Mark Scheme (Results)

January 2020

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks – can only be awarded when relevant M marks have been gained
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - cso – correct solution only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers may score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  If a candidate misreads a number from the question: eg. uses 252 instead of 255; follow through their working and deduct 2A marks from any gained provided the work has not been simplified. (Do not deduct any M marks gained.)

  If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used

  Examiners should send any instance of a suspected misread to review (but see above for simple misreads).

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right), \text{ where } |pq|=|c| \text{ leading to } x=....$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right) \text{ where } |pq|=|c| \text{ and } |mn|=|a| \text{ leading to } x=....$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$$x^2+bx+c=0: \quad \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0, \quad q\neq 0 \qquad \text{leading to } x=....$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\rightarrow x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\rightarrow x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication

from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

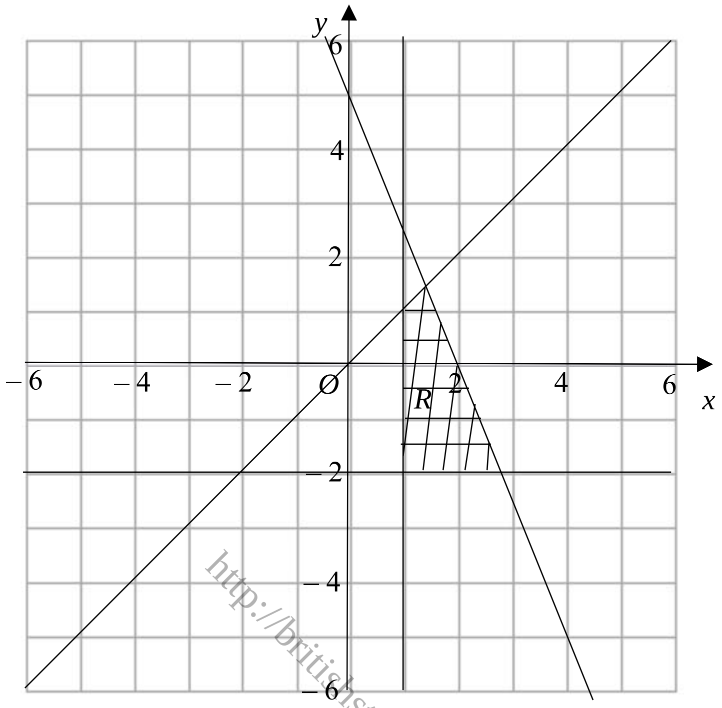
Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the is rule may allow the mark to be awarded before the final answer is given.

International GCSE Further Pure Mathematics – Paper 1 mark scheme

Question number	Scheme	Marks
1 (a) (i)	$a + d + a + 8d = 0$ $\quad a + 3d + a + 5d + a + 9d = 14$ Solve simultaneously $d = 4$	M1 M1 A1
(ii)	$a = -18$	A1 (4)
(b)	$\frac{3n}{2}[48 + 6(n-1)] = \frac{2n}{2}[48 + 6(2n-1)]$	M1 A1
	$3n(42 + 6n) = 2n(42 + 12n) \Rightarrow 6n^2 - 42n = 0$	A1
	$6n(n-7) = 0 \Rightarrow n = [0,\ 7]$	M1
	$n = 7$	A1 (5)
		[9]

Part	Mark	Additional Guidance
(a)	M1	For writing down both correct expressions in terms of a and d $a + d + a + 8d = 0$ $a + 3d + a + 5d + a + 9d = 14$
	M1	For attempting to solve their simultaneous equations for a and d $2a+9d=0$ $3a+17d=14$
	A1 (i)	For $d = 4$ * **This is a show question – there must be no errors for the award of this mark**
	B1 (ii)	For $a = -18$ **This is an A mark in Epen**
(b)	M1	For the **correct** use of the **correct** summation formula on **one of** the LHS or the RHS of the following equation. $\frac{3n}{2}\left(2\times 24+6[n-1]\right)=\frac{2n}{2}\left(2\times 24+6[2n-1]\right)$ No simplification is required for this mark.
	A1	For a fully correct equation as shown above – simplified or unsimplified
	A1	For reaching a correct 2TQ equation in n $126n+18n^2=84n+24n^2 \Rightarrow 6n^2-42n=0$
	M1	For attempting to solve **their** quadratic (See General Guidance for the definition of an attempt) $6n^2-42n=0 \Rightarrow 6n(n-7)=0 \Rightarrow n=[0,\ 7]$
	A1	$n = 7$ Condone the value of 0 for this mark
	ALT	
	M1	For the **correct** use of the **correct** summation formula on **one of** the LHS or the RHS of the following equation. $\frac{3n}{2}\left(2\times 24+6[n-1]\right)=\frac{2n}{2}\left(2\times 24+6[2n-1]\right)$ No simplification is required for this mark.
	A1	For a fully correct equation as shown above – simplified or unsimplified
	A1	Divides through n to reach a linear equation to give $6n = 42$ oe
	M1	Solves their linear equation in n
	A1	$n = 7$

Paper 1		
Question number	Scheme	Marks
2 (a)		B1 B1 (2)
(b)		B1 B1 (2) **[4]**

Part	Mark	Additional Guidance
(a)	B1 (i)	For **either** correct line drawn The correct intersection on the axis for $5x+2y=10$ are $(2,\ 0)$ and $(0,\ 5)$ Coordinates for the $y=x$ are $(0,\ 0)$ $(1,\ 1)$ $(2,\ 2)$ $(3,\ 3)$ etc
	B1 (ii)	For **both** lines drawn correctly
(b)	B1	For both lines $y=-2$ and $x=1$ drawn correctly
	B1	The correct region shaded in or out You do not need to see the label *R*

Question number	Scheme	Marks
3 (a)	$x = 4$ $64p - 496 + 100 + 12 = 0$ $64p = 384$ $p = 6$ *	 M1 A1 (2)
(b)	$(x-4)(6x^2-7x-3)$ $(x-4)(2x-3)(3x+1)$ $x = 4, \frac{3}{2}, -\frac{1}{3}$	M1 A1 M1 A1 (4) **[6]**

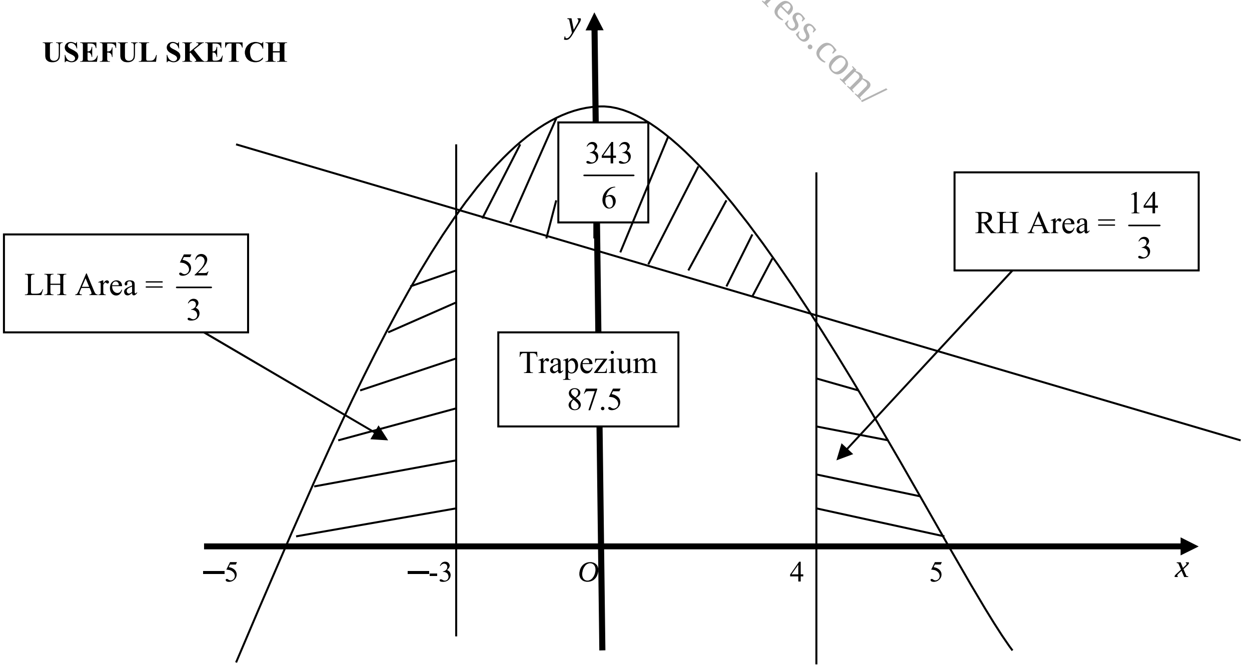
Part	Mark	Additional Guidance
(a)	M1	For substituting $x = 4$ into the **given** expression, equating the expression = 0 **and** attempting to solve for p
	A1	For $p = 6$ * This is a show question so every step must be seen
(b)	M1	For attempting to divide $6x^3-31x^2+25x+12$ by $(x-4)$ (k is an integer) $\Rightarrow x-4 \overline{\big)6x^3-31x^2+25x+12}$ giving $6x^2-7x+k$
	A1	For finding the correct 3TQ $6x^2-7x-3$
	dM1	For an attempt to factorise their 3TQ to give $6x^2-7x-3=(2x-3)(3x+1)$ — Condone $\left(x-\frac{3}{2}\right)\left(x+\frac{1}{3}\right)$
	A1	For the correct solution seen: $x = 4.\ \frac{3}{2},\ -\frac{1}{2}$
	ALT – equates coefficients	
	M1	For stating $6x^3-31x^2+25x+12=(x-4)(Ax^2+Bx+C)\Rightarrow$ $6x^3-31x^2+25x+12=Ax^3+x^2(B-4A)+x(C-4B)-4C$ Minimum required is $A=6,\ B=-7$ and $C=k$
	A1	For $A=6,\ B=-7$ and $C=-3$
	dM1	For an attempt to factorise their 3TQ to give $6x^2-7x-3=(2x-3)(3x+1)$
	A1	For the correct solution seen: $x = 4.\ \frac{3}{2},\ -\frac{1}{2}$
	ALT – by inspection	
	M1	For finding the **quadratic** factor minimum required is $[(x-4)](6x^2-7x+k)$
	A1	For finding the correct 3TQ $6x^2-7x-3$
	dM1	For an attempt to factorise their 3TQ to give $6x^2-7x-3=(2x-3)(3x+1)$
	A1	For the correct solution seen: $x = 4.\ \frac{3}{2},\ -\frac{1}{2}$
	Evidence of the 3TQ seen is required in part (b) $(x-4)(2x-3)(3x+1)=0\Rightarrow x=4,\ \frac{3}{2},\ -\frac{1}{3}$ **is M0**	

Question number	Scheme	Marks
4	Area of sector = $0.4r^2$	B1
	$BC = r\tan 0.8$	B1
	Area of triangle = $\frac{1}{2}r^2\tan 0.8$	B1 ft
	Shaded region = $\frac{1}{2}r^2\tan 0.8 - 0.4r^2 = 101$	M1 M1
	$r^2 = \frac{101}{\frac{1}{2}\tan 0.8 - 0.4}$	
	$r = 29.7$	A1
		[6]

Mark	Additional Guidance
Accept angle converted to degrees $0.8^c = 45.84°$ throughout $\tan(0.8) = \tan(45.8...)^0 = 1.0296$	
B1	For the correct area of the sector = $\frac{0.8}{2}r^2$ oe (need not be simplified)
B1	For $BC = r\tan 0.8$ oe e.g. accept $\tan\left(\frac{4}{5}\right) = \frac{BC}{r}$ This may be embedded in $'\frac{r\times r\tan 0.8}{2}' - '\frac{0.8}{2}r^2' = 101$ Award when seen.
B1ft	$A = \frac{r\times r\tan 0.8}{2}$
M1	Shaded region = $'\frac{r\times r\tan 0.8}{2}' - '\frac{0.8}{2}r^2' = 101$ Ft their expressions for the areas of the sector and triangle provided they are as a minimum $kr^2\tan 0.8$ and lr^2 where k and l are constants
dM1	**This mark is dependent on the previous M mark** For attempting to solve their equation $r = \sqrt{\frac{101}{\frac{1}{2}\tan 0.8 - 0.4}} = (29.658...)$ **This is an A mark in Epen**
A1	$r = 29.7$ only

Question number	Scheme	Marks
5 (a)	$25 - x^2 = 13 - x$	M1
	$x^2 - x - 12 = 0$	
	$(x-4)(x+3) = 0$	M1
	$A = (-3, 16)$	A1
	$B = (4, 9)$	A1
		(4)
(b)	$\int_{-5}^{5}(25 - x^2)\mathrm{d}x - \left[\int_{-3}^{4}(25 - x^2)\,\mathrm{d}x - \frac{1}{2}(16+9)\times 7\right]$	M1 A1
	$\left[25x - \frac{x^3}{3}\right]_{-5}^{5} - \left\{\left[25x - \frac{x^3}{3}\right]_{-3}^{4} - 87.5\right\}$	M1 A1
		B1
	$\left(\frac{250}{3} + \frac{250}{3}\right) - \left[\left(100 - \frac{64}{3}\right) - (-75 + 9) - 87.5\right]$	M1
	$\frac{219}{2} = (109.5)$	A1
		(7)
		[11]
	Alternative (b)	
	$\int_{-5}^{5}(25 - x^2)\mathrm{d}x - \int_{-3}^{4}(12 - x^2 + x)\,\mathrm{d}x$	M1 A1
	$\left[25x - \frac{x^3}{3}\right]_{-5}^{5} - \left[12x - \frac{x^3}{3} + \frac{x^2}{2}\right]_{-3}^{4}$	M1 A1
		A1
	$\left(\frac{250}{3} + \frac{250}{3}\right) - \left(\frac{104}{3} + \frac{45}{2}\right)$	M1
	$\frac{219}{2} = (109.5)$	A1
		(7)

USEFUL SKETCH



Part	Mark	Additional Guidance
(a)	M1	For setting the given equation of the curve = given equation of the line $25 - x^2 = 13 - x$ and attempting to form a 3TQ $x^2 - x - k = 0$ (k is an integer) Ignore the absence of = 0 if further work shows that they are attempting to solve a 3TQ = 0
	M1	For attempting to solve their 3TQ See general guidance for the definition of an attempt.
	A1	For **either** $(-3,\ 16)$ or $(4,\ 9)$
	A1	For **both** $(-3,\ 16)$ and $(4,\ 9)$
(b)	There are two ways to calculate this area. In each case; The first M mark is for a correct strategy (allow ft from (a) in their limits) The first A mark (M mark in Epen) is a fully correct strategy with correct limits The second M mark is for an attempt to integrate The second A mark is for a fully correct integration – ignore limits for this mark. The B mark (and A mark in Epen) is for the area of the trapezium of 87.5 seen anywhere. The third M mark is for substituting in their limits The final A mark is the correct answer only.	
	Method 1 – Trapezium + two sides	
	M1	For an attempt at the correct strategy to find the area. Allow for this mark a correct statement with using their limits correctly. This may well be seen at the end when they combine individual areas. $(A=)\frac{1}{2}('16'+'9')\times'7'+\int_{'4'}^{'5'}(25-x^2)\ \mathrm{d}x+\int_{'-5'}^{'-3'}(25-x^2)\ \mathrm{d}x$ OR $(A=)\int_{-3}^{4}(13-x)\ \mathrm{d}x+\int_{'4'}^{'5'}(25-x^2)\ \mathrm{d}x+\int_{'-5'}^{'-3'}(25-x^2)\ \mathrm{d}x$
	A1	Fully correct expression with correct limits. $(A=)\frac{1}{2}(16+9)\times 7+\int_{4}^{5}(25-x^2)\ \mathrm{d}x+\int_{-5}^{-3}(25-x^2)\ \mathrm{d}x$ OR $(A=)\int_{-3}^{4}(13-x)\ \mathrm{d}x+\int_{'4'}^{'5'}(25-x^2)\ \mathrm{d}x+\int_{'-5'}^{'-3'}(25-x^2)\ \mathrm{d}x$
	M1	For an attempt to integrate their expression for area. (Follow General Guidance for the definition of an attempt) Ignore limits for this mark
	A1	For a fully correct integrated expression for the Area with a correct expression for the trapezium (**Ignore limits for this mark**.) $\left[13x-\frac{x^2}{2}\right],\ \left[25x-\frac{x^3}{3}\right],\ \left[25x-\frac{x^3}{3}\right]$ OR $\frac{1}{2}(16+9)\times 7,\ \left[25x-\frac{x^3}{3}\right],\ \left[25x-\frac{x^3}{3}\right]$
	B1	For the correct area of the trapezium of 87.5. Award wherever seen. $\frac{343}{6}$ seen implies B1 If not seen explicitly, this can be implied from a correct final answer. **This is an A mark in Epen**

	M1	For an attempt to substitute **their** limits into **their integrated** expression.
	A1	For the correct final area of $A=\dfrac{219}{2}$ oe
	Method 2 – Using the area under the whole curve between −5 and 5; minus the area of the curve between −4 and 3; plus the area of the trapezium	
	M1	For an attempt at the correct strategy to find the area Allow for this mark, the correct strategy with **their** limits This may well be seen at the end when they combine individual areas. $(A=)\int_{-5}^{5}(25-x^2)\ \mathrm{d}x-\int_{'-3'}^{'4'}(25-x^2)\ \mathrm{d}x+\frac{1}{2}('16'+'9')\times'7'$ OR $(A=)\int_{-5}^{5}(25-x^2)\ \mathrm{d}x-\int_{'-3'}^{'4'}(25-x^2)\ \mathrm{d}x+\int_{'-3'}^{'4'}(13-x)\ \mathrm{d}x$ OR $(A=)\int_{-5}^{5}(25-x^2)\ \mathrm{d}x-\int_{'-3'}^{'4'}(12+x-x^2)\ \mathrm{d}x$
	A1	For the correct expression with correct limits $(A=)\int_{-5}^{5}(25-x^2)\ \mathrm{d}x-\int_{'-3'}^{'4'}(25-x^2)\ \mathrm{d}x+\frac{1}{2}('16'+'9')\times'7'$ OR $(A=)\int_{-5}^{5}(25-x^2)\ \mathrm{d}x-\int_{'-3'}^{'4'}(25-x^2)\ \mathrm{d}x+\int_{'-3'}^{'4'}(13-x)\ \mathrm{d}x$ OR $(A=)\int_{-5}^{5}(25-x^2)\ \mathrm{d}x-\int_{'-3'}^{'4'}(12+x-x^2)\ \mathrm{d}x$
	M1	For an attempt to integrate their expression for area. (Follow General Guidance for the definition of an attempt) Ignore limits for this mark
	A1	For a fully correct integrated expression for the Area with a correct expression for the trapezium. Accept this seen as individual parts Ignore limits for this mark. $\left[25x-\frac{x^3}{3}\right],\ \left[25x-\frac{x^3}{3}\right],\ \left[13x-\frac{x^2}{2}\right]$ OR $\left[25x-\frac{x^3}{3}\right],\ \left[25x-\frac{x^3}{3}\right],\ \frac{1}{2}(16+9)\times 7$ OR $\left[25x-\frac{x^3}{3}\right],\ \left[12x+\frac{x^2}{2}-\frac{x^3}{3}\right]$
	B1	For the correct area of the trapezium of 87.5 $\dfrac{343}{6}$ seen implies B1 Award wherever seen. If not seen explicitly, this can be implied from a correct final answer. **This is an A mark in Epen**
	M1	For an attempt to substitute **their** limits into **their integrated** expression or individual parts..
	A1	For the correct final area of $A=\dfrac{219}{2}$ oe

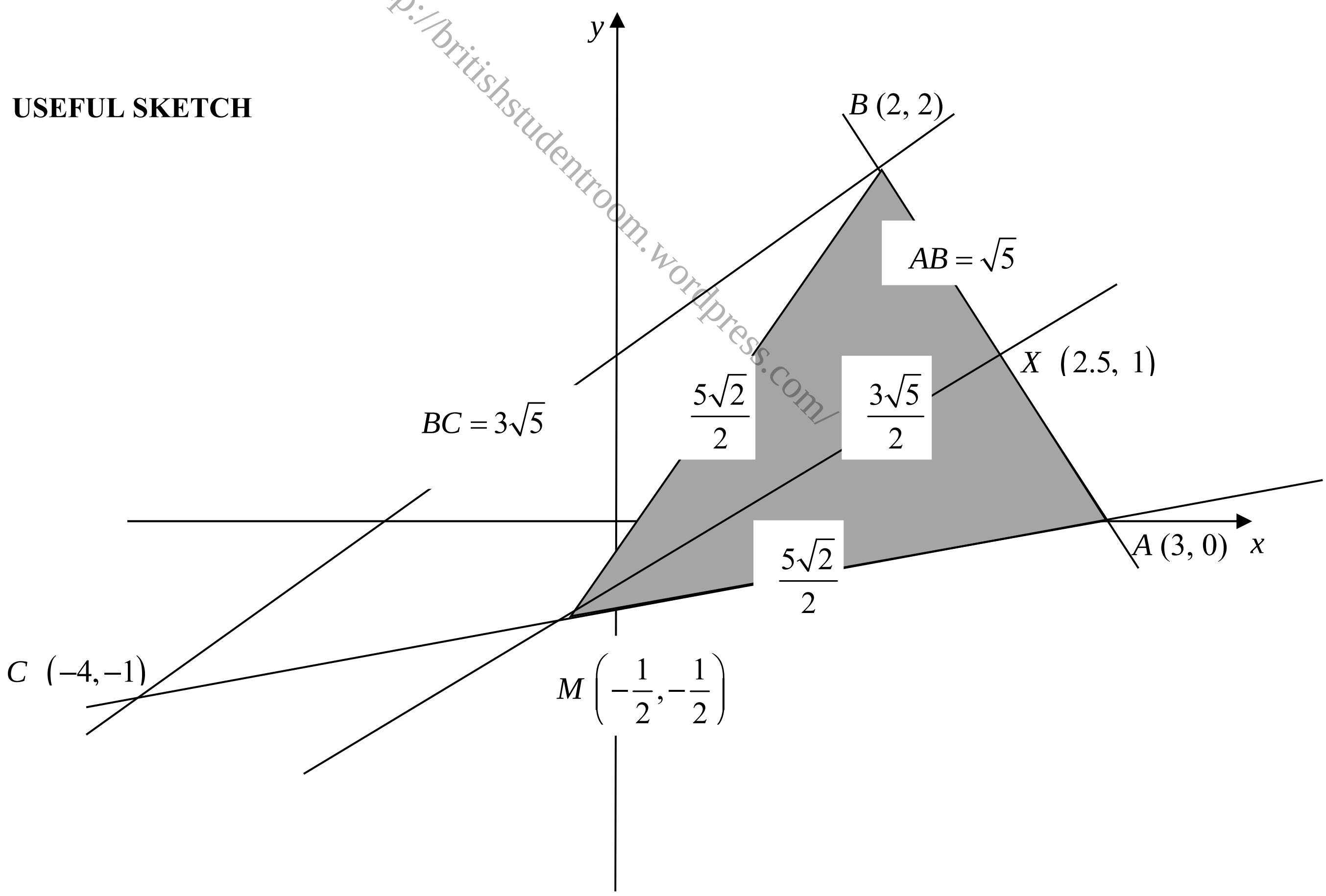
Question number	Scheme	Marks
6 (a)	$\frac{\text{Change in } y}{\text{Change in } x} = \frac{2-0}{2-3} = -2$ $2 = 1 + c$ $c = 1$ $x - 2y + 2 = 0$	M1 A1 M1 A1 ft A1 (5)
(b)	$2y - 2 = 7y + 3$ $-5y = 5$ $y = -1$ When $y = -1$ $x = 2 \times -1 - 2 = -4$ So $C = (-4, -1)$ $\left(\frac{3-4}{2}, \frac{0-1}{2}\right) = \left(-\frac{1}{2}, -\frac{1}{2}\right)$	M1 A1 A1 M1A1 (5)
(c)	$AB = \sqrt{5}$ $BC = \sqrt{45}$ Area $= \frac{1}{2} \times \sqrt{5} \times \sqrt{45} \times \frac{1}{2}$ 3.75 **Alternative c** $\pm\frac{1}{2}\begin{vmatrix} 3 & 0 & 1 \\ 2 & 2 & 1 \\ -4 & -1 & 1 \end{vmatrix}$ $\pm\frac{1}{2}\left[3\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 2 \\ -4 & -1 \end{vmatrix}\right]$ $\pm\frac{1}{2}[3(2+1) + (-2+8)] \times \frac{1}{2}$ 3.75	M1 A1 M1 A1 (4) **(14)** M1 A1 M1 A1

Part	Mark	Additional Guidance
(a)	M1	For an attempt to find the gradient using the given coordinates **and** a correct attempt to find the perpendicular gradient. Accept either $\frac{2-0}{2-3} = (-2)$ or $\frac{0-2}{3-2} = (-2) \Rightarrow m_p = -\frac{1}{'-2'}$
	A1	For $m = \frac{1}{2}$
	dM1	For a correct method to find the equation of a line $y - 2 = '\frac{1}{2}'(x-2)$ or $y - 0 = '\frac{1}{2}'(x-3)$ **The gradient must come from a correct attempt to find the gradient and the gradient of the perpendicular** If $y = mx + c$ is used, then they must use the correct values of x and y and a value for c must be reached before this mark is awarded.
	A1	For the correct equation in any form $y - 2 = \frac{1}{2}(x-2)$ or $y - 0 = \frac{1}{2}(x-3)$ or $y = \frac{1}{2}x + 1$ oe

	A1	For the correct equation in the required form $x-2y+2=0$ **oe** arranged in any order but all one side (e.g. accept even $\frac{x}{2}-y+1=0$)	
(b)	M1	Sets $L_1 = L_2$ **and** attempts to solve for *y* or *x*	
		$2y-2=7y+3$ $-5y=5 \Rightarrow y=...$	$\frac{x+2}{2}=\frac{x-3}{7} \Rightarrow 7x+14=2x-6$ $5x=-20 \Rightarrow x=...$
	A1	$y=-1$	$x=-4$
	A1	$x=-4$	$y=-1$
	M1	For any **correct** method to find the coords of *M* using **their** values for C of *x* and *y* and the given coordinates of *A* (3, 0) $\left(\frac{3+['-4']}{2},\frac{0+['-1']}{2}\right)$ **This is a B mark in Epen**	
	A1	$\left(-\frac{1}{2},-\frac{1}{2}\right)$ **This is a B mark in Epen**	
(c)	M1	For attempting to find the length *AB* **and** *BC* $AB=\sqrt{(3-2)^2+(0-2)^2}$ and $BC=\sqrt{(2--4)^2+(2--1)^2}$ **This is a B mark in Epen**	
	A1	For both $AB=\sqrt{5}$ and $BC=\sqrt{45}$ **This is a B mark in Epen**	
	M1	$\frac{1}{2}('\sqrt{5}'\times'\sqrt{45}')\times\frac{1}{2}$	For using a correct method to find the area of the triangle using correct lengths. i.e. they must be using *BC* and *AB*
	A1	For *A* = 3.75	
	ALT – using determinants		
	M1	For using a correct method with their coordinates for *C* in any order (it is a triangle), but they must start and finish with the same coordinates $A=\frac{1}{2}\begin{pmatrix} 3 & 2 & '-\frac{1}{2}' & 3 \\ 0 & 2 & '-\frac{1}{2}' & 0 \end{pmatrix}$ **This is a B mark in Epen**	
	A1	For using the correct coordinates $A=\frac{1}{2}\begin{pmatrix} 3 & 2 & -\frac{1}{2} & 3 \\ 0 & 2 & -\frac{1}{2} & 0 \end{pmatrix}$ **This is a B mark in Epen**	
	M1	For a correct evaluation using their coordinates $A=\frac{1}{2}\left(\left[3\times2+2\times'-\frac{1}{2}'+'-\frac{1}{2}'\times0\right]-\left[2\times0+'-\frac{1}{2}'\times2+3\times'-\frac{1}{2}'\right]\right)=...$	

	A1	For $A = 3.75$
	ALT	
	M1	For finding the length $AB = \sqrt{(3-2)^2 + (0-2)^2}$ **and** $MX = \frac{1}{2}\sqrt{3^2 + \left(\frac{3}{2}\right)^2}$ (Let X be midpoint of AB so MX is height of triangle ABM)
	A1	$AB = \sqrt{5}$ $\quad MX = \frac{3\sqrt{5}}{2}$
	M1	Area of $\Delta ABM = \frac{1}{2} \times AB \times MX = \frac{1}{2} \times \sqrt{5} \times \frac{3\sqrt{5}}{2} = \left(\frac{15}{4}\right)$
	A1	Area of $\Delta ABM = \frac{15}{4} = 3.75$
If they use trigonometry, please send to review		

USEFUL SKETCH



Question number	Scheme	Marks
7	$\frac{\log_7 x^2}{\log_7 49}$	B1
	$\log_7\left(\frac{8x^2-6x+3}{x}\right)$, $\log_7 2^3$	M1 A1
	$\frac{8x^2-6x+3}{x} = 2^3$	
	$8x^2 - 14x + 3 = 0$	M1
	$(4x-1)(2x-3) = 0$	
	$x = \frac{1}{4}, \frac{3}{2}$	A1
		[5]

Mark	Additional Guidance	
B1	For changing the base of the log either to base 7 or base 49	
	$\log_{49} x^2 = \frac{\log_7 x^2}{\log_7 49} = \frac{\log_7 x^2}{2}$ OR $\log_{49} x^2 = \frac{2\log_7 x}{\log_7 49} = \log_7 x$	$\log_7\left(8x^2-6x+3\right) = \frac{\log_{49}\left(8x^2-6x+3\right)}{\log_{49} 7}$ $= 2\log_{49}\left(8x^2-6x+3\right)$ **AND** $\log_7 2 = \frac{\log_{49} 2}{\log_{49} 7} = 2\log_{49} 2$
M1	For combining the LHS together into one log and dealing with the powers on both sides	
	$\left[\frac{1}{2}\log_7 x^2 = \log_7 x\right] \Rightarrow$ $\log_7\left(\frac{8x^2-6x+3}{x}\right)$, $\log_7 2^3$	$\log_{49}\left(\frac{\left[8x^2-6x+3\right]^2}{x^2}\right)$, $\log_{49} 2^6$
dM1	For forming a 3TQ with their expressions which must have come from an acceptable attempt to deal with the logs **This is an A mark in Epen**	
	$8x^2 - 14x + 3 = 0$	$\left(8x^2-6x+3\right)^2 = 64x^2 \Rightarrow$ $8x^2 - 6x + 3 = \pm 8x \Rightarrow 8x^2 - 14x + 3 = 0$ If this method is used they must reject the negative root of $64x^2$ $\left(\text{i.e } -8x\right)$ because it will form a quadratic equation with no real roots. $\left\{8x^2 + 2x + 3 = 0 \Rightarrow b^2 - 4ac = -92\right\}$
dM1	For attempting to solve **thei**r 3TQ $8x^2 - 14x + 3 = \left(4x-1\right)\left(2x-3\right) = 0 \Rightarrow x = \ldots,\ldots$	
A1	$x = \frac{3}{2}, \quad \frac{1}{4}$	

Question number	Scheme	Marks
8 (a)	$2x - 75 = -31, 211$ $x = 22, 143$	M1A1 A1 (3)
(b)	$2\frac{\sin y°}{\cos y°} + 5\sin y° = 0$ $\sin y°\left(\frac{2}{\cos y°} + 5\right) = 0$ $\cos y° = -\frac{2}{5}$ $(\sin y° = 0)$ $y = 113.6°$ $y = 0°, 180°$	M1 M1 A1 B1 (4)
(c)	$3(1 - \sin^2\theta) - 3\sin^2\theta + \sin\theta + 12 = 0$ $6\sin^2\theta - \sin\theta - 15 = 0$ $(2\sin\theta + 3)(3\sin\theta - 5) = 0$ $\sin\theta = -\frac{3}{2}$ $\sin\theta = \frac{5}{3}$ As $-1 \le \sin\theta \le 1$ no such values for θ exist	M1 M1 A1 B1 (4) **[11]**

Part	Mark	Additional Guidance
(a)	M1	For finding at least one correct value of $(2x-75) = -31^{\circ}$ or 211° and attempting to find one value of x $\Rightarrow x = \frac{-31+75}{2}$ or $x = \frac{211+75}{2}$
	A1	For $x = 22$ or 143
	A1	For $x = 22$ and 143 — Extra values within range – A0; Extra values outside of the range - ignore
.(b)	M1	For using the identity $\tan y^{\circ} = \frac{\sin y^{\circ}}{\cos y^{\circ}}$
	M1	For factorising their expression and finding values for sin y° and cos y° $\sin y^{\circ}\left(\frac{2}{\cos y^{\circ}}+5\right) = 0 \Rightarrow \sin y^{\circ} = 0,\ \cos y^{\circ} = -\frac{2}{5} \Rightarrow y = ...$
	A1	For $y = 113.6$ if there are extra values within range – A0
	B1	For both $y = 0$ and 180 **Both required**
	ALT	
	M1	For multiplying $\sin y \times \frac{\cos y}{\cos y} \Rightarrow \tan y \cos y \Rightarrow (2\tan y + 5\tan y \cos y = 0)$
	M1	For factorising the above expression and finding values for tan y° and cos y° $2\tan y + 5\tan y\cos y = 0 \Rightarrow \tan y(2+5\cos y) = 0$ $\Rightarrow \tan y = 0,\quad \cos y = -\frac{2}{5} \Rightarrow y = ...$
	A1	For $y = 113.6$ Extra values within range – A0; Extra values outside of the range - ignore
	B1	For both $y = 0$ and 180 **Both required**
	SC	$2\frac{\sin y}{\cos y} = -5\sin y \Rightarrow \cos y = -\frac{2}{5} \Rightarrow y = 113.6$ no evidence of factorising – award M1M0A1B0 only (unless there is later recovery)
(c)	M1	For using the identity $\cos^2\theta + \sin^2\theta = 1 \Rightarrow 3(1-\sin^2\theta) - 3\sin^2\theta + \sin\theta + 12 = 0$ to form a 3TQ in terms of $\sin\theta$ Minimally acceptable attempt is $6\sin^2\theta \pm \sin\theta \pm 15 = 0$
	M1	For an attempt to solve their 3TQ (see general guidance for the definition of an attempt $6\sin^2\theta - \sin\theta - 15 = 0 \Rightarrow (2\sin\theta+3)(3\sin\theta-5) = 0 \Rightarrow \sin\theta = ...,...$
	A1	$\sin\theta = -\frac{3}{2},\ \frac{5}{3}$
	B1	For the conclusion; $\lvert\sin\theta\rvert > 1$ therefore no values exist for $\sin\theta$ Do not accept 'undefined' without an explanation that $\lvert\sin\theta\rvert > 1$
Penalise rounding only once in this question		

Question number	Scheme	Marks
9 (a)	$1+\frac{1}{2}(-4x)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)(-4x)^2}{2!}+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-4x)^3}{3!}$	M1
	$1-2x-2x^2-4x^3$	A1 A1 (3)
(b)	$x = 0.06$	B1
	$1 - 0.12 - 0.0072 - 0.000864$	M1
	0.8719	A1 (3)
(c)	$\sqrt{\frac{76}{100}}=\frac{1}{5}\sqrt{19}$	M1
	$\sqrt{19}=0.8719\times 5$	
	4.360	A1 (2)
		[8]

Part	Mark	Additional Guidance
(a)	M1	For an attempt at a Binomial expansion. A attempt is defined as the following • The expansion must start with 1 • The powers of x must be correct • $-4x$ must be used at least once • The denominators (2! And 3!) must be seen. Accept 2 and 6 $(1-4x)^{\frac{1}{2}}=1+\left(\frac{1}{2}\right)(-4x)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(-4x)^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-4x)^3$
	A1	For at least one term in x correct and fully simplified. $1-2x-2x^2-4x^3$
	A1	For the expansion fully correct and simplified
(b)	B1	For finding the value of $x = 0.06$
	M1	For substituting **their** value of x into the expansion **provided** $\|x\| \leqslant 0.25$ Use of **their expansion** or the correct expansion must be seen explicitly here
	A1	0.8719
(c)	M1	For using their value from (b) in $\sqrt{0.76}=\frac{\sqrt{19}}{5}\Rightarrow\sqrt{19}=5\sqrt{0.76}=5\times'0.8719'$
	A1	For 4.360 rounded correctly

Penalise rounding once only in this question. Answers must round to the given answers.

Question number	Scheme	Marks
10 (a) (i)	$\mathbf{a}+\mathbf{c}$	B1
(ii)	$\frac{1}{2}(\mathbf{c}-\mathbf{a})$	B1 (2)
(b)	$\overrightarrow{OX} = OA + AM + \lambda MN$ $\mathbf{a}+\frac{1}{2}\mathbf{c}+\lambda(\frac{1}{2}\mathbf{c}-\frac{1}{2}\mathbf{a})$	M1 A1
	$\mu(\mathbf{a}+\mathbf{c})$	B1
	$\mathbf{a}+\frac{1}{2}\mathbf{c}+\lambda\left(\frac{1}{2}\mathbf{c}-\frac{1}{2}\mathbf{a}\right)=\mu(\mathbf{a}+\mathbf{c})$	M1
	$1-\frac{1}{2}\lambda=\mu \quad \frac{1}{2}+\frac{1}{2}\lambda=\mu$	M1
	$1-\frac{1}{2}\lambda=\frac{1}{2}+\frac{1}{2}\lambda$	M1
	$\lambda=\frac{1}{2} \quad \mu=\frac{3}{4}$	A1 A1 (8)
(c)	Triangle $XBN=\frac{1}{8}$ of $\frac{1}{2}$ the parallelogram Quadrilateral $OXNC=\frac{7}{8}$ of $\frac{1}{2}$ the parallelogram So Quadrilateral $OXNC=\frac{7}{16}$ of the parallelogram ∴ 7 : 16	M1 M1 A1 (3) **[13]**

Part	Mark	Additional Guidance	
(a)(i)	B1	For the correct vector $\mathbf{a}+\mathbf{c}$	
(ii)	B1	For the correct vector $\frac{1}{2}(\mathbf{c}-\mathbf{a})$	
(b)	M1	For the correct vector statement $\overrightarrow{OX}=\overrightarrow{OA}+\overrightarrow{AM}+\lambda\overrightarrow{MN}$	
	A1	For the correct vector (need not be simplified) $\overrightarrow{OX}=\mathbf{a}+\frac{1}{2}\mathbf{c}+\frac{\lambda}{2}(\mathbf{c}-\mathbf{a})$ or $\overrightarrow{OX}=\mathbf{a}+\frac{1}{2}\mathbf{c}+\lambda\left(\frac{\mathbf{c}}{2}-\frac{\mathbf{a}}{2}\right)$	
	B1ft	For $\overrightarrow{OX}=\mu(\mathbf{a}+\mathbf{c})$ ft their $\overrightarrow{OB}=\text{'}\mathbf{a}+\mathbf{c}\text{'}$	
	M1	For equating their two vector statements for $\overrightarrow{OX}$ $\mathbf{a}+\frac{1}{2}\mathbf{c}+\frac{\lambda}{2}(\mathbf{c}-\mathbf{a})=\mu(\mathbf{a}+\mathbf{c})$	
	M1	For equating coefficients of $\mathbf{a}$ and $\mathbf{c}$ $\mathbf{a}+\frac{1}{2}\mathbf{c}+\frac{\lambda}{2}(\mathbf{c}-\mathbf{a})=\mu(\mathbf{a}-\mathbf{c})\Rightarrow\mathbf{a}\left(1-\frac{\lambda}{2}\right)+\mathbf{c}\left(\frac{1}{2}+\frac{\lambda}{2}\right)=\mu\mathbf{a}+\mu\mathbf{c}$ $\Rightarrow\mu=1-\frac{\lambda}{2},\ \mu=\frac{1}{2}+\frac{\lambda}{2}$	
	M1	For attempting to solve their two simultaneous equations in terms of λ and μ.	
		$1-\frac{\lambda}{2}=\frac{1}{2}+\frac{\lambda}{2}\Rightarrow\lambda=\ldots\Rightarrow\mu=\ldots$	$1-\mu=\mu-\frac{1}{2}\Rightarrow\mu=\ldots\Rightarrow\lambda=\ldots$

	A1	For **either** $\lambda = \frac{1}{2}$ **or** $\mu = \frac{3}{4}$
	A1	For **both** $\lambda = \frac{1}{2}$ **and** $\mu = \frac{3}{4}$
	ALT	
	M1	For the correct vector statement $\overrightarrow{MX} = \overrightarrow{MO} + \overrightarrow{OX}$
	A1	For the correct vector (need not be simplified) $\overrightarrow{MX} = -\frac{\mathbf{c}}{2} - \mathbf{a} + \mu(\mathbf{a} + \mathbf{c})$
	B1ft	$\overrightarrow{MX} = \frac{\lambda}{2}(\mathbf{c} - \mathbf{a})$ ft their $\overrightarrow{MN} = '\frac{1}{2}(\mathbf{c} - \mathbf{a})'$
	M1	For equating the two vector statements for $\overrightarrow{MX}$ $-\frac{\mathbf{c}}{2} - \mathbf{a} + \mu(\mathbf{a} + \mathbf{c}) = \frac{\lambda}{2}(\mathbf{c} - \mathbf{a})$
	M1	For equating coefficients of $\mathbf{a}$ and $\mathbf{c}$ $-\frac{\mathbf{c}}{2} - \mathbf{a} + \mu(\mathbf{a} + \mathbf{c}) = \frac{\lambda}{2}(\mathbf{c} - \mathbf{a}) \Rightarrow \mathbf{c}\left(-\frac{1}{2} + \mu\right) + \mathbf{a}(\mu - 1) = \mathbf{c}\frac{\lambda}{2} - \mathbf{a}\frac{\lambda}{2}$ $\Rightarrow \frac{\lambda}{2} = \mu - \frac{1}{2}$ and $-\frac{\lambda}{2} = \mu - 1$
	M1	For attempting to solve their two simultaneous equations in terms of λ and μ. $\mu - \frac{1}{2} = -(\mu - 1) \Rightarrow \mu = \left(\frac{3}{4}\right)\ \frac{\lambda}{2} = 1 - '\frac{3}{4}' \Rightarrow \lambda = \left(\frac{1}{2}\right)$
	A1	For **either** $\lambda = \frac{1}{2}$ **or** $\mu = \frac{3}{4}$
	A1	For **both** $\lambda = \frac{1}{2}$ **and** $\mu = \frac{3}{4}$
(c)	M1	For area of $\Delta XBN = \frac{1}{8}\Delta OBC$ so $\frac{1}{8}$ of $\frac{1}{2}$ of the area of parallelogram *OABC* $\left(\begin{array}{l}\Delta OBC = \frac{1}{2} \times OB \times BC \times \sin \angle XBN \\ \Delta XBN = \frac{1}{2} \times \frac{1}{4} OB \times \frac{1}{2} BC \times \sin \angle XBN = \frac{1}{8}\Delta OBC\end{array}\right)$
	M1	Therefore Quadrilateral *OXNC* = $\frac{7}{8}$ of $\frac{1}{2}$ of the area of parallelogram *OABC* ft their fraction from the first M mark provided it is < $\frac{1}{2}$
	A1	Quadrilateral *OXNC* = $\frac{7}{16}$ of the area of parallelogram *OABC* so ratio is 7:16

Question number	Scheme	Marks
11 (a)	Let x = the length of the side of the triangle and h = the length of the prism	
	$\frac{1}{2}x^2 \sin 60\, h = 72$ or $\frac{1}{2}\left(\sqrt{\left(x^2 - \left(\frac{1}{2}x\right)^2\right)}\right)xh = 72$	M1
	$\frac{\sqrt{3}x^2h}{4} = 72$	M1
	$h = \frac{288}{\sqrt{3}x^2}$	A1
	$S = 2 \times \frac{1}{2}x^2 \sin 60 + 3xh$ or $2 \times \frac{1}{2}\left(\sqrt{\left(x^2 - \left(\frac{1}{2}x\right)^2\right)}\right)x + 3xh$	M1
	$S = \frac{\sqrt{3}x^2}{2} + 3x\left(\frac{288}{\sqrt{3}x^2}\right)$	M1
	$S = \frac{\sqrt{3}x^2}{2} + \frac{288\sqrt{3}}{x}$ *	A1 cso (6)
(b)	$\frac{dS}{dx} = \sqrt{3}x - \frac{288\sqrt{3}}{x^2} (= 0)$	M1
	$x^3 = 288$	dM1
	$x = \sqrt[3]{288} = 6.604$	A1
	$\frac{d^2S}{dx^2} = \sqrt{3} + \frac{576\sqrt{3}}{x^3}$	ddM1
	$\frac{d^2S}{dx^2} > 0$ (when x =6.6) $\therefore$ value is a minimum	A1 (5)
(c)	Substitutes their x into $S = \frac{\sqrt{3}x^2}{2} + \frac{288\sqrt{3}}{x}$	M1
	$S = 113$	A1 (2)
		[13]

Part	Mark	Additional Guidance
(a)	M1	For the correct expression for the volume of the prism in terms of x and h (or other letter for the length, e.g. l) Simplification not required for this mark $72 = \left(\frac{1}{2} \times x \times x \times \sin 60^\circ\right) \times h$ or $72 = \left(\frac{1}{2} \times x \times \sqrt{x^2 - \frac{x^2}{4}}\right) \times h$ or $72 = \left(\frac{\sqrt{3}}{4}x^2\right) \times h$
	M1	For an attempt to find an expression for h in terms of x Accept as a minimum $h = \frac{k}{x^2}$ where k is a positive integer
	A1	For $h = \frac{288}{\left(\sqrt{3}\right)x^2}$ or $h = \frac{96\sqrt{3}}{x^2}$

	M1	For an expression for S in terms of x and h (ft their area of the triangle) $S=2\left(\frac{1}{2}\times x\times x\times\sin 60^{\circ}\right)+3xh$ or $S=2\left(\frac{1}{2}\times x\times\sqrt{x^2-\frac{x^2}{4}}\right)+3xh$ $\left(S=\frac{\sqrt{3}}{2}x+3xh\right)$
	M1	For substituting their h into their S $S=2\left(\frac{1}{2}\times x\times x\times\sin 60^{\circ}\right)+\left(3x\times\frac{288}{\left(\sqrt{3}\right)x^2}\right)$ or $S=2\left(\frac{1}{2}\times x\times\sqrt{x^2-\frac{x^2}{4}}\right)+\left(3x\times\frac{288}{\left(\sqrt{3}\right)x^2}\right)$
	A1	For the correct expression for S as given. **The expression must be set equal to S.** $S=\frac{\sqrt{3}x^2}{2}+\frac{288\sqrt{3}}{x}$ exactly as seen here. * This is a given result so full working must be seen.
(b)	M1	For an attempt to differentiate the **given** expression for S $\frac{dS}{dx}=\sqrt{3}x-\frac{288\sqrt{3}}{x^2}$ or $\frac{dS}{dx}=\sqrt{3}x-288\sqrt{3}x^{-2}$ (See General Guidance for the definition of an attempt)
	dM1	For setting their differentiated expression = 0 and attempting to solve for x $\sqrt{3}x-\frac{288\sqrt{3}}{x^2}=0\Rightarrow x^3=288\Rightarrow x=(6.604)$ (rounded correctly) This mark is dependent on the first M mark in (b)
	A1	For $x = 6.604$ rounded correctly
	dM1	For attempting the second derivative (usual definition of an attempt) $\frac{d^2S}{dx^2}=\sqrt{3}+\frac{576\sqrt{3}}{x^3}$ This mark is dependent on first M mark in (b)
	A1ft	Concludes either that $\frac{d^2S}{dx^2}>0$ for all **positive** values of x **or** substitutes in their value of x to show that $\frac{d^2S}{dx^2}=5.19.....$ hence positive so must be a minimum. Only ft if the final conclusion is a minimum provided their $\frac{d^2S}{dx^2}$ is algebraically correct
	ALT – for justifying the minimum using their derivative	
	dM1	Chooses a value either side of their value of x and substituting them into their $\frac{dS}{dx}$ e.g. $x = 6$ and 7 $\frac{dS}{dx}=\sqrt{3}\times 6-\frac{288\sqrt{3}}{6^2}=-3.46...$ and $\frac{dS}{dx}=\sqrt{3}\times 7-\frac{288\sqrt{3}}{7^2}=1.944...$
	A1ft	Concludes that the gradient function moves from negative to positive hence must be a minimum.
(c)	M1	Substitutes their value of x into the given expression for S $S=\frac{\sqrt{3}\,'6.604'^2}{2}+\frac{288\sqrt{3}}{'6.604'}=...$
	A1	For $S = 113$ rounded correctly

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