in their expansion from (c) and having the coefficient of x^3 as $-\frac{8}{3}$. If the coefficient of x^2 is not correct then the substitution must be seen to award this mark. Note: this is an M mark in epen.
	M1	For an attempt to integrate their expansion which must have 4 terms. At least two powers of *x* to increase by 1 (including $c \to cx$). No power of *x* to reduce.
	A1ft	For a correctly integrated expression follow through their 4 term polynomial. Simplified integrated expression is $2x - \frac{2x^2}{3} + \frac{44x^3}{81} - \frac{2}{3}x^4$
	M1	For substituting in 0.2 and 0.1 the correct way around in their integrated expression. Expression should not have been differentiated and needs to have changed. Not dependent on first M. Must show the substitution if their integral or their limits are incorrect. Correct awrt 0.1828 following correct integral with correct limits implies this mark.
	A1	For the correct value of awrt 0.1828 NB: The calculator value is 0.18301744

Question	Scheme	Marks
10(a)	(i) $\overrightarrow{AB}=-\overrightarrow{OA}+\overrightarrow{OB}\Rightarrow\overrightarrow{AB}=-2\mathbf{a}+4\mathbf{b}$ (ii) $\overrightarrow{MY}=\overrightarrow{MA}+\frac{3}{4}\left(\overrightarrow{AB}\right)=\mathbf{a}+\frac{3}{4}(-2\mathbf{a}+4\mathbf{b})=-\frac{\mathbf{a}}{2}+3\mathbf{b}$	B1 M1A1 [4]
(b)	$\overrightarrow{OX}=\mu\overrightarrow{OB}=\mu 4\mathbf{b}$ $\overrightarrow{OX}=\overrightarrow{OM}+\overrightarrow{MX}=\overrightarrow{OM}+\lambda\overrightarrow{MY}=\mathbf{a}+\lambda\left(-\frac{\mathbf{a}}{2}+3\mathbf{b}\right)=\mathbf{a}\left(1-\frac{\lambda}{2}\right)+3\lambda\mathbf{b}$ $\Rightarrow\mu 4\mathbf{b}=\mathbf{a}\left(1-\frac{\lambda}{2}\right)+3\mathbf{b}$ $\Rightarrow 1-\frac{\lambda}{2}=0\Rightarrow\lambda=2$ $\Rightarrow 4\mu=3\lambda\Rightarrow\mu=\frac{6}{4}=\frac{3}{2}$ $OB:OX=2:3$ oe	M1 M1 dM1 ddM1 A1 [5]
	ALT – working with alternative vector within triangle *OMX*	
	$\overrightarrow{MX}=\overrightarrow{MO}+\overrightarrow{OB}+\mu\overrightarrow{OB}=-\boldsymbol{a}+4\boldsymbol{b}+\mu 4\boldsymbol{b}$ $\overrightarrow{MX}=\lambda\left(-\frac{\boldsymbol{a}}{2}+3\boldsymbol{b}\right)$ $\Rightarrow-\boldsymbol{a}=-\frac{\lambda\boldsymbol{a}}{2}\quad\Rightarrow\quad\lambda=2$ $\Rightarrow 4\boldsymbol{b}+\mu 4\boldsymbol{b}=3\lambda\mathbf{b}\quad\Rightarrow\quad\mu=\frac{1}{2}$ $OB:OX=2:3$ oe	[M1 M1 dM1 ddM1 A1]
(c)	$\frac{\Delta YBX}{\Delta ABX}=\frac{1}{4}$ $\frac{\Delta ABX}{\Delta OAX}=\frac{1}{3}$ $\Rightarrow\frac{\Delta YBX}{\Delta OAX}=\frac{1}{4}\times\frac{1}{3}=\frac{1}{12}\Rightarrow\Delta YBX:\Delta OAX=1:12$	M1 M1A1 [3]
	ALT – working with relative areas of triangles	
	Area $\Delta YBX=a$ Area $\Delta ABX=4a$ Area $\Delta OMX=6a$ Area $\Delta AYX=3a$ Area $\Delta OAY=6a$ Area $\Delta OYB=2a$ Area $\Delta OAX=12a$ $\Delta YBX:\Delta OAX=1:12$	[M1 M1 A1]
		Total 11 marks

Part	Mark	Notes
(a)	B1	For the correct simplified vector $\overrightarrow{AB}$
	M1	For the correct vector statement for $\overrightarrow{MY}$
	A1	For the correct simplified vector for $\overrightarrow{MY}$
(b)	M1	For the statement $\overrightarrow{OX} = \mu 4\mathbf{b}$ Note: this is a B mark on epen.
	M1	For the correct vector for $\overrightarrow{OX}$ (ft their *MY*)
	dM1	For equating both vectors for $\overrightarrow{OX}$ and for comparing coefficients of **a** and **b** Dep on M1M1
	ddM1	For finding a value for their parameter for μ Note: there is no mark for only finding λ, they must find μ Dep on M1M1M1
	A1	For the correct ratio $OB : OX = 2 : 3$ Allow equivalent ratios e.g. 4 : 6, 1 : 1.5
	ALT – working with alternative vector within triangle *OMX*	
	M1	For a vector statement which includes $\mu 4\boldsymbol{b}$ (for *OX* or *BX*) (ft their *MY*) Note: this is a B mark on epen.
	M1	For a correct second vector equation for the same vector (ft their *MY*)
	dM1	For equating both vectors and comparing coefficients of **a** and **b** Dep on M1M1
	ddM1	For finding a value for their parameter for μ Note: there is no mark for only finding λ, they must find μ Dep on M1M1M1
	A1	For the correct ratio $OB : OX = 2 : 3$ Allow equivalent ratios e.g. 4 : 6, 1 : 1.5 Condone $OX : OB = 3 : 2$ if clearly stated, but not just 3:2
(c)	M1	For either the relationship between the of areas of triangles *BYX* and *ABX* **or** the relationship between the areas of triangles *ABX* and *OAX*
	M1	For finding the relationship between the of areas of triangles *BYX* and *OAX*
	A1	For the correct ratio [1:12] Allow equivalent ratios. Note: do not penalise answer given as a fraction i.e. $\frac{1}{12}$ if already penalised in (b).
	ALT – working with relative areas	
	M1	For assigning a value to one triangle area and writing a second area in terms of this. Note: This could also follow from working with area of a triangle $= \frac{1}{2}ab\sin C$

		e.g. $\Delta YBX = ½\, yz \sin B$ and $\Delta ABX = \frac{1}{2}y(4z)\sin B$
	M1	For finding the relationship between the of areas of triangles *BYX* and *OAX* Note: This could also follow from working with area of a triangle $= \frac{1}{2}ab\sin C$ e.g. $\Delta YBX = ½\, yz \sin B$ and $\Delta ABX = \frac{1}{2}y(4z)\sin B$
	A1	For the correct ratio [1:12] Allow equivalent ratios. Note: do not penalise answer given as a fraction i.e. $\frac{1}{12}$ if already penalised in (b).

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