

Please check the examination details below before entering your candidate information

Candidate surname

Other names

**Pearson Edexcel  
International GCSE**

Centre Number

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Candidate Number

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**Thursday 20 June 2019**

Morning (Time: 2 hours)

Paper Reference **4PM1/02**

**Further Pure Mathematics**

**Paper 2**



**Calculators may be used.**

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.  
Anything you write on the formulae page will gain NO credit.

### Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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## International GCSE in Further Pure Mathematics Formulae sheet

### Mensuration

Surface area of sphere =  $4\pi r^2$

Curved surface area of cone =  $\pi r \times$  slant height

Volume of sphere =  $\frac{4}{3}\pi r^3$

### Series

#### Arithmetic series

Sum to  $n$  terms,  $S_n = \frac{n}{2}[2a + (n - 1)d]$

#### Geometric series

Sum to  $n$  terms,  $S_n = \frac{a(1 - r^n)}{(1 - r)}$

Sum to infinity,  $S_\infty = \frac{a}{1 - r}$   $|r| < 1$

#### Binomial series

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$$

### Calculus

#### Quotient rule (differentiation)

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

### Trigonometry

#### Cosine rule

In triangle  $ABC$ :  $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

### Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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- 2 Oil is leaking from a pipe and forms a circular pool on a horizontal surface. The area of the surface of the pool is increasing at a constant rate of  $8 \text{ cm}^2/\text{s}$ . Find, in  $\text{cm/s}$  to 3 significant figures, the rate at which the radius of the pool is increasing when the area of the pool is  $50 \text{ cm}^2$

(6)

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**Question 2 continued**

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**(Total for Question 2 is 6 marks)**





**Question 3 continued**

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**(Total for Question 3 is 6 marks)**



4 In triangle  $ABC$ ,  $AB = 5x$  cm,  $BC = (3x - 1)$  cm,  $AC = (2x + 5)$  cm and angle  $ABC = 60^\circ$

Find, to 3 significant figures, the value of  $x$ .

(5)

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**Question 4 continued**

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**(Total for Question 4 is 5 marks)**



5 Use algebra to solve the equations

$$xy = 36$$

$$xy + x + 2y = 53$$

(6)

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**Question 5 continued**

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**(Total for Question 5 is 6 marks)**



6 (a) Given that  $y = (4x - 3)e^{2x}$

(i) find  $\frac{dy}{dx}$  (3)

(ii) show that  $(4x - 3)\frac{dy}{dx} = (8x - 2)y$  (2)

(b) Differentiate  $\frac{\sin 5x}{(x - 3)^2}$  with respect to  $x$  (3)

Dotted lines for writing answers.



**Question 6 continued**

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**(Total for Question 6 is 8 marks)**



7 The sum of the first  $n$  terms of an arithmetic series is  $A_n$  where

$$A_n = \sum_{r=1}^n (4r + 5)$$

(a) For this arithmetic series, find

- (i) the first term,
- (ii) the common difference.

(2)

The sum of the first  $n$  terms of a geometric series is  $G_n$  where

$$G_n = \sum_{r=1}^n 4(3)^{r-1}$$

(b) For this geometric series, find

- (i) the first term,
- (ii) the common ratio.

(2)

(c) Find the value of  $n$  for which  $A_{14} - 6 = G_n$

(5)

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**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

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**(Total for Question 7 is 9 marks)**



8 The point  $A$  has coordinates  $(2, 6)$ , the point  $B$  has coordinates  $(6, 8)$  and the point  $C$  has coordinates  $(4, 2)$ .

(a) Find the exact length of

- (i)  $AB$       (ii)  $BC$       (iii)  $AC$  (4)

(b) Find the size of each angle of triangle  $ABC$  in degrees. (3)

The points  $A, B$  and  $C$  lie on a circle with centre  $P$ .

(c) Find the coordinates of  $P$ . (2)

(d) Find the exact length of the radius of the circle in the form  $\sqrt{a}$ , where  $a$  is an integer. (2)

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**Question 8 continued**

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**Question 8 continued**

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**Question 8 continued**

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**(Total for Question 8 is 11 marks)**



9 The curve  $C$ , with equation  $y = f(x)$ , passes through the point with coordinates  $\left(-2, -\frac{28}{3}\right)$

Given that  $f'(x) = x^3 - x^2 - 4x + 4$

(a) show that  $C$  passes through the origin. (4)

(b) (i) Show that  $C$  has a minimum point at  $x = 2$  and a maximum point at  $x = 1$

(ii) Find the exact value of the  $y$  coordinate at each of these points. (7)

The curve has another turning point at  $A$ .

(c) (i) Find the coordinates of  $A$ .

(ii) Determine the nature of this turning point. (3)

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**Question 9 continued**

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**Question 9 continued**

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**Question 9 continued**

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**(Total for Question 9 is 14 marks)**



10 The roots of the equation  $x^2 + 3x - 5 = 0$  are  $\alpha$  and  $\beta$ .

(a) Without solving the equation, find

(i) the value of  $\alpha^2 + \beta^2$

(ii) the value of  $\alpha^4 + \beta^4$

(5)

Given that  $\alpha > \beta$  and without solving the equation

(b) show that  $\alpha - \beta = \sqrt{29}$

(2)

(c) Factorise  $\alpha^4 - \beta^4$  completely.

(3)

(d) Hence find the exact value of  $\alpha^4 - \beta^4$

(2)

Given that  $\beta^4 = p + q\sqrt{29}$  where  $p$  and  $q$  are positive constants

(e) find the value of  $p$  and the value of  $q$ .

(3)

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**Question 10 continued**

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**Question 10 continued**

Handwriting practice area with 25 horizontal dotted lines.

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**Question 10 continued**

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**(Total for Question 10 is 15 marks)**



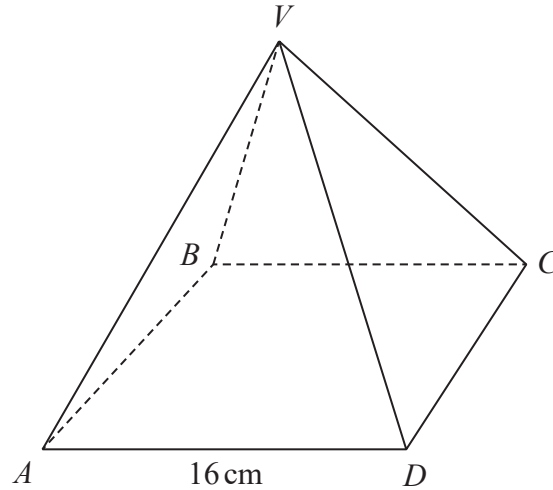


Diagram NOT accurately drawn

Figure 1

Figure 1 shows a right pyramid with vertex  $V$  and square base,  $ABCD$ , of side 16 cm.

The size of angle  $AVC$  is  $90^\circ$

(a) Show that the height of the pyramid is  $8\sqrt{2}$  cm. (4)

(b) Find, in cm, the length of  $VA$ . (3)

(c) Find, in cm, the exact length of the perpendicular from  $D$  onto  $VA$ . (3)

Find, in degrees to one decimal place, the size of

(d) the angle between the plane  $VAB$  and the base  $ABCD$ , (3)

(e) the obtuse angle between the plane  $VAB$  and the plane  $VAD$ . (3)

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(Total for Question 11 is 16 marks)

**TOTAL FOR PAPER IS 100 MARKS**

