Pearson Edexcel

Mark Scheme (Results)

January 2020

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 01R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks – can only be awarded when relevant M marks have been gained
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - cso – correct solution only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers may score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  If a candidate misreads a number from the question: eg. uses 252 instead of 255; follow through their working and deduct 2A marks from any gained provided the work has not been simplified. (Do not deduct any M marks gained.)

  If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used

  Examiners should send any instance of a suspected misread to review (but see above for simple misreads).

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = ...$

$(ax^2 + bx + c) = (mx + p)(nx + q)$ where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = ...$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x = ...$

3. Completing the square:

$x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$ leading to $x = ...$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n \to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n \to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication
from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the is rule may allow the mark to be awarded before the final answer is given.

MARK SCHEME

Question number	Scheme	Marks
1	$\dfrac{\left(a+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)}\times\dfrac{\left(2+\sqrt{3}\right)}{\left(2+\sqrt{3}\right)}=\dfrac{2a+\sqrt{3}(a+2)+3}{1}$	M1
	$2a+\sqrt{3}(a+2)+3=11+b\sqrt{3}\Rightarrow 11=2a+3,\ \ b=a+2$	M1M1
	Solves the equations in a and b $\Rightarrow a=4,\ \ b=6$	A1
	ALT	[4]
	$\dfrac{a+\sqrt{3}}{2-\sqrt{3}}=11+b\sqrt{3}\Rightarrow a+\sqrt{3}=\left(2-\sqrt{3}\right)\left(11+b\sqrt{3}\right)$	{M1}
	$\Rightarrow a+\sqrt{3}=(22-3b)+(2b-11)\sqrt{3}$	
	$\Rightarrow a=22-3b$ and $1=2b-11$	{M1}{M1}
	Solves the equations in a and b $\Rightarrow a=4,\ \ b=6$	{A1}
		[4]
		Total 4 marks

(a)	
M1	Multiply by $\dfrac{\left(2+\sqrt{3}\right)}{\left(2+\sqrt{3}\right)}$
M1	For either $11=2a+3$ **or** $b=a+2$
M1	For $11=2a+3$ **and** $b=a+2$
A1	$a=4,\ \ b=6$
ALT	
M1	Multiply by $2-\sqrt{3}$
M1	For either $a=22-3b$ **or** $1=2b-11$
M1	For $a=22-3b$ **and** $1=2b-11$
A1	$a=4,\ \ b=6$

Question number	Scheme	Marks
2 (a)	$7+4x-x^2=11-(x-2)^2$ [$a = 11$, $b = 1$, $c = -2$] **ALT** $7+4x-x^2=a-b(x^2+2cx+c^2)$ $a-bc^2=7 \quad b=1 \quad bc=4$ So $a = 11$, $b = 1$, $c = -2$ $7+4x-x^2=11-(x-2)^2$	M1A1A1 [3] {M1} {A1}{A1} [3]
(b)	(i) 11 (ii) 2	B1ft B1ft [2]
		Total 5 marks

(a)

M1 An attempt to factorise to make x^2 positive e.g. $-(x \pm p)^2 \pm q$

A1 Complete the square to obtain an expression in the form $-(x \pm 2)^2 \pm q$ **NB** Any expression in this form will score M1A1

A1 $11-(x-2)^2$ or $a = 11$, $b = 1$, $c = -2$

ALT

M1 Expands $a-b(x+c)$

A1 $a-bc^2=7 \quad b=1 \quad bc=4$

A1 $11-(x-2)^2$ or $a = 11$, $b = 1$, $c = -2$

(b) (i) **Mark parts b(i) and b(ii) together**

B1ft 11 follow through their a

(b) (ii)

B1ft 2 follow through their c

NB Answer of Max = (2, 11) score B1B1

Question number	Scheme	Marks
3(a)	$\frac{dy}{dx} = 2e^{2x}\left(x^2+1\right) + e^{2x}\left(2x\right)$	M1A1A1 [3]
(b)	When $x = 0$ $\frac{dy}{dx} = 2\times1\times1+1\times0 = 2$ $\quad y = e^{2\times0}\left(0+1\right) = 1$ $y-1 = 2\left(x-0\right) \Rightarrow y = 2x+1$	B1B1 B1 [3]
		Total 6 marks

(a)	
M1	Attempted use of the product rule. Sum of two terms (either way round) with $x^n \to x^{n-1}$ (Condone e^{2x} instead of $2e^{2x}$) Once the correct answer is seen ISW. This mark may be implied by the sum of two terms with one of the two terms correct.
A1	Either term correct
A1	Both terms correct
(b)	
B1	When $x = 0$ $\frac{dy}{dx} = 2$
B1	When $x = 0$ $y = 1$
B1	$y = 2x+1$

Question number	Scheme	Marks
4 (a)	$f(2) = 2\times 2^3 + a\times 2^2 + b\times 2 + 18 = 0$ $f'(x) = 6x^2 + 2ax + b \Rightarrow f'(2) = 6\times 2^2 + 2\times a\times 2 + b = 5$ $4a + 2b + 34 = 0$ $4a + b + 19 = 0$ $\Rightarrow b = -15,\ a = -1$	M1 M1M1 A1 M1A1 [6]
(b)	$x-2\overline{)2x^3 - x^2 - 15x + 18}$ $\quad 2x^2 + 3x - 9$ $2x^2 + 3x - 9 = (x+3)(2x-3)$ $\Rightarrow (x-2)(x+3)(2x-3)$	M1 M1A1 [3]
(c)	$x = 2,\ -3,\ \frac{3}{2}$	B2ft [2]
		Total 11 marks

(a)

M1 $f(2) = 0$ leading to an equation in a and b

M1 Attempt to differentiate

M1 $f'(2) = 5$ leading to an equation in a and b

A1 $4a + 2b + 34 = 0$ and $4a + b + 19 = 0$

M1 Solving simultaneously

A1 $b = -15,\ a = -1$

(b)

M1 Dividing by $x - 2$ to obtain a 3TQ

M1 Factorising the 3TQ

A1 All 3 terms correct

(c)

B2 ft $x = 2,\ -3,\ \frac{3}{2}$

(**B1** for 2 correct)

Question number	Scheme	Marks
5 (a)	$\log_4 32 = \dfrac{\log_2 32}{\log_2 4} = \dfrac{5}{2}*$ or $\log_4 32 = \log_4 4^{\frac{5}{2}} = \dfrac{5}{2}*$ or $\log_4 32 = \log_{2^2} 2^5 = \dfrac{5}{2}*$ **ALT** $\log_4 32 = a \quad \Rightarrow 4^a = 32 \quad \Rightarrow a = \dfrac{5}{2} \quad *$	M1A1cso [2] {M1}{A1} cso [2]
(b)	$\log_2 x - \log_4 32 + \dfrac{1}{4}\log_x 16 = 0$ Let $\log_2 x = y$ $y - \dfrac{5}{2} + \dfrac{1}{4}\left(\dfrac{\log_2 16}{\log_2 x}\right) = 0$ or $y - \dfrac{5}{2} + \dfrac{1}{\log_2 x} = 0$ $\Rightarrow y - \dfrac{5}{2} + \dfrac{1}{y} = 0$ $\Rightarrow 2y^2 - 5y + 2 = 0$ $\Rightarrow (2y-1)(y-2) = 0$ $\Rightarrow y = \log_2 x = \dfrac{1}{2}$ or 2 $\Rightarrow x = 2^{\frac{1}{2}} = \sqrt{2}$ and $x = 2^2 = 4$	 M1 M1A1 M1 M1 M1A1 [7]
		Total 9 marks

(a)	
M1	For $\log_4 32 = \dfrac{\log_2 32}{\log_2 4}$ or $\log_4 32 = \log_4 4^{\frac{5}{2}}$ or $\log_4 32 = \log_{2^2} 2^5$
ALT	
M1	For $4^a = 32$
A1 cso	Obtains the **given** answer with no errors in the working
(b)	
M1	Use of $\log_a x = \dfrac{\log_b x}{\log_b a}$ or $\log_a b = \dfrac{1}{\log_b a}$
M1	Forming a 3TQ
A1	$2y^2 - 5y + 2 = 0$
M1	Solving the 3TQ
M1	For $y = \log_2 x = \dfrac{1}{2}$ or 2
M1	Either $x = 2^{\frac{1}{2}} = \sqrt{2}$ or $x = 2^2 = 4$
A1	Both $x = 2^{\frac{1}{2}} = \sqrt{2}$ and $x = 2^2 = 4$

Question number	Scheme	Marks
6 (a)	<table><tr><td>x</td><td>0.5</td><td>1</td><td>1.5</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>y</td><td>−11.5</td><td>**-2**</td><td>**0.2**</td><td>1.3</td><td>2.7</td><td>**3.8**</td><td>**4.9**</td><td>5.9</td></tr></table>	B2 [2]
(b)	Points plotted within half of a square Points joined together in a smooth curve	B1ft B1ft [2]
(c)	$\frac{x^3-3}{x^2}=ax+b\Rightarrow x^3-3=ax^3+bx^2\Rightarrow 0=x^3(a-1)+bx^2+3$ Comparing coefficients $x^3(a-1)-bx^2+3=2x^3+6x^2+3$ $\Rightarrow a=3, b=-6$ so line required is $y=3x-6$ Draws the line $y=3x-6$ and identifies two intersections with the curve when $x=0.8/0.9$ and $x=2.8/2.9$	M1 M1A1 M1 A1 (both) [5]
	ALT	
	$2x^3-6x^2+3=0\Rightarrow 2x-6=-\frac{3}{x^2}$	{M1}
	$3x-6=x-\frac{3}{x^2}$ so line required is $y=3x-6$	{M1}{A1}
	Draws the line $y=3x-6$ and identifies two intersections with the curve when $x=0.8/0.9$ and $x=2.8/2.9$	{M1} {A1} both [5]
		Total 9 marks

(a)	
B2	All 4 points correct (**B1** for 3 points correct)
(b)	
B1ft	Points plotted ft their table allow half a square tolerance
B1ft	Points joined together with a smooth curve ft their table
(c)	
M1	Setting $x-\frac{3}{x^2}=ax+b$ and simplifying to $x^3\left(a-1\right)+bx^2+3$
M1	Comparing coefficients
A1	Identifying that the line required is $y=3x-6$
M1	$y=3x-6$ drawn intersecting the curve in two places
A1	$x=0.8/0.9$ and $x=2.8/2.9$
ALT	
M1	Subtracting 3 from both sides and dividing by x^2
M1	Adding x to both sides
A1	Identifying that the line required is $y=3x-6$
M1	$y=3x-6$ drawn intersecting the curve in two places
A1	$x=0.8/0.9$ and $x=2.8/2.9$

Question number	Scheme	Marks
7 (a)(i)	$a+4d=4x+6$ $a+7d=7x+3$ $\Rightarrow 3d=3x-3\Rightarrow d=x-1$*	M1A1cso
(ii)	$a+7(x-1)=7x+3\Rightarrow a=10$ or $a+4(x-1)=4x+6\Rightarrow a=10$	M1A1 [4]
(b)	$42=10+8(x-1)\Rightarrow x=5$	M1A1 [2]
(c)	$d=x-1\Rightarrow d=5-1=4$ $S_{n+1}=12U_n+18\Rightarrow \frac{n+1}{2}\left(2\times 10+\left[(n+1)-1\right]4\right)=12\left[10+(n-1)4\right]+18$ $\Rightarrow n^2-18n-40=0$ $\Rightarrow (n-20)(n+2)=0\Rightarrow n=20$	B1 M1 M1 M1A1 [5]
		Total 11 marks

(a) (i)
M1 $a+4d=4x+6$ and $a+7d=7x+3$
A1 cso Obtains the **given** answer with no errors in the working
(a) (ii)
M1 Substitution of $d=x-1$
A1 $a=10$
(b)
M1 Use of $a+8d=42$
A1 $x=5$
(c)
B1 $d=4$
M1 Use of $\frac{n}{2}(2a+(n-1)d)$
M1 Simplifying to $n^2-18n-40=0$
M1 Solving the 3TQ
A1 $n=20$ if shown must reject $n=-2$

Question number	Scheme	Marks
8	$s = \int \left(3 + 5t - 2t^2\right) \mathrm{d}t = 3t + \frac{5t^2}{2} - \frac{2t^3}{3} + c$	M1
	when $t = 0$ $s = 5$	
	$5 = 0 + 0 - 0 + c$	B1
	$s = 5 + 3t + \frac{5t^2}{2} - \frac{2t^3}{3}$	A1
	When $s = x$	
	$\frac{\mathrm{d}x}{\mathrm{d}t} = 3 + 5t - 2t^2 = 0 \Rightarrow (2t+1)(t-3) = 0 \Rightarrow t = 3$	M1A1
	$\Rightarrow x = 5 + 3\times 3 + \frac{5\times 3^2}{2} - \frac{2\times 3^3}{3} = \frac{37}{2}$ oe	A1
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 5 - 4t$ when $t = 3$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -7 \Rightarrow$ max	M1A1
		[8]
		Total 8 marks

M1	Attempt to integrate
B1	$c = 5$
A1	$s = 5 + 3t + \frac{5t^2}{2} - \frac{2t^3}{3}$
M1	Solving $3 + 5t - 2t^2 = 0$
A1	$t = 3$ if shown must reject $t = -\frac{1}{2}$
A1	$x = \frac{37}{2}$ oe
M1	Differentiates to obtain $\left(\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \right) 5 - 4t$
A1	Establish that the maximum has been obtained and give a conclusion

Question number	Scheme	Marks
9 (a)	$a=-1,\ b=-2$	B1,B1 [2]
(b)	Gradient of $l_1=-2,\Rightarrow$ Gradient of $l_2=\dfrac{1}{2}$	B1,B1
	$180=(x+1)^2+(y-6)^2$	M1
	$\dfrac{1}{2}=\dfrac{y-6}{x+1}\Rightarrow x=2y-13$	M1
	Solves simultaneous equations; $180=([2y-13]+1)^2+(y-6)^2\Rightarrow 0=5y^2-60y$ or $\quad 180=(x+1)^2+\left(\dfrac{1}{2}x+\dfrac{13}{2}-6\right)^2\Rightarrow 0=x^2+2x-143=0$	M1M1
	$y=0,\ y=12\Rightarrow x=-13,\ x=11\quad$ or $\quad x=-13,\ x=11\Rightarrow y=0,\ y=12$ Coordinates are $(-13,\ 0)$ and $(11,\ 12)$	A1A1 [8]
(c)	Area of triangle PQR $PQ=\sqrt{(6+2)^2+(-1-3)^2}=4\sqrt{5}$	M1
	Area $=\dfrac{1}{2}\times 4\sqrt{5}\times 6\sqrt{5}=60$ (units)^2	M1A1 [3]
	ALT Area $=\dfrac{1}{2}\begin{pmatrix}-13 & -1 & 3 & -13\\ 0 & 6 & -2 & 0\end{pmatrix}=\dfrac{1}{2}(-78+2+0-0-18-26)=-60$	{M1}{M1}
	$\Rightarrow 60$ (units)^2	{A1} [3]
(d)	Coordinates of R required are $(-13,\ 0)$	
	$\angle RPQ=90^\circ$ so RQ is a diameter	
	$\left(\dfrac{-13+3}{2},\dfrac{0-2}{2}\right)\Rightarrow(-5,\ -1)$	M1A1 [2]
		Total 15 marks
(a) **B1** **B1**	$a=-1$ $b=-2$	

(b)	
B1	Gradient of $l_1 = -2$
B1	Gradient of $l_2 = \frac{1}{2}$
M1	Use of $PR = 6\sqrt{5}$ to obtain an equation
M1	Use of gradient of the perpendicular to obtain an equation
M1	Solves simultaneously
M1	Simplifies to $5y^2 - 60y = 0$ or $x^2 + 2x - 143 = 0$
A1	All 4 values identified 0, 12, –13, 11
A1	$(11,\ 12)$ and $(-13,\ 0)$ (must be paired correctly and if written as a coordinate then must be in the correct order)
(c)	
M1	$PQ = 4\sqrt{5}$
M1	Use of Area $= \frac{1}{2} \times PQ \times PR$
A1	60 (units)^2
ALT	
M1	Use of Area $= \frac{1}{2}\begin{pmatrix} -13 & -1 & 3 & -13 \\ 0 & 6 & -2 & 0 \end{pmatrix}$ ft R provided $e < 0$
M1	$\frac{1}{2}(-78 + 2 + 0 - 0 - 18 - 26)$ ft R provided $e < 0$
A1	60 (units)^2
(d)	
M1	$\left(\frac{-13+3}{2}, \frac{0-2}{2}\right)$ ft R provided $e < 0$
A1	$(-5,\ -1)$

Question number	Scheme	Marks
10 (a)	$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -(2\mathbf{a} - \mathbf{b}) + 3\mathbf{a} + \mathbf{b} = \mathbf{a} + 2\mathbf{b}$	M1A1 [2]
(b)	$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = 3\mathbf{a} + \mathbf{b} - \mathbf{a} + 3\mathbf{b} = 2\mathbf{a} + 4\mathbf{b} = 2(\mathbf{a} + 2\mathbf{b})$ Conclusion required; same direction and $\overrightarrow{OC}$ is a multiple of $\overrightarrow{AB}$ therefore $\overrightarrow{OC}$ is parallel to $\overrightarrow{AB}$.	M1 A1 [2]
(c)	$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \mathbf{a} + 2\mathbf{b} + (-\mathbf{a} + 3\mathbf{b}) = 5\mathbf{b}$ $\overrightarrow{AX} = \mu\overrightarrow{AC} = \mu 5\mathbf{b}$ $\overrightarrow{AX} = \overrightarrow{AO} + \overrightarrow{OX} = -(2\mathbf{a} - \mathbf{b}) + \lambda(3\mathbf{a} + \mathbf{b})$ $\Rightarrow \mu 5\mathbf{b} = -(2\mathbf{a} - \mathbf{b}) + \lambda(3\mathbf{a} + \mathbf{b})$ $\Rightarrow -2 + 3\lambda = 0 \Rightarrow \lambda = \frac{2}{3}$ $\Rightarrow 5\mu = 1 + \lambda \Rightarrow \mu = \frac{1}{3}$ $\Rightarrow AX : XC = 1 : 2$	B1 B1 M1 M1 M1 A1 A1 [7]
		Total 11 marks

(a)

M1 Use of $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

A1 $\mathbf{a} + 2\mathbf{b}$

(b)

M1 Use of $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$

A1 Correct conclusion i.e. $\overrightarrow{OC} = 2\overrightarrow{AB}$ $\therefore \overrightarrow{OC}$ is parallel to $\overrightarrow{AB}$

(c)

B1 $\overrightarrow{AC} = 5\mathbf{b}$ may be implied by 2nd B1

B1 $\overrightarrow{AX} = \mu 5\mathbf{b}$ or $\overrightarrow{XC} = \lambda 5\mathbf{b}$

M1 A correct vector for $\overrightarrow{AX}$ or $\overrightarrow{OX}$ or $\overrightarrow{BX}$ or $\overrightarrow{CX}$ or $\overrightarrow{BC}$ including an unknown multiple of a vector e.g. $\overrightarrow{AX} = -(2\mathbf{a} - \mathbf{b}) + \lambda(3\mathbf{a} + \mathbf{b})$

M1 Equate 2 forms of the same vector e.g. $\mu 5\mathbf{b} = -(2\mathbf{a} - \mathbf{b}) + \lambda(3\mathbf{a} + \mathbf{b})$

M1 Comparing coefficients

A1 $\lambda = \frac{2}{3}$ or $\mu = \frac{1}{3}$

A1 $AX : XC = 1 : 2$

Question number	Scheme	Marks
11 (a)	at P $\quad b=\sqrt{a-2} \Rightarrow b^2=a-2$*	M1A1cso [2]
(b)	At P $y=b \Rightarrow y^2=b^2 \Rightarrow y^2=a-2$	B1
	$V=\pi\int_a^{16}\left(\sqrt{x-2}\right)^2\,\mathrm{d}x-\pi\int_a^{16}(a-2)\,\mathrm{d}x=\pi\int_a^{16}(x-2)\,\mathrm{d}x-\pi\int_a^{16}(a-2)\,\mathrm{d}x$	M1
	$\Rightarrow \pi\int_a^{16}(x-a)\,\mathrm{d}x$ or $\pi\int_a^{16}\left(\sqrt{x-2}\right)^2\,\mathrm{d}x-\pi(a-2)(16-a)$	
	$50\pi=\pi\left[\frac{x^2}{2}-ax\right]_a^{16}$ or $50\pi=\pi\left[\frac{x^2}{2}-2x\right]_a^{16}-\pi(a-2)(16-a)$	depM1A1
	$50\pi=\pi\left[\left(\frac{256}{2}-16a\right)-\left(\frac{a^2}{2}-a^2\right)\right]$	depM1
	or $50\pi=\pi\left[\left(96-\frac{a^2}{2}+2a\right)-\left(18a-a^2-32\right)\right]$	
	$\Rightarrow a^2-32a+156=0$	M1
	$\Rightarrow (a-6)(a-26)=0 \Rightarrow a<16$ so $a=6$	
	$b^2=a-2 \Rightarrow b^2=4 \Rightarrow b=2$	M1A1 A1 [9]
		Total 11 marks

(a)	
M1	$b=\sqrt{a-2}$
A1 cso	Obtains the **given** answer with no errors in the working
(b)	
B1	$y^2=a-2$
M1	Use of $V=\pi\int_a^{16}\left(\sqrt{x-2}\right)^2\,\mathrm{d}x-\pi\int_a^{16}(a-2)\,\mathrm{d}x$ or $V=\pi\int_a^{16}(x-a)\,\mathrm{d}x$ or $\pi\int_a^{16}\left(\sqrt{x-2}\right)^2\,\mathrm{d}x-\pi(a-2)(16-a)$ Ignore limits
depM1	Attempts to integrate. Ignore limits (Dependent on previous M1)
A1	Correct integration. Ignore limits
depM1	Correct substitution of limits (Dependent on previous M1)
M1	Obtaining the 3TQ
M1	Solving the 3TQ
A1	$a=6$
A1	$b=2$

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