Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/02**

Further Pure Mathematics
PAPER 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \dfrac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$

Geometric series

Sum to n terms, $S_n = \dfrac{a(1 - r^n)}{(1 - r)}$

Sum to infinity, $S_\infty = \dfrac{a}{1 - r} \quad |r| < 1$

Binomial series

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule
In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$ $\qquad$ $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$ $\qquad$ $\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ $\qquad$ $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

Logarithms

$\log_a x = \dfrac{\log_b x}{\log_b a}$

1
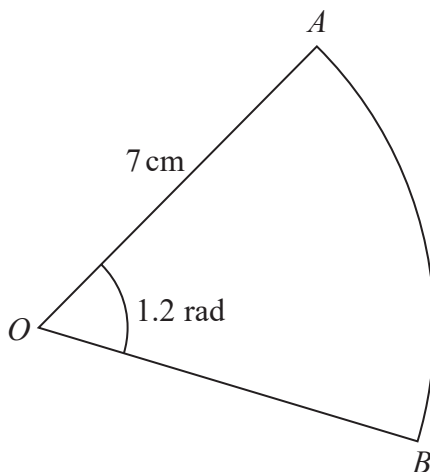
Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows sector AOB of a circle with centre O and radius 7 cm.

The angle AOB is 1.2 radians.

Calculate

(a) the area of sector AOB

(2)

(b) the perimeter of sector AOB

(2)

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(Total for Question 1 is 4 marks)

P 6 6 3 0 8 A 0 3 3 6

2 Solve the equation

$$\sin(2\theta - 20)° - \sqrt{3}\cos(2\theta - 20)° = 0 \quad \text{for } 0 \leqslant \theta \leqslant 180$$

(5)

Question 2 continued

(Total for Question 2 is 5 marks)

3 The curve C has equation $y = 9 - x^2$

Use algebraic integration to find the area of the finite region bounded by C and the x-axis.

(5)

Question 3 continued

(Total for Question 3 is 5 marks)

4

Diagram **NOT**
accurately drawn

Figure 2

Figure 2 shows a solid cube $ABCDEFGH$ with sides of length 10 cm.

(a) Find, in cm, the exact length of FC

(2)

(b) Find, in degrees to one decimal place, the size of $\angle FCA$

(2)

(c) Find, to the nearest degree, the size of the obtuse angle between the plane BFH and the plane BHC

(5)

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Question 4 continued

Question 4 continued

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(Total for Question 4 is 9 marks)

P 6 6 3 0 8 A 0 1 1 3 6

5   A particle P is moving along the x-axis. At time t seconds ($t \geqslant 0$) the velocity, v m/s, of P is given by $v = 3t^2 - 23t + 30$

(a) Find the values of t when P is instantaneously at rest.

(3)

At time t seconds the acceleration of P is a m/s^2

(b) Find the range of values of t for which $a > 0$

(2)

When $t = 0$, P is at the point with coordinates $(d, 0)$

Given that, when $t = 8$, P is at the point with coordinates $(26, 0)$

(c) find the value of d

(4)

Question 5 continued

P 6 6 3 0 8 A 0 1 3 3 6

Question 5 continued

(Total for Question 5 is 9 marks)

P 6 6 3 0 8 A 0 1 5 3 6

6 The sum of the first n terms of an arithmetic series is S_n where $S_n = n(3 + 2n)$

(a) Find the value of S_{20}

(2)

Given that $S_n = \sum_{r=1}^{n} (Ar + B)$

(b) find the value of A and the value of B

(6)

A different arithmetic series has first term 7 and common difference 4

The sum of the first n terms of this series is T_n

(c) Use algebra to find the value of n for which $T_n = S_n + 252$

(5)

P 6 6 3 0 8 A 0 1 6 3 6

P 6 6 3 0 8 A 0 1 7 3 6

Question 6 continued

P 6 6 3 0 8 A 0 1 8 3 6

(Total for Question 6 is 13 marks)

7

$$f(x) = 2x^2 + px + q \qquad \text{where } p \text{ and } q \text{ are integers}$$
$$g(x) = 14x^2 + 37x + 14$$

The equation $f(x) = 0$ has roots α and β

The equation $g(x) = 0$ has roots $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$

Given that $p + q = -4$ where $p > 0$ and without solving the equation $g(x) = 0$

(a) find

    (i) the value of p

    (ii) the value of q

(9)

Given also that $\alpha > \beta$

(b) show that $\alpha^2 - \beta^2 = -\dfrac{3\sqrt{65}}{4}$

(4)

Question 7 continued

P 6 6 3 0 8 A 0 2 1 3 6

Question 7 continued

P 6 6 3 0 8 A 0 2 2 3 6

(Total for Question 7 is 13 marks)

P 6 6 3 0 8 A 0 2 3 3 6

8 (a) Given that x is measured in radians, complete the table of values for

$$y = \sin x - 3\cos 2x - 0.5$$

Give your answers to one decimal place.

x	0	0.5	0.8	1	1.6	2	2.5	3
y			0.3	1.6			-0.8	-3.2

(2)

(b) On the grid opposite, draw the graph of $y = \sin x - 3\cos 2x - 0.5$ for $0 \leqslant x \leqslant 3$

(2)

(c) Use a formula from page 2 to show that $\cos 2A = 1 - 2\sin^2 A$

(2)

$$f(x) = 2\sin x + 12\sin^2 x - x - 5$$

(d) By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, of the roots of the equation $f(x) = 0$ in the interval $0 \leqslant x \leqslant 3$

(5)

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Question 8 continued

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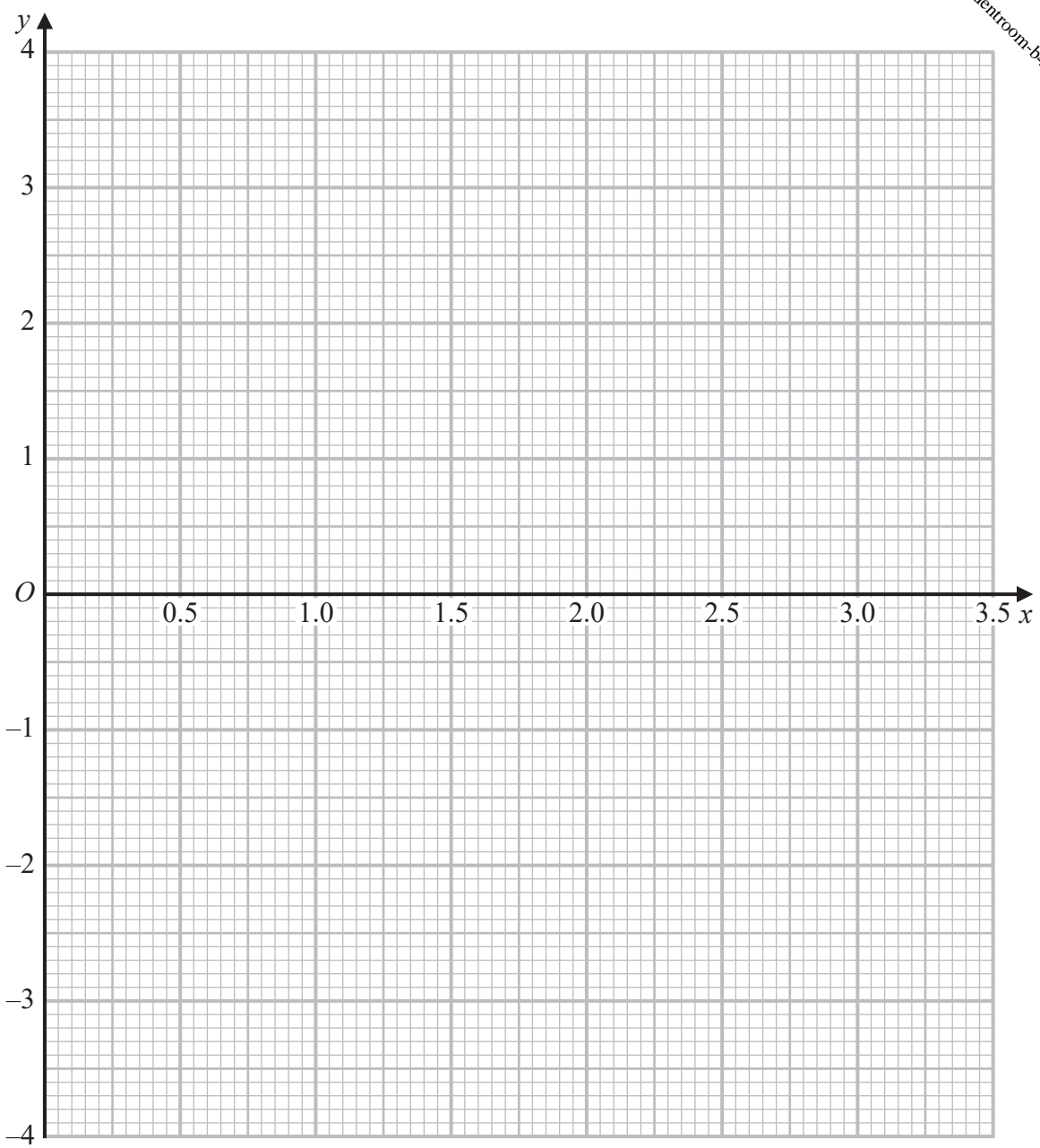
Turn over for a spare grid if you need to redraw your graph.
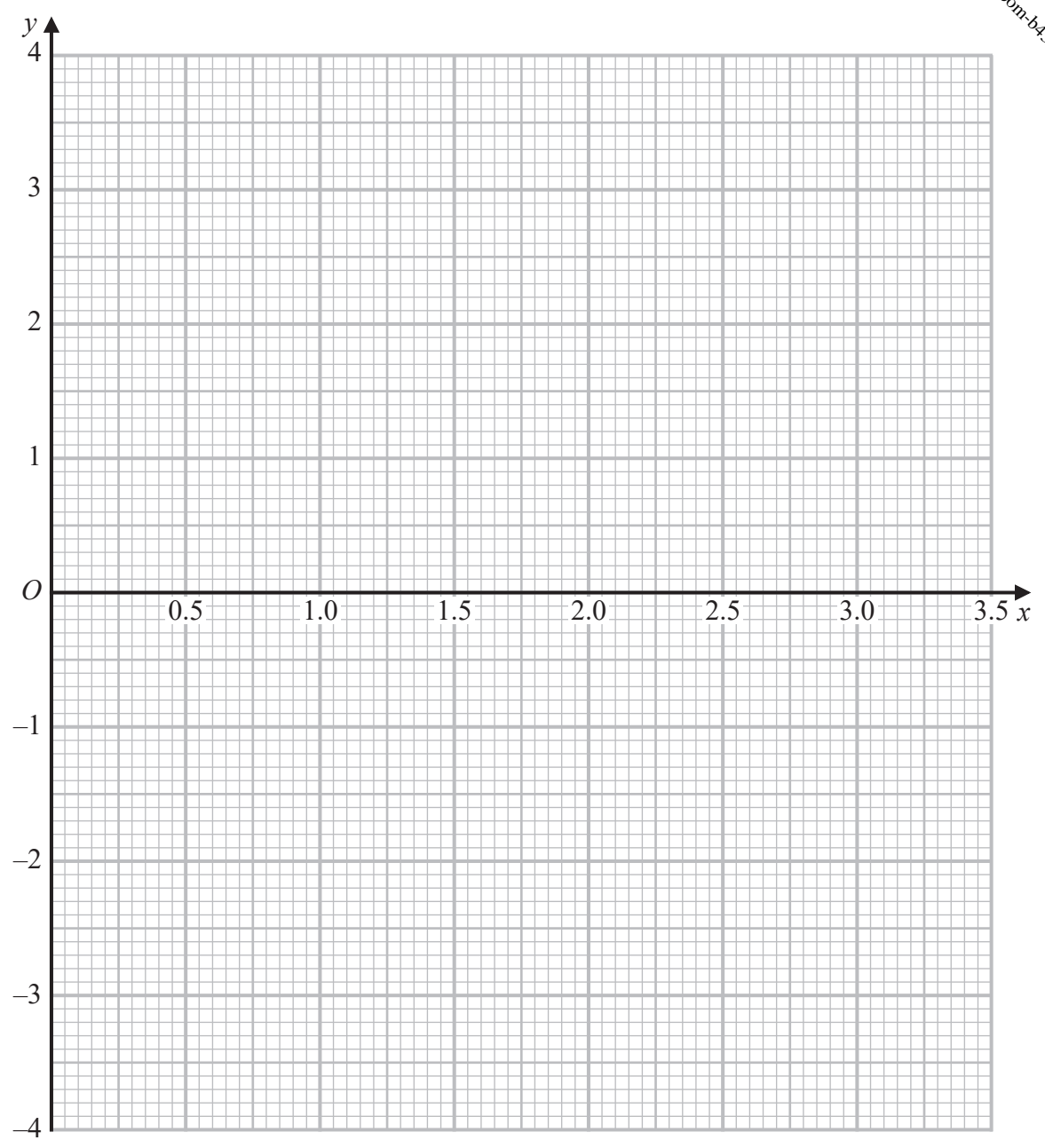
Question 8 continued

P 6 6 3 0 8 A 0 2 6 3 6

Question 8 continued

Only use this grid if you need to redraw your graph.

(Total for Question 8 is 11 marks)

9

Diagram **NOT** accurately drawn
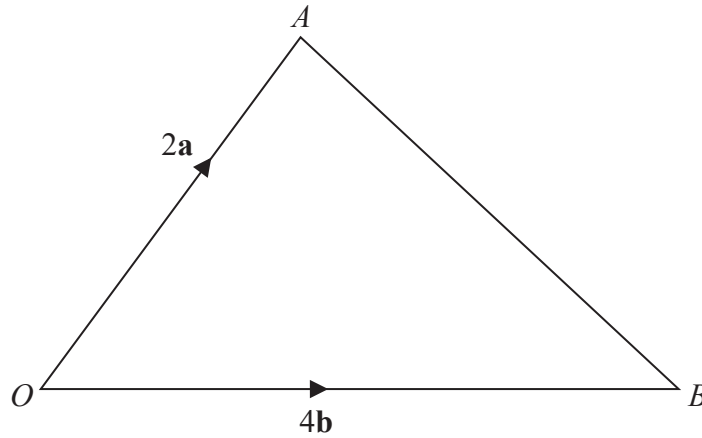
Figure 3

Figure 3 shows the triangle OAB with

$$\overrightarrow{OA} = 2\mathbf{a} \quad \text{and} \quad \overrightarrow{OB} = 4\mathbf{b}$$

(a) Find $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

(2)

The point P is the midpoint of AB

(b) Find $\overrightarrow{OP}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$

(2)

The point Q lies on OP such that $OQ : QP = 3 : 1$

(c) Find $\overrightarrow{AQ}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$

(3)

The point R lies on OB such that AQR is a straight line.

(d) Find in its simplest form the ratio $OR : RB$

(6)

P 6 6 3 0 8 A 0 3 0 3 6

Question 9 continued

(Total for Question 9 is 13 marks)

10 The curve C has equation $y = \dfrac{2x-1}{x+4}$ $\qquad x \neq -4$

(a) Write down an equation of the asymptote to C that is

(i) parallel to the x-axis,

(ii) parallel to the y-axis.

(2)

(b) Find the coordinates of the points of intersection of C with the coordinate axes.

(2)

(c) Using the axes on the opposite page, sketch C, showing clearly the asymptotes and the coordinates of the points of intersection of C with the coordinate axes.

(3)

The line with equation $y = x + k_1$ is the tangent to C at the point P

The line with equation $y = x + k_2$ is the tangent to C at the point Q

Given that the x coordinate of P is greater than the x coordinate of Q

(d) using calculus, find the coordinates of

(i) P

(ii) Q

(8)

(e) Hence find the value of

(i) k_1

(ii) k_2

(3)

Question 10 continued

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Turn over for spare axes if you need to redraw your sketch.

Question 10 continued

Question 10 continued

Only use these axes if you need to redraw your sketch.
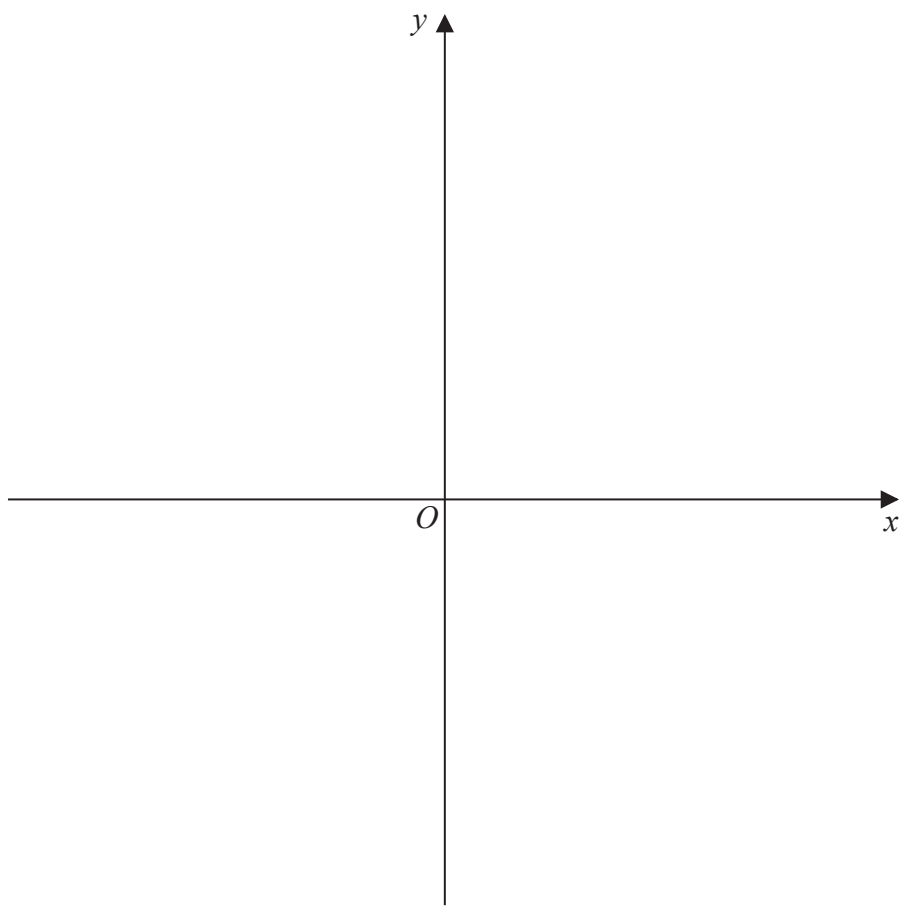
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Question 10 continued

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(Total for Question 10 is 18 marks)

TOTAL FOR PAPER IS 100 MARKS