Pearson Edexcel

Mark Scheme (Results)

November 2020

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 02

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=(x+p)(x+q), \text{ where } |pq|=|c| \quad \text{leading to } x=\ldots.$$

$$\left(ax^2+bx+c\right)=(mx+p)(nx+q) \text{ where } |pq|=|c| \text{ and } |mn|=|a| \quad \text{leading to } x=\ldots.$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*
leading to $x=\ldots.$

3. Completing the square:

$$x^2+bx+c=0: \ \left(x\pm\tfrac{b}{2}\right)^2 \pm q \pm c=0, \quad q\neq 0 \qquad \text{leading to } x=\ldots.$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n \to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n \to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:
When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)
Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question Number	Scheme	Marks
1	$(v=)8+2t-t^2$	B1
	$8+2t-t^2=(2+t)(4-t)=0 \Rightarrow t=4$	M1A1
	Distance $=3+8\times4+4^2-\frac{1}{3}4^3=29\frac{2}{3}$ m	A1 (4)
	(accept 29.7 or better or a recurring decimal)	
		[4]

B1	Correct differentiation
M1	Equate their differentiated expression (min 2 correct terms) to 0 (= 0 may be implied by their solution) and attempt to solve the 3 TQ by any valid method. Must reach t = ... Calculator solution: Allow M1A1 if their equation and its roots are correct , otherwise M0A0
A1	Correct value of t (Ignore t = -2 if shown)
A1	Correct distance, exact or min 3 s f Award A0 if value when t = -2 is also offered (and not excluded) If there is an error in the solution of their equation but t = 4 is used to obtain the correct answer this mark cannot be awarded.

Question Number	Scheme	Marks
2	$\text{Vol} = \pi\int_0^3 \left(e^{3x}\right)^2 dx \left(= \pi\int_0^3 e^{6x} dx\right)$	M1
	$\pi\left[\frac{1}{6}e^{6x}\right]_0^3, = \left(\frac{1}{6}e^{18} - \frac{1}{6}\right)\pi$ oe	dM1A1,A1 (4)
		[4]

M1	Use $\text{Vol} = \pi\int y^2 dx$ Award if pi missing here but reappears later. Limits not needed, ignore any shown. dx may be missing.
dM1	Square correctly and attempt the integration. $e^{6x} \to ke^{6x}$ where $k = \pm\frac{1}{6}$ or ± 1 Limits and dx may be missing. Award if pi missing here but reappears later.
A1	Correct integration including correct limits
A1	Substitute the limits and obtain the correct answer

Question Number	Scheme	Marks
3(a)	$(1+px)^{-5}=1+(-5)(px)+\frac{(-5)(-6)(px)^2}{2!}+\frac{(-5)(-6)(-7)(px)^3}{3!}$ $+\frac{(-5)(-6)(-7)(-8)(px)^4}{4!}+\ldots$	M1
	$=1-5px+15p^2x^2-35p^3x^3+70p^4x^4+\ldots$	A1A1 (3)
(b)	$70p^4+2\times35p^3=0$	M1
	$p=-1$	A1 (2) [5]

(a)	
M1	Attempt the binomial expansion up to and including the term in x^4. Must start with 1 and (px) must appear in at least one term. Ignore terms beyond x^4. 2! or 2, 3! or 6, 4! or 24 accepted.
A1	Any 2 correct algebraic terms, simplified (1 is not algebraic) Numbers must be simplified but $(px)^n$, n = 2,3,4 allowed
A1	Fully correct simplified expansion as shown but allow terms such as +(- 5px) etc
(b)	
M1	Use their coefficients and the **given** equation to form an equation in p (If powers of x included give M0)
A1	Correct value of p $p=-1$ only Must have come from correct working

Question Number	Scheme	Marks
4(i)	$\dfrac{16}{\log_4 r} = \log_4 r \Rightarrow 16 = (\log_4 r)^2 \Rightarrow \log_4 r = \pm 4$	M1
	$r = 4^4 = 256 \quad \text{or } r = 4^{-4} = \dfrac{1}{256}$	A1 (2)
(ii)	$\log_5 9 + \log_5 12 + \log_5 15 + \log_5 18 = \log_5 (9\times12\times15\times18) = \log_5 29\,160$	M1
	$1 + \log_5 x + \log_5 x^2 = \log_5 5 + \log_5 x + \log_5 x^2 = \log_5 5x^3$	M1A1
	$5x^3 = 29\,160$	dM1
	$x = 18$	A1 (5) [7]
ALT 1		
	$\text{LHS} = \log_5 29\,160$	M1
	$\text{RHS} = 1 + \log_5 x^3$	M1
	$\left(\dfrac{\log_{10} 29\,160}{\log_{10} 5}\right) = 6.387...\left(= \log_5 x^3 + 1\right)$	A1
	$5.387... = 3\log_5 x$	dM1
	$\log_5 x = 1.795...$	
	$x = 18$	A1
ALT 2	$\text{LHS} = \log_5 29\,160$	M1
	$\text{RHS} = \log_5 5 + \log_5 x^3$	M1A1
	$\log_5 29\,160 = \log_5 5 + \log_5 5832$	
	$5832 = x^3$	dM1
	$x = 18$	A1
ALT 3	$\text{LHS} = \log_5 5832 + \log_5 5$	M1
	$\text{RHS} = 1 + \log_5 x^3$	M1
	$\text{LHS} = \log_5 5832 + 1$	A1
	$\log_5 5832 = \log_5 x^3$	
	$5832 = x^3$	dM1
	$x = 18$	A1
ALT 4	$\log_5 29\,160 - \log_5 x^3 = 1$	M1M1
	$\log_5 \dfrac{29\,160}{x^3} = 1$	A1
	$\dfrac{29\,160}{x^3} = 5 \Rightarrow x^3 = 5832$	dM1
	$x = 18$	A1

(i)	
M1	Change base (can have base 4 or base *r* provided the same for both logs), multiply to remove the fraction and solve to $\log_4 r = ...$ (or $\log_r 4 = ...$) (One answer only is sufficient)
A1	Complete to the correct answers, **both** needed
(ii)	
M1	Combine the LHS logs to a single log. Numbers should be multiplied – if added award M0
M1	Change 1 to $\log_5 5$ and obtain a single log for the RHS
A1	Correct single log for RHS (Requires second M mark, not first)
dM1	Use LHS = RHS to obtain an equation without logs Depends on both previous M marks
A1	Correct answer
ALT 1	
M1	Combine the LHS logs to a single log. Numbers should be multiplied – if added award M0
M1	Combine the two logs on RHS
A1	Correct numerical value for **LHS**. This will need a calculator so change of base need not be seen. Equation need not be formed yet. Correct final answer implies correct value here. Otherwise min 3 sf needed This mark requires the **first** M mark to have been given – the second M mark can be M0
dM1	Use LHS = RHS to obtain a value for $3\log_5 x$ or $\log_5 x$ Depends on both previous M marks
A1	Correct answer. This will be exact if all numbers stored on the calculator so accept 18 only.
ALT 2	
M1	Combine the LHS logs to a single log. Numbers should be multiplied – if added award M0 **Alternatively** we may see $\text{LHS} = \log_5 5 + \log_5 5832$ without ever seeing $\text{LHS} = \log_5 29\,160$
M1	Combine the 2 logs on RHS and change 1 to $\log_5 5$
A1	Correct RHS (Requires second M mark, not first)
dM1	Use LHS = RHS to obtain a value for x^3 Depends on both previous M marks
A1	Correct answer
ALT 3	
M1	Split $\log_5 15$ and combine all logs apart from $\log_5 5$ to a single log
M1	Combine the two logs on RHS
A1	Change $\log_5 5$ to 1 and have the correct log on LHS This mark requires the **first** M mark to have been given – the second M mark can be M0
dM1	Use LHS = RHS to obtain a value for x^3 Depends on both previous M marks
A1	Correct answer
ALT 4	
M1	Combine the LHS logs to a single log. Numbers should be multiplied – if added award M0
M1	Combine the two logs from the RHS
A1	Obtain the equation shown
dM1	Obtain a value for x^3 Depends on both previous M marks
A1	Correct answer

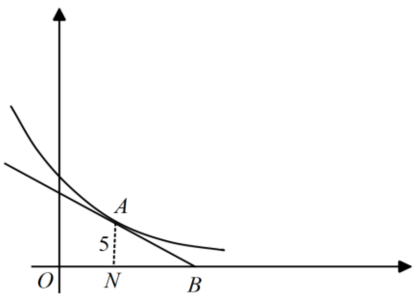
Question Number	Scheme	Marks
5(a)	$\sum_{r=1}^{n}(3r+5)=8+11+14+...+(3n+5)=\frac{1}{2}n(8+3n+5)=\frac{1}{2}n(3n+13)$ *	M1M1A1cso (3)
ALT	$\sum_{r=1}^{n}(3r+5)=\sum_{1}^{n}3r+5n=\frac{n}{2}(3+3n)+5n=\frac{n}{2}(13+3n)$ *	M1M1A1cso
(b)	$\sum_{r=35}^{50}(3r+5)=\frac{50}{2}(13+150)-\frac{34}{2}(13+102)$	M1
	$=2120$	A1 (2)
(c)	$\frac{n}{2}(13+3n)=385$	
	$3n^2+13n-770=0$	M1
	$(3n+55)(n-14)=0 \quad n=14$	M1A1 (3)
		[8]

(a)	
M1	Evaluate **either** first and last terms **or** first and common difference
M1	Use either sum formula. Can be shown explicitly or implied by a correct, full substitution of n and their a and their d or l
A1cso	Reach the **given** result with no errors in the working. Must be the complete result, not just the RHS or there must be a conclusion eg "shown"
ALT	
M1	Split the $(3r + 5)$ into 2 parts and deal with the 5 correctly
M1	Sum $(3r)$ either by using a summation formula or by using the standard result
A1	Reach the **given** result with no errors in the working
(b)	
M1	Express the required sum as the difference of 2 sums. Second sum must have 34 terms. Use the result given in (a). **Using a standard formula** with first term and either last term or common difference scores 0/2 as question states "hence". Calculator solutions (without showing the difference of the 2 sums first) score M0
A1	Correct answer.
(c)	
M1	Use the result in (a) or some other valid method to form a 3 term quadratic in n
M1	Solve their 3TQ by any valid means. Must reach n = ... Negative value need not be seen
A1	Correct answer. n = 14 and no other Correct quadratic followed by correct answer scores 3/3 Incorrect quadratic solved by calculator M0A0

Question Number	Scheme	Marks
6(a)	$\alpha+\beta=\dfrac{3}{4}\quad \alpha\beta=-\dfrac{5}{4}$	B1
	$\dfrac{2\alpha}{\beta}+\dfrac{2\beta}{\alpha}=\dfrac{2\left(\alpha^2+\beta^2\right)}{\alpha\beta}=\dfrac{2\left(\left(\alpha+\beta\right)^2-2\alpha\beta\right)}{\alpha\beta}$	M1
	$=\dfrac{2\left(\dfrac{9}{16}+\dfrac{5}{2}\right)}{-\dfrac{5}{4}}=-\dfrac{49}{10}$	A1
	$\dfrac{2\alpha}{\beta}\times\dfrac{2\beta}{\alpha}=4$	B1
	$x^2+\dfrac{49}{10}x+4\ (=0)$	M1
	$10x^2+49x+40=0$	A1 (6)
(b)	$4\left(\alpha+\beta\right)=3=-\dfrac{p}{4}\quad p=-12$	M1A1
	$\left(3\alpha+\beta\right)\times\left(\alpha+3\beta\right)=3\left(\alpha^2+\beta^2\right)+10\alpha\beta$	M1
	$=3\left(\dfrac{9}{16}+\dfrac{5}{2}\right)-10\times\left(\dfrac{5}{4}\right)=-\dfrac{53}{16}$ oe	A1
	$\dfrac{q}{4}=-\dfrac{53}{16}\quad q=-\dfrac{53}{4}$ oe	A1 (5)
		[11]
(a)B1	Correct values for $\alpha+\beta$ **and** $\alpha\beta$. Award if values not seen explicitly but embedded in the sum/product calculations for the new equation.	
M1	Attempt the sum of the roots of the new equation. Must reach a **correct** expression ready for substitution of values of $\alpha+\beta$ **and** $\alpha\beta$	
A1	Correct value for the sum. Allow if $\alpha+\beta=-\dfrac{3}{4}$ has been used.	
B1	Correct value for the product of roots of the new equation.	
M1	Use $x^2-\left(\text{sum of roots}\right)x+\text{product of roots}$ "= 0" may be missing	
A1	Correct final equation, as shown or an **integer** multiple of this. Must have = 0 now. Allow if $\alpha+\beta=-\dfrac{3}{4}$ has been used.	
(b)		
M1	Use their value of $\alpha+\beta$ to obtain a value for the sum of the roots of $g\left(x\right)=0$ and equate to $\pm p/4$	
A1	Obtain the correct value of p	
M1	Attempt the product of the roots of $g\left(x\right)=0$ and obtain an expression ready for substitution of values. May use work from (a) for value of $\alpha^2+\beta^2$ so the expression shown is sufficient.	
A1	Obtain the correct value for the product	
A1	Correct value of q	

Question Number	Scheme	Marks
7(a)	$\dfrac{(x+1)}{(x-3)}=\dfrac{(4x-2)}{(x+1)}$ or $(x+1)^2=(x-3)(4x-2)$	M1
	$(x+1)^2=(4x-2)(x-3)\Rightarrow 3x^2-16x+5(=0)$	A1
	$(x-5)(3x-1)=0\Rightarrow x=5,\ \dfrac{1}{3}$	M1A1A1 (5)
(b)	$x=\dfrac{1}{3}\Rightarrow\ r=\dfrac{x+1}{x-3}=\dfrac{\frac{4}{3}}{\frac{-8}{3}}$	M1
	$r=-\dfrac{1}{2}\quad\therefore$ convergent as $-1<r<1$ *	A1cso (2)
(c)	$S=\dfrac{a}{1-r}=\dfrac{-\frac{8}{3}}{1+\frac{1}{2}}=-\dfrac{16}{9}$ oe	M1A1 (2)
(d)	$\dfrac{S}{S_n}=\dfrac{a}{1-r}\times\dfrac{1-r}{a(1-r^n)}=\dfrac{1}{(1-r^n)}=\dfrac{256}{255}$	M1
	$255=256(1-r^n)$	
	$256r^n=1\quad\left(-\dfrac{1}{2}\right)^n=\dfrac{1}{256}$ oe $\quad n=8$	dM1A1 (3) [12]

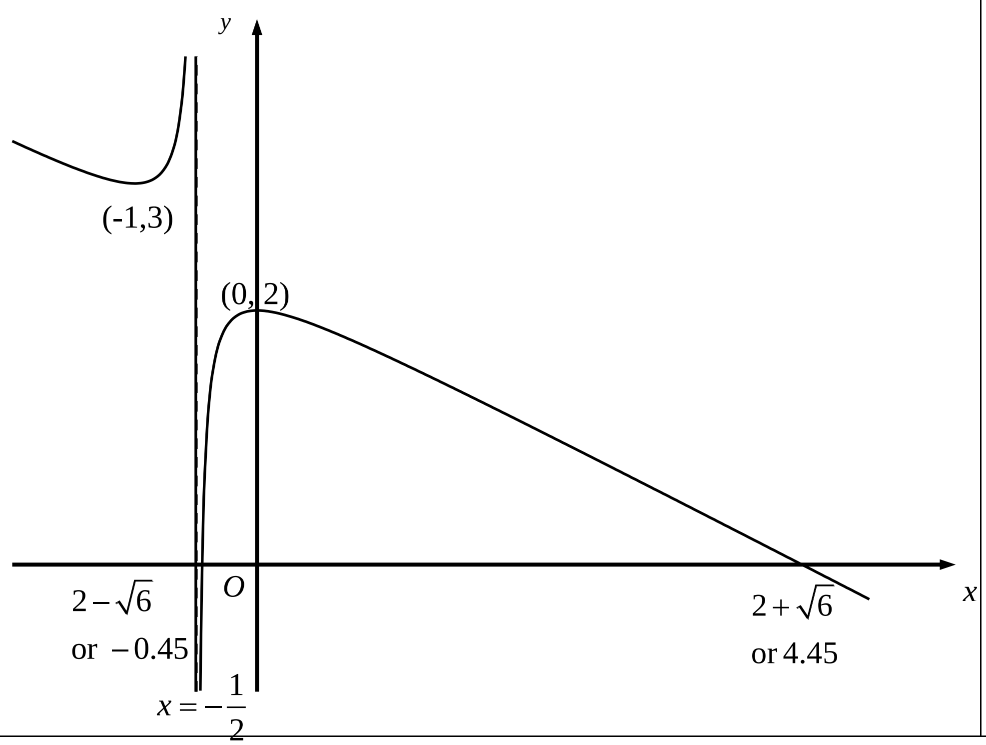
(a)M1	Form an equation using the given information about the terms
A1	Simplify their equation to a correct 3TQ, terms in any order Condone missing = 0
M1	Attempt to solve their 3TQ by any valid method. Must reach x = ... (at least one root) Calculator solutions: *Both* roots correct from a correct equation scores M1A1A1 Incorrect equation or incorrect roots scores M0A0A0
A1	One correct value of x
A1	Both correct values of x
(b)	
M1	Use either of their values of x, provided it is < 1, to find the corresponding value of r. No need to simplify
A1cso	Correct value of r **and** the conclusion including the reason
(c)	
M1	Use their value of r (not x) provided $-1<r<1$ (as found in (b) or here) and the formula for the sum to infinity to obtain a value for S
A1	Correct value
(d)	
M1	Obtain an equation in r and n. May use the formulae to cancel a or may sub values of a and r in the formulae for the LHS Must equate to 256/255 Value of r not needed for this mark so allow any value used.
dM1	Solve their equation of the form $r^n=...$ where $-1<r<1$ (r not x). May use trial and improvement or logs. This mark can be given if the equation and value of r are incorrect. Evidence of method needed if final answer is incorrect. If logs used condone log $(-1/2)^n$
A1	Correct value from correct working

Question Number	Scheme	Marks
8(a)	$5e^{-2x}+4=e^{2x} \quad 5e^{-2x}+4-e^{2x}=0$ OR $y=\frac{5}{y}+4 \Rightarrow y^2-4y-5=0$	M1
	$(5e^{-x}-e^{x})(e^{-x}+e^{x})=0$ | $(y-5)(y+1)=0$	M1
	$5e^{-x}=e^{x} \quad e^{2x}=5 \quad x=\frac{1}{2}\ln 5$ (oe eg $\ln\sqrt{5}$) | $y=5$	A1
	($e^{-x}=-e^{x}$ not possible) | $e^{2x}=5 \quad x=\frac{1}{2}\ln 5$	
	A is $\left(\frac{1}{2}\ln 5, 5\right)$	A1 (4)
(b)	$y=5e^{-2x}+4 \Rightarrow \frac{dy}{dx}=-10e^{-2x}$	M1
	At A $\frac{dy}{dx}=-10e^{-2x}=-10\times\frac{1}{5}=-2$	A1ft
	Eqn tgt: $y-5=-2\left(x-\frac{1}{2}\ln 5\right)$	dM1A1
	$y=0 \Rightarrow x=\frac{1}{2}(5+\ln 5)$ (= x coordinate of B)*	A1cso (5)
ALT	For last 3 marks:	
	Hence $\frac{5}{NB}=2 \Rightarrow NB=\frac{5}{2}$	dM1A1
	$ON=\frac{1}{2}\ln 5$	
	$OB=\frac{1}{2}\ln 5+\frac{5}{2}=\frac{1}{2}\ \ 5+\ln 5$ * 	A1cso
(c)	$C_2 : \frac{dy}{dx}=2e^{2x} \Rightarrow$ grad tgt at A is $2\times 5=10$	B1ft
	Eqn tgt: $y-5=10\left(x-\frac{1}{2}\ln 5\right)$	M1
	At D: $x=\frac{1}{2}(-1+\ln 5)$	A1
	Area $\Delta ABD=\frac{1}{2}\left(\frac{1}{2}(5+\ln 5)-\frac{1}{2}(-1+\ln 5)\right)\times 5$	M1A1
	$=\frac{15}{2}$ or $7\frac{1}{2}$ (units^2)	A1 (6)
	See notes for area by "determinant" method	

ALT	For second and third marks: $\dfrac{5}{ND}=10 \Rightarrow ND=\dfrac{1}{2}$ $OD=\dfrac{1}{2}\ln 5-\dfrac{1}{2}$	 M1 A1	 [15]

(a)	
M1	Equate the 2 curve equations. No need to simplify
M1	Factorise their equation
A1	Obtain the one possible value for x (other root need not be seen; if seen it must be rejected) **Must be exact**
A1	Obtain the corresponding value for y. **Must be exact.** Need not be shown in coordinate brackets. Use of $e^{2x}=5$ leads to $y=5$ without use of a value of x, so M1M1A0A1 can be scored. There must only be one correct y shown. Accept $y=e^{\ln 5}$
(b)	
M1	Differentiate the equation of C_1 $5e^{-2x} \to ke^{-2x}$ where $k=\pm 5$ or ± 10 and no integration seen
A1ft	Grad at A = -2 follow through their x coordinate
dM1	Obtain the equation of the tangent at A using their gradient and their coordinates of A. Can be in any form but if $y=mx+c$ is used a value for c must be found. Gradient of the tangent must be numerical.
A1	Correct equation in any form
A1cso	Correct x coordinate of B obtained from correct working.
ALT	For last 3 marks
dM1	Use their y coordinate of A and their (numerical) gradient of the tangent to find the length NB (where N is the foot of the perpendicular from A to the x-axis)
A1	Correct length of NB
A1cso	Add the x coordinate of A to obtain the x coordinate of B
(c)	
B1ft	Correct gradient of tangent to C_2 at A follow through their x coordinate
M1	Obtain an equation for the tangent using their gradient and their coordinates of A Gradient of the tangent must be numerical.
A1	Correct x coordinate of D (exact or minimum 3 sf)
M1	Use a correct formula for the area of a triangle with their y coordinate of A, their x coordinate of D and the **given** x coordinate of B
A1	Correct, unsimplified area Allow use of correct but non-exact coordinates
A1	Correct area Accept only $7\frac{1}{2}, \frac{15}{2}$ or 7.5 **Heron's formula:** Nos which may be seen: $AB=\frac{5\sqrt{5}}{2}, AD=\frac{\sqrt{101}}{2}, BD=3, s=\frac{1}{2}(a+b+c)=6.8$
ALT	For second and third marks:
M1	Use their y coordinate of A and their gradient of the tangent to find the length ND
A1	Use the x coordinate of A to obtain the x coordinate of D

ALT	Area by "determinant" method:
M1	Eg Area $=\frac{1}{2}\begin{vmatrix}\frac{1}{2}\ln 5 & \frac{1}{2}(5+\ln 5) & \frac{1}{2}(\ln 5-1) & \frac{1}{2}\ln 5 \\ 5 & 0 & 0 & 5\end{vmatrix}$ *y* coordinates of *B* and *D* must be 0
	Must include the ½ and have 4 sets of coordinates with first and last the same.
A1	$=\frac{1}{2}\left(\frac{1}{2}(\ln 5-1)\times 5-\frac{1}{2}(\ln 5+1)\right)$ Allow use of correct but non-exact coordinates
A1	Correct area Accept only $7\frac{1}{2}, \frac{15}{2}$ or 7.5 **Must be positive**

Question Number	Scheme	Marks
9(a)	$y=\dfrac{2+4x-x^2}{2x+1} \Rightarrow x^2-4x-2+2yx+y\ (=0)$ $x^2+(2y-4)x+(y-2)=0$	M1A1A1 (3)
(b)	$(2y-4)^2 \geq 4(y-2)$ $4y^2-16y+16=4y-8 \Rightarrow 4y^2-20y+24=0$ $y\leq 2$ or $y \geq 3$ *	M1 M1A1 A1cso (4)
(c)	$y=\dfrac{2+4x-x^2}{2x+1}$ $\dfrac{dy}{dx}=\dfrac{(4-2x)(2x+1)-2(2+4x-x^2)}{(2x+1)^2}$ **See notes for product rule method** $\dfrac{dy}{dx}=0 \Rightarrow (4-2x)(2x+1)-2(2+4x-x^2)=0$ $2x(x+1)=0 \Rightarrow x=0,\ -1$ stationary points are $(0,2)\ (-1,3)$	 M1A1A1 M1 A1 A1 (6)
ALT	$x^2+(2y-4)x+(y-2)=0 \Rightarrow 2x+(2y-4)+2\dfrac{dy}{dx}x+\dfrac{dy}{dx}=0$ $\dfrac{dy}{dx}=0 \Rightarrow x+y=2$ (using (b)) stationary points are $(0,2)\ (-1,3)$	M1A1A1 M1 A1A1
(d)		B1 curve (i) M1A1 (M1 finding coords, A1 correct (oe or min 2dp) and on diagram (ii) B1 (iii) B1ft (5) [18]

(a)M1	Re-write the equation of C without fractions and rearrange to the required form.
A1	Correct value for a and either b or c. These values need not be stated explicitly.
A1	Fully correct equation. Values of a, b and c need not be stated explicitly. Condone missing brackets round "$y - 2$". Award this mark when the equation is reached – isw any listing of values with incorrect signs.
(b)	
M1	Use "$b^2 \geqslant 4ac$" for their equation
M1	Re-arrange their inequality to a 3TQ in y. Allow an equation here.
A1	Correct 3TQ, as shown or any equivalent
A1cso	Deduce the CVs (need not be shown explicitly) and state the (given) inequalities. There must be no errors in the working but the equation, if correct, can be solved easily so no working need be shown. Condone use of "and" instead of "or".
(c)	
M1	Differentiate the equation of C using the quotient rule. The denominator must be correct and the numerator must consist of the difference of 2 terms of the type shown. The product rule can be used.
A1	Either numerator term correct
A1	Fully correct numerator
ALT:	**Product Rule** $y=\left(2+4x-x^2\right)\left(2x+1\right)^{-1}$
M1A1 **A1**	$\frac{dy}{dx}=\left(4-2x\right)\left(2x+1\right)^{-1}-2\left(2+4x-x^2\right)\left(2x+1\right)^{-2}$
	M1: rewrite without denominator and attempt product rule. Difference of 2 terms of the form shown needed (Difference because of the negative power) A1 Either term correct A1 Second term correct
M1	Equate the numerator of their derivative to 0 and solve to $x = ...$(any valid method of solving a quadratic with 2 or 3 terms) If product rule used must multiply through by $\left(2x+1\right)^2$
A1	Both x values correct
A1	Both stationary points correct
ALT	
M1	Use implicit differentiation on the re-arranged equation
A1	Correct derivative of $\left(2y-4\right)x$ (inc use of product rule)
A1	Fully correct derivative
M1	Set $\frac{dy}{dx}=0$ and use the result from (b) to obtain solutions
A1	One correct stationary point
A1	Both correct stationary points
(d)	
B1	Shape: Two parts, one above $y = 3$ and the other below $y = 2$
(i)M1	Attempt to find the x coordinates of the crossing points
A1	Correct coordinates shown on their sketch, 2 crossing points only. $y = 0$ need not be seen. There must be a curve through these points.
(ii)B1	The asymptote must be drawn and labelled (by its equation or by showing the x coordinate of the point where it crosses the x-axis). There must be at least one part of the curve which is asymptotic to the line $x=-\frac{1}{2}$. No part of the curve should touch/cross the asymptote or curve dramatically away from the line.
(iii)B1ft	Label the stationary points with their coords. Follow through provided the result is sensible.

Question Number	Scheme	Marks
10(a)	$\cos(A+B)+\cos(A-B)=\cos A\cos B-\sin A\sin B+\cos A\cos B+\sin A\sin B$	M1
	$=2\cos A\cos B$ *	A1 cso (2)
(b)	Let $A+B=P,\quad A-B=Q\ \Rightarrow A=\frac{1}{2}(P+Q),\quad B=\frac{1}{2}(P-Q)$	M1
	As $\cos(A+B)+\cos(A-B)=2\cos A\cos B$	
	$\cos P+\cos Q=2\cos\frac{1}{2}(P+Q)\cos\frac{1}{2}(P-Q)$ *	M1A1cso (3)
ALT		
	Let $A=\frac{1}{2}(P+Q),\quad B=\frac{1}{2}(P-Q)\Rightarrow A+B=P,\ A-B=Q$	M1
	As $2\cos A\cos B=\cos(A+B)+\cos(A-B)$	
	$2\cos\frac{1}{2}(P+Q)\cos\frac{1}{2}(P-Q)=\cos P+\cos Q$ *	M1A1cso
(c)	$\cos 5\theta+\cos 7\theta=2\cos 6\theta\cos\theta=0$	M1
	$\cos 6\theta=0\Rightarrow 6\theta=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2}\Rightarrow\theta=\frac{\pi}{12},\frac{\pi}{4}\ \left(\text{or }\frac{3\pi}{12}\right),\frac{5\pi}{12}$	A1A1
	$\cos\theta=0\Rightarrow\theta=\frac{\pi}{2}$	A1 (4)
(d)	$\cos 8x+2\cos 6x+\cos 4x=(\cos 8x+\cos 6x)+(\cos 6x+\cos 4x)$	
	$=2\cos 7x\cos x+2\cos 5x\cos x$	M1
	$=2\cos x(\cos 7x+\cos 5x)=2\cos x\times 2(\cos 6x\cos x),\ =4\cos 6x\cos^2 x$ *	dM1,A1cso (3)
ALT 1	$\cos 8x+2\cos 6x+\cos 4x=(\cos 8x+\cos 4x)+2\cos 6x$	
	$=2\cos 6x\cos 2x+2\cos 6x$	M1
	$=2\cos 6x(\cos 2x+1)=2\cos 6x(2\cos^2 x-1+1)=4\cos 6x\cos^2 x$ *	dM1A1cso (3)
ALT 2	Working from right to left	
	$4\cos 6x\cos^2 x=4\cos 6x\times\frac{1}{2}(\cos 2x+1)=2\cos 6x\cos 2x+2\cos 6x$	M1
	$=\cos 8x+\cos 4x+2\cos 6x$ *	dM1A1cso (3)

(e)	$\int_0^{\frac{\pi}{3}} \cos 6x \cos^2 x \, dx = \frac{1}{4}\int_0^{\frac{\pi}{3}} (\cos 8x + 2\cos 6x + \cos 4x)\, dx$	M1	
	$= \frac{1}{4}\left[\frac{1}{8}\sin 8x + \frac{1}{3}\sin 6x + \frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{3}}$	A1	
	$= \frac{1}{4}\left[\frac{1}{8}\sin\frac{8\pi}{3} + \frac{1}{3}\sin 2\pi + \frac{1}{4}\sin\frac{4\pi}{3} - 0\right] = -\frac{\sqrt{3}}{64}$ (-0.02706... scores M1A0)	dM1A1	(4)
			[16]
ALT	For the first M mark Award M1 only when the integrand has been changed (by a valid method) to a function which can be integrated $\int_0^{\frac{\pi}{3}} \cos 6x \cos^2 x \, dx = \int_0^{\frac{\pi}{3}} \cos 6x \times \frac{1}{2}(\cos 2x + 1)\, dx$		
	$= \frac{1}{2}\int_0^{\frac{\pi}{3}} (\cos 6x \times \cos 2x + \cos 6x)\, dx = \frac{1}{2}\int_0^{\frac{\pi}{3}} \frac{1}{2}(\cos 8x + \cos 4x + 2\cos 6x)\, dx$ Rest as above.	M1	

(a)	
M1	Use the standard formulae. This is a "show that" question so these formulae must be written out in full. Both sides of the result must be seen although the working may appear between them as seen in the scheme.
A1cso	Final **given** result. Do not award if the expansions shown are not in the correct order (this suggests incorrect signs in the formulae).
(b)	
M1	Use $A + B = P, \quad A - B = Q$ to obtain A and B in terms of P and Q
M1	Substitute in the result from (a) to obtain an identity in P and Q only
A1cso	Correct result reached with no errors seen
ALT	Working right to left: Notes similar to above
(c)	
M1	Use the result from (b) to show that $(2)\cos 6\theta \cos(\pm\theta) = 0$
A1	Obtain one correct solution of $\cos 6\theta = 0$. Allow if in decimal form but must be radians.
A1	Two further correct solutions **and no more within the range.** If any of the 3 solutions of $\cos 6\theta = 0$ is not exact do not award this mark.
A1	State the solution of $\cos(\pm\theta) = 0$. Must be exact unless this is penalised above.
NB	If answers are given in degrees deduct 2A marks from any that would otherwise have been given. (Answers in degrees but then changed to radians are acceptable – mark the radian answers.) Ignore extra answers outside the stated range – any within are incorrect.
(d)	
M1	Use the result from (a) or (b) once. Allow if one or both "2"s are missing
dM1	Use the result from (a) or (b) again. Must include both "2"s this time. Depends on previous M mark.
A1cso	Reach the **given** result with no errors seen

ALT 1	
M1	Use the result from (a) or (b). Allow if the “2” is missing.
dM1	Factorise and use the *correct* double angle formula on $\cos 2x$ Depends on the previous M mark
A1cso	Reach the **given** result with no errors seen
ALT 2	
M1	Use the *correct* half angle formula on $\cos^2 x$
dM1A1	M1: Use the result from (a) or (b)
cso	A1: reach the **given** result with no errors seen
(e)M1	Obtain a function which can be integrated **either** by using the **given** result from (d) **OR**. deriving the same result. Allow if 1/4 is missing. (Integration by Parts – send to review)
A1	Correct integration (must have included the ¼) Limits not needed for these 2 marks.
dM1	Correct use of the given limits. All sines are 0 at the lower limit so these need not be shown
A1	Correct final answer which must be **exact and stated as a single fraction**.