

Please check the examination details below before entering your candidate information

Candidate surname

Other names

**Pearson Edexcel  
International GCSE**

Centre Number

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Candidate Number

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**Monday 20 January 2020**

Morning (Time: 2 hours)

Paper Reference **4PM1/02R**

**Further Pure Mathematics**

**Paper 2R**



**Calculators may be used.**

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.  
Anything you write on the formulae page will gain NO credit.

### Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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Pearson

**International GCSE in Further Pure Mathematics Formulae sheet**

**Mensuration**

**Surface area of sphere** =  $4\pi r^2$

**Curved surface area of cone** =  $\pi r \times$  slant height

**Volume of sphere** =  $\frac{4}{3}\pi r^3$

**Series**

**Arithmetic series**

Sum to  $n$  terms,  $S_n = \frac{n}{2}[2a + (n - 1)d]$

**Geometric series**

Sum to  $n$  terms,  $S_n = \frac{a(1 - r^n)}{(1 - r)}$

Sum to infinity,  $S_\infty = \frac{a}{1 - r}$   $|r| < 1$

**Binomial series**

$(1 + x)^n = 1 + nx + \frac{n(n - 1)}{2!}x^2 + \dots + \frac{n(n - 1)\dots(n - r + 1)}{r!}x^r + \dots$  for  $|x| < 1, n \in \mathbb{Q}$

**Calculus**

**Quotient rule (differentiation)**

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

**Trigonometry**

**Cosine rule**

In triangle  $ABC$ :  $a^2 = b^2 + c^2 - 2bc \cos A$

$\tan \theta = \frac{\sin \theta}{\cos \theta}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

**Logarithms**

$\log_a x = \frac{\log_b x}{\log_b a}$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1

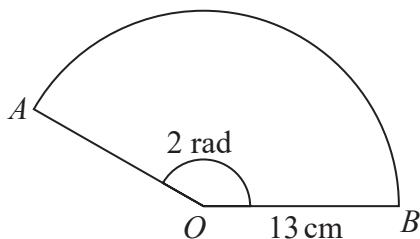


Diagram NOT accurately drawn

Figure 1

Figure 1 shows the sector  $AOB$  of a circle with centre  $O$ .  
The radius of the circle is 13 cm and angle  $AOB = 2$  radians.

- (a) Find the length of the arc  $AB$ . (1)
  
- (b) Find the area of the sector  $AOB$ . (2)

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(Total for Question 1 is 3 marks)



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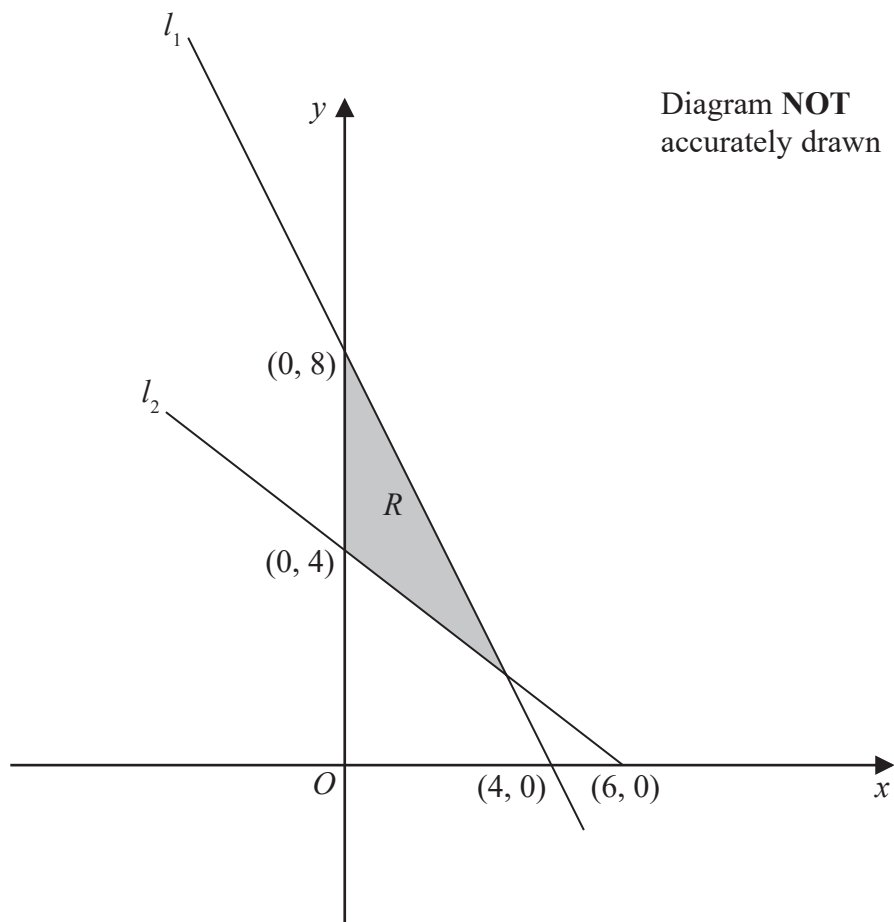


Figure 2

Figure 2 shows the shaded region  $R$  bounded by the line  $l_1$ , the line  $l_2$  and the  $y$ -axis.

The points with coordinates  $(0, 8)$  and  $(4, 0)$  lie on  $l_1$

The points with coordinates  $(0, 4)$  and  $(6, 0)$  lie on  $l_2$

(a) Find, in the form  $ax + by = c$ , where  $a$ ,  $b$  and  $c$  integers, an equation of

(i)  $l_1$

(ii)  $l_2$

(3)

(b) Hence write down three inequalities that define the region  $R$ .

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**Question 2 continued**

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**(Total for Question 2 is 6 marks)**



P 6 0 4 7 6 A 0 5 4 0

3 In triangle  $ABC$ ,  $AB = 11$  cm and  $BC = 12$  cm.

The area of triangle  $ABC = 33$  cm<sup>2</sup>

Find, in cm to 3 significant figures, the two possible lengths of  $AC$ .

(5)

Handwriting practice area consisting of multiple horizontal dotted lines for writing the answer.

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**Question 3 continued**

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**(Total for Question 3 is 5 marks)**



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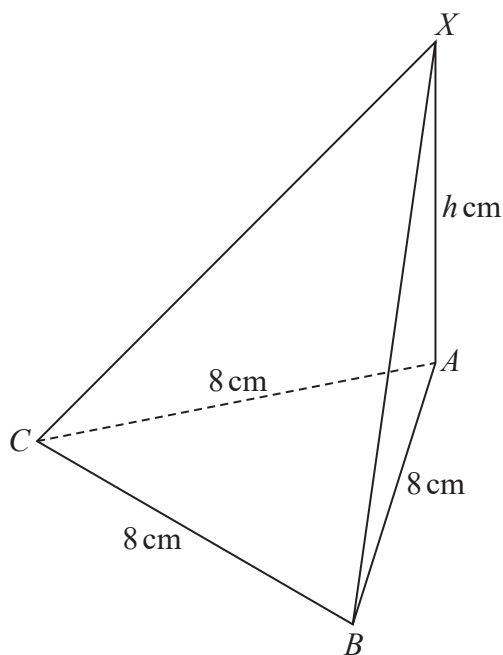


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**Figure 3**

Figure 3 shows a triangular pyramid  $ABCX$ .

The base  $ABC$  of the pyramid is an equilateral triangle where  $AB = BC = CA = 8$  cm. The vertex  $X$  of the pyramid is such that  $AX$  is perpendicular to the base of the pyramid and  $AX = h$  cm.

The volume of the pyramid is  $48\sqrt{3}$  cm<sup>3</sup>

- (a) Show that  $h = 9$  (3)
- (b) Find, in degrees to one decimal place, the size of angle  $BXC$ . (3)
- (c) Find, in degrees to one decimal place, the size of the angle between the plane  $BCX$  and the base  $ABC$  of the pyramid. (3)

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**Question 4 continued**

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**Question 4 continued**

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**Question 4 continued**

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**(Total for Question 4 is 9 marks)**



5 (a) Show that  $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \alpha^3 + \beta^3$  (2)

The roots of the equation  $2x^2 + 3x + 6 = 0$  are  $\alpha$  and  $\beta$

Without solving the equation,

(b) find the value of  $\alpha^3 + \beta^3$  (2)

(c) Show that  $(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = \alpha^4 + \beta^4$  (2)

(d) Form a quadratic equation with integer coefficients that has roots  $(\alpha^3 - \beta)$  and  $(\beta^3 - \alpha)$  (6)

Area with horizontal dotted lines for writing answers.

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**Question 5 continued**

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**Question 5 continued**

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**Question 5 continued**

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**(Total for Question 5 is 12 marks)**



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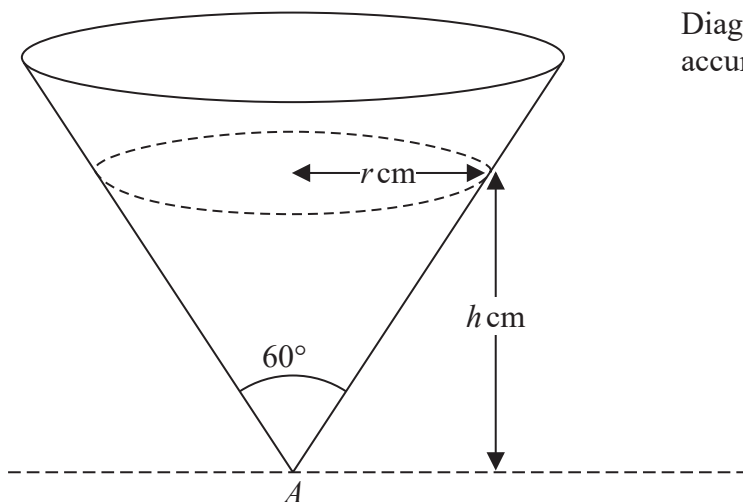


Diagram NOT accurately drawn

Figure 4

Figure 4 shows a hollow right circular cone fixed with its axis of symmetry vertical.

The cone is inverted and contains liquid, which is dripping out of a small hole at the vertex  $A$  of the cone at a constant rate of  $0.9 \text{ cm}^3/\text{s}$ .

At time  $t$  seconds after the liquid starts to drip from the cone, the height of the liquid is  $h \text{ cm}$  above  $A$ . The volume of liquid in the cone at time  $t$  seconds is  $V \text{ cm}^3$

The vertical angle of the cone is  $60^\circ$

(a) Show that  $V = \frac{1}{9}\pi h^3$  (2)

(b) Find, in  $\text{cm/s}$  to 3 significant figures, the rate at which the height of the liquid is decreasing when the height of the liquid in the cone above the vertex is  $1.2 \text{ cm}$ . (4)

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**Question 6 continued**

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**Question 6 continued**

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**Question 6 continued**

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**(Total for Question 6 is 6 marks)**



7 The geometric series  $G$  has first term  $a$ , common ratio  $r$  and  $n$ th term  $u_n$

Given that  $u_4 = e^{x+2}$  and that  $u_7 = e^{\frac{2x+1}{2}}$

(a) show that  $r = e^{-\frac{1}{2}}$  (3)

(b) Hence find  $a$  in terms of  $e$  and  $x$ . (3)

Given that the sum to infinity of  $G$  can be written as  $\frac{e^p}{e^{\frac{1}{2}} - 1}$

(c) find an expression for  $p$  in terms of  $x$ . (3)

Given that  $u_{18} > 1.6$  and that  $x$  is an integer,

(d) find the least value of  $x$ . (4)

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**Question 7 continued**

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Area with horizontal dotted lines for writing.



**Question 7 continued**

A large rectangular area with a rounded border, containing 25 horizontal dotted lines for writing.

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**Question 7 continued**

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**(Total for Question 7 is 13 marks)**







**Question 8 continued**

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**Question 8 continued**

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**Question 8 continued**

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**(Total for Question 8 is 10 marks)**



9 Given that  $\frac{1}{(2-x)^3}$  can be written as  $p(1-qx)^{-3}$

(a) find the value of  $p$  and the value of  $q$ . (2)

(b) Expand  $\frac{1}{(2-x)^3}$  in ascending powers of  $x$  up to and including the term in  $x^3$  and express each coefficient as an exact fraction in its lowest terms. (3)

$$f(x) = \frac{a+bx}{(2-x)^3} \text{ where } a \text{ and } b \text{ are integers}$$

The first three terms of the expansion of  $f(x)$  are  $\frac{3}{8} - \frac{43}{16}x + cx^2$

(c) Find the value of  $a$  and the value of  $b$ . (3)

(d) Find the exact value of  $c$ . (2)

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**Question 9 continued**

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**Question 9 continued**

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**Question 9 continued**

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**(Total for Question 9 is 10 marks)**



10 The equation of a curve  $C$  is  $y = f(x)$  where  $f'(x) = 3x^2 - 4x - p$  and  $p \neq 0$

The points with coordinates  $(2, 0)$  and  $(-1, 9)$  lie on  $C$ .

(a) Show that  $C$  has equation  $y = x^3 - 2x^2 - 4x + 8$  (6)

The straight line  $l$  has equation  $y = 8 - 4x$

(b) Use algebraic integration to find the exact area of the finite region bounded by  $C$  and  $l$ . (6)

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**Question 10 continued**

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### Question 10 continued

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**Question 10 continued**

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**(Total for Question 10 is 12 marks)**



11 The curve  $C$  has equation  $y = \frac{3x - 2}{x + 1}$

(a) Write down an equation of the asymptote to  $C$  which is parallel to the

- (i)  $x$ -axis
  - (ii)  $y$ -axis
- (2)

(b) Find the coordinates of the point where  $C$  crosses the

- (i)  $x$ -axis
  - (ii)  $y$ -axis
- (2)

(c) Sketch  $C$ , showing clearly the asymptotes and the coordinates of the points where  $C$  crosses the coordinate axes.

(3)

The straight line  $l$  has equation  $y = mx + 4$

Given that there are **no** points of intersection between  $l$  and  $C$ ,

(d) show algebraically that the range of possible values of  $m$  can be written as

$$a - 2\sqrt{b} < m < a + 2\sqrt{b}$$

where  $a$  and  $b$  are integers whose values need to be found.

(7)

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**Question 11 continued**

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Handwriting practice area with 25 horizontal dotted lines.



Question 11 continued

Area for writing answers, consisting of multiple horizontal dotted lines.

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**Question 11 continued**

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**Question 11 continued**

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**(Total for Question 11 is 14 marks)**

**TOTAL FOR PAPER IS 100 MARKS**

