Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel International GCSE In Further Pure Mathematics (4PM1) Paper 02

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2019
Publications Code 4PM1_02_2019_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent

- indep – independent
- awrt – answer which rounds to
- eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified.
- If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used.
- Examiners should send any instance of a suspected misread to review. If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes it clear the method has been used.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
- It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
- Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$(x^2+bx+c)=(x+p)(x+q), \text{ where } |pq|=|c| \quad \text{leading to } x=\ldots.$$

$$(ax^2+bx+c)=(mx+p)(nx+q) \text{ where } |pq|=|c| \text{ and } |mn|=|a| \quad \text{leading to } x=\ldots.$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=\ldots.$

3. Completing the square:

$$x^2+bx+c=0:\ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0 \qquad \text{leading to } x=\ldots.$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the is rule may allow the mark to be awarded before the final answer is given.

June 2019
4PM1 Paper 2
Mark Scheme

Question Number	Scheme	Marks
1(a)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}, = -3\mathbf{i} + 4\mathbf{j}$	M1,A1 (2)
(b)	$\left\lvert\overrightarrow{AB}\right\rvert = 5$	
	Unit vector $= \frac{1}{5}(-3\mathbf{i} + 4\mathbf{j})$ or $-\frac{1}{5}(-3\mathbf{i} + 4\mathbf{j})$ oe Accept column vectors	M1A1 (2)
		[4]
(a)		
M1	For $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ seen, or $\overrightarrow{AB} = (\mathbf{i} + 7\mathbf{j}) - (4\mathbf{i} + 3\mathbf{j})$ or equivalent in column form	
A1	Correct simplified answer as shown or equivalent but NOT a column vector	
(b)		
M1	Correct modulus of their $\overrightarrow{AB}$ **and** divide $\pm$ their $\overrightarrow{AB}$ by it Correct unit vector, as shown or equivalent inc column vector	
A1	$\pm\frac{1}{5}(-3\mathbf{i} + 4\mathbf{j})$ scores M1A0 $\frac{1}{5} - 3\mathbf{i} + 4\mathbf{j}$ scores M1A0	
NB:	If $\overrightarrow{BA}$ is found in (a) both (b) marks are still available	

Question Number	Scheme	Marks
2	$\frac{\mathrm{d}A}{\mathrm{d}t}=8$	B1
	$\frac{\mathrm{d}A}{\mathrm{d}r}=2\pi r$	M1
	$A=50 \quad r=\sqrt{\frac{50}{\pi}} \ (3.989...)$	M1
	$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\mathrm{d}r}{\mathrm{d}A}\times\frac{\mathrm{d}A}{\mathrm{d}t},=\frac{1}{2\pi\sqrt{\frac{50}{\pi}}}\times 8,=0.319\ (\text{cm/s})$	M1,A1ft,A1 [6]
NB	For either method, accept *A* or *S* for area, *r* for radius. Any other letters used for area and/or radius must be defined.	
B1	$\frac{\mathrm{d}A}{\mathrm{d}t}=8$ seen explicitly or used	
M1	Attempt to differentiate πr^2 to obtain $\frac{\mathrm{d}A}{\mathrm{d}r}$ Power of *r* must decrease	
M1	Attempt to obtain *r* when A = 50 cm^2 (ie solve $50=\pi r^2$)	
M1	For a correct, useful, chain rule. Derivatives can appear in any order	
A1ft	Substitute their known quantities and rearrange to $\frac{\mathrm{d}r}{\mathrm{d}t}=...$ if not in this form already. **All 3 M marks needed**	
A1	Correct answer, **must be 3 sf**	
ALT	$\frac{\mathrm{d}A}{\mathrm{d}t}=8$	B1
	$r=\sqrt{\frac{a}{\pi}}$ oe	M1
	$\frac{\mathrm{d}r}{\mathrm{d}A}=\frac{1}{2\sqrt{\pi}}A^{-\frac{1}{2}}$	M1
	$\frac{\mathrm{d}A}{\mathrm{d}t}\times\frac{\mathrm{d}r}{\mathrm{d}A}=\frac{\mathrm{d}r}{\mathrm{d}t}$	M1
	$=8\times\frac{1}{2\sqrt{\pi}}A^{-\frac{1}{2}}=8\times\frac{1}{2\sqrt{\pi}}\times\frac{1}{\sqrt{50}}$	A1ft
	= 0.3191…= 0.319 (cm/s)	A1
B1	$\frac{\mathrm{d}A}{\mathrm{d}t}=8$ seen explicitly or used	
M1	Attempt to find *r* in terms of *A*	
M1	Attempt to differentiate their expression for *r* to obtain $\frac{\mathrm{d}r}{\mathrm{d}A}$ power of *A* must decrease	
M1	For a correct, useful, chain rule. Derivatives can appear in any order	
A1ft	Substitute their known quantities and rearrange to $\frac{\mathrm{d}r}{\mathrm{d}t}=...$ if not in this form already. All 3 M marks needed	
A1	Correct answer, **must be 3 sf**	

Question Number	Scheme	Marks
3(a)	$\frac{dv}{dt} = 2t - 4$	M1
	$\text{Accel} = 2 \ \left(\text{m/s}^2\right)$	A1 (2)
(b)	$s = \int_0^6 \left(t^2 - 4t + 7\right) dt = \left[\frac{t^3}{3} - 2t^2 + 7t\right]_0^6$	M1A1
	$= \frac{6^3}{3} - 2\times 6^2 + 7\times 6 = 42 \ (\text{m})$	dM1 A1cao (4) [6]
(a) M1 A1 (b) M1 A1 dM1 A1cao NB	Attempt to differentiate the expression for v. Power of t to decrease in at least one term and increase in none. Substitute $t = 3$ and obtain correct acceleration – units may be missing Attempt to integrate the expression for v. Power of t to increase in at least one term and decrease in none. Ignore limits if shown. Constant not needed for indefinite integration. Correct integration. Limits/constant not needed. **Either** substitute the limits 0 and 6 **or** use $s = 0$, $t = 0$ to obtain a value for the constant and substitute $t = 6$ in the complete expression. (Substitution of 0 can be implied if the result would have been 0) Depends on the previous M mark If more values of t are substituted **and results used** award M0 $S = 42$ (m) Ans 42 w/o working scores 4/4 (Done on a calculator)	
4	$(2x+5)^2 = (3x-1)^2 + (5x)^2 - 2\times(3x-1)\times 5x\cos 60°$	M1A1
	$15x^2 - 21x - 24 (= 0) \quad \left(5x^2 - 7x - 8 = 0\right)$	A1
	$x = \frac{21 \pm \sqrt{21^2 + 4\times 15\times 24}}{30}$	M1
	$x = 2.1456......\ (\text{or } -0.7456...)$	
	$\therefore x = 2.15$	A1 **[5]**
M1 A1 A1 M1 A1cao	Use the cosine rule in either form. Rule to be correct either by quoting and using the general formula or by implication from a correct substitution. Correct substitution in their cosine rule. Simplify to obtain a 3TQ. Terms in any order. $= 0$ may be missing Solve their 3TQ by formula (correct general formula or correct substitution for their equation) or completing the square. Reach a positive value for x. Negative need not be seen. **Calculator solutions:** Correct answer from correct equation scores M1A1, otherwise M0A0 Correct value for x. **Must** be 3 sf Negative value (if shown) must be eliminated or positive clearly identified as the required value..	

Question Number	Scheme	Marks
5	$(x+2y=17)\quad x=\frac{36}{y}\quad \left(\text{or } y=\frac{36}{x}\right)$	M1
	$\frac{36}{y}+2y=17,\quad 36+2y^2=17y\quad \left(\text{or } 72+x^2=17x\right)$	M1
	$2y^2-17y+36\ \ (=0)\qquad \left(\text{or } x^2-17x+72=0\right)$	A1
	$(y-4)(2y-9)=0\qquad \left(\text{or } (x-8)(x-9)\right)$	
	$y=4\quad x=9$	dM1A1
	$y=4\frac{1}{2}\quad x=8$	A1 (6)
M1 M1 A1 M1 A1 A1	Rearrange $xy=36$ to $x=\ldots$ or $y=\ldots$ Eliminate x or y from the linear equation and obtain a 3TQ, $=0$ not needed Correct 3TQ, terms in any order. $=0$ not needed Solve their 3TQ by any valid method. Obtain at least one value for y or x **Either** 2 correct values for x or y **or** a correct (x, y) pair Both pairs correct and pairing clear.	
ALT:	The following method may possibly be seen: $xy+x+2y=53 \Rightarrow 36+x+2y=53 \Rightarrow x+2y=17$ and $xy=36$ or $x\times 2y=72$	M1
	Hence x and $2y$ are the roots of the equation $z^2-17z+72=0$	M1A1
	$(z-9)(z-8)=0 \Rightarrow z=9$ or 8	M1
	So $x=8\quad y=4.5$ or $x=9\quad y=4$	A1A1 [6]
M1 M1 A1 M1 A1 A1	Substitute $xy=36$ in the linear equation to obtain $x+2y=17$ and $xy=36$ oe Obtain a 3TQ with roots x and $2y$ Correct 3TQ Solve their 3TQ by any valid method. Obtain at least one value for *for the roots* **Either** 2 correct values for x or y **or** a correct (x, y) pair Both pairs correct and pairing clear.	
	Special Case $x+2y=17\quad xy=36$ Use $xy=36$ in the other equation to obtain $x+2y=17$ $\Rightarrow\ x=9\quad y=4$ By inspection: Score M1M0A0M1A1A0 (Must see $x+2y=17$; otherwise no marks) If the second answer is also obtained correctly by inspection, award all marks	

Question Number	Scheme	Marks
6(a)(i)	$\frac{dy}{dx} = 4e^{2x} + 2(4x-3)e^{2x}$	M1A1A1 (3)
(ii)	$(4x-3)\frac{dy}{dx} = (4x-3)(8x-2)e^{2x} = (8x-2)y$ *	M1A1cso (2)
(b)	$\frac{dy}{dx} = \frac{5\cos 5x \times (x-3)^2 - \sin 5x \times 2(x-3)}{(x-3)^4}$	M1A1A1 (3)
ALT	Using product rule:	
	$y = (x-3)^{-2}\sin 5x$	M1
	$\frac{dy}{dx} = -2(x-3)^{-3}\sin 5x + 5(x-3)^{-2}\cos 5x$	A1A1
		[8]

(a)(i)	
M1	Use product rule to differentiate the given expression. Must have 2 terms added. One to be of the form ke^{2x} and the other of the form $k'(4x-3)e^{2x}$ where k' = 1 or 2
A1	Either term correct
A1	Second term correct
NB	No simplification needed for these 3 marks
ALT	$y = 4xe^{2x} - 3e^{2x} \Rightarrow \frac{dy}{dx} = 4e^{2x} + 8xe^{2x} - 6e^{2x}$
	M1 Expand the given expression and differentiate using the product rule for $4xe^{2x}$ A1 Any 2 terms correct; A1 Third term correct.
NB	No simplification needed for these 3 marks
(ii)	
M1	Use their result from (i) to obtain an expression for $(4x-3)\frac{dy}{dx}$. No need to simplify.
A1cso	Correct **given** result obtained with no errors in the working. Can start with LHS and show equal to the RHS or vice versa or can start with each side and "meet in the middle"
(b)	
M1	Attempt the quotient rule. The denominator must be $(x-3)^4$ and the numerator must be of the form $\left(k\cos 5x \times (x-3)^2 - \sin 5x \times l(x-3)\right)$ $k = \pm 5$ or ± 1, $l = 1$ or 2 (ie sine may have been differentiated to − cosine)
A1	One fully correct term in numerator.
A1	All fully correct.
ALT	
M1	Rewrite without a quotient and apply the product rule obtaining 2 terms of the form shown
A1	Either term correct
A1	Second term correct No need to simplify

Question Number	Scheme	Marks
7(a)	(i) $a=9$	B1
	(ii) $d=4$	B1 (2)
(b)	(i) $a=4$	B1
	(ii) $r=3$	B1 (2)
(c)	$A_{14}=\frac{14}{2}(2\times9+13\times4)$ or $\frac{14}{2}(9+61)$, $=490$	M1, A1
	$"490"-6=\frac{4(3^n-1)}{3-1}$	M1
	$3^n=243 \quad n=5$	ddM1A1 (5) [9]

(a)	
B1	Correct value, no working or explanation needed
B1	Correct value, no working or explanation needed
(b)	
B1	Correct value, no working or explanation needed
B1	Correct value, no working or explanation needed
(c)	
M1	Use either formula for the sum of an arithmetic series with their *a* and *d* (if needed) and obtain a value for the sum of the first 14 terms
A1	Correct value for the sum
M1	Subtract 6 from their sum (explicitly or implicitly) and equate to the sum of the first *n* terms of the geometric series obtained using their *a* and *r*
ddM1	Solve their equation by a correct method. No method need be shown but must reach $n=$ ... Depends on both M marks above
A1	Correct value for *n* obtained
ALT	For the last 3 marks:
M1	Subtract 6 from their sum and generate at least the first 5 terms of the geometric series.
ddM1	Sum their terms until at least “484” is reached
A1	Correct answer (5) obtained from correct work.

Question Number	Scheme	Marks
8(a)	$AB^2 = 4^2 + 2^2,\ BC^2 = 2^2 + 6^2,\ AC^2 = 2^2 + 4^2$	M1 (any one)
	(i) $AB = \sqrt{20}$ (ii) $BC = \sqrt{40}$ (iii) $AC = \sqrt{20}$ or equivalents (4.47) (6.32) (4.47)	A1A1A1 (4)
(b)	Any complete method for finding one of the angles: eg $AB^2 + AC^2 = BC^2 \Rightarrow \angle A = 90°$ or use trigonometry	M1
	$\angle A = 90°, \angle B = \angle C = 45°$	A1, A1 (3)
I	(centre at midpoint of BC) $(5,5)$	M1A1 (2)
(d)	Radius $= \frac{1}{2}BC = \frac{1}{2}\sqrt{40} = \sqrt{10}$ (Working for (d) may be seen in a previous part)	M1A1 (2) [11]

(a) M1	Use Pythagoras with a plus sign to obtain AC^2, BC^2 or AC^2. If the answer is incorrect it must be clear that the correct coordinates have been used correctly.
A1A1A1	Award A1 for each correct length. Ignore labels (i), (ii) and (iii). Award M1A1A1A1 / M1A1A1A0 / M1A1A0A0 as appropriate.
	If there is no working shown but at least one length is correct, award M1 and deduct one A mark for each incorrect length. (no length correct and no working $\Rightarrow$ M0)
SC:	If all 3 lengths are correct to at least 3 sf, award M1A1A1A0 If 2 are correct to at least 3 sf, award M1A1A0A0
(b) M1	Attempt to obtain any of the required angles. Method must be complete (ie reach a value for one angle) and formula used must be correct and values must be substituted into a correct formula.
A1	$\angle A = 90°$ Any labelling given can be ignored.
A1	$\angle B = \angle C = 45°$ All 3 correct w/o working scores M1A1A1
(c) M1	For indicating that the centre is at the midpoint of BC. This can be stated explicitly or used by attempting to find the midpoint. **OR:** Find equations for perpendicular bisectors of 2 of the sides and find the point of intersection
A1	Both coordinates correct. Correct answer written down w/o working scores M1A1
(d) M1	For indicating that the radius is half the length of BC. This can be stated explicitly or used by attempting to find half of their BC (not nec in the required form).
A1	Correct length of the radius, in the required form.
NB	If half the length of BC has been found earlier the marks for (d) can only be awarded if the length of the radius has been written in (d).

Question Number	Scheme	Marks
9(a)	$\mathrm{f}(x)=\frac{1}{4}x^4-\frac{1}{3}x^3-2x^2+4x\ \ (+c)$	M1A1
	$x=-2\ \ y=-\frac{28}{3}\ \ \Rightarrow c=0$	M1
	$\left(\mathrm{f}(x)=\frac{1}{4}x^4-\frac{1}{3}x^3-2x^2+4x\right)\ \ \therefore C$ passes through O	A1 cso (4)
(b)(i)	$x=2\ \ \mathrm{f}'(x)=8-4-8+4=0$	M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=3x^2-2x-4$	M1
	$x=2\quad \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=12-4-4>0\quad \therefore$ min at $x=2$	A1cso
	$x=1\ \ \mathrm{f}'(x)=1-1-4+4=0$	M1
	$x=1\quad \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=3-2-4<0\quad \therefore$ max at $x=1$	A1 cso
ALT	$\mathrm{f}'(x)=(x-2)(x-1)(x+2)\ \ (=0)$ factorise	M1
	$x=2,1,(-2)$ solve (solutions to be 2,1 (and another))	M1
	OR: $\mathrm{f}'(x)\ (=0)$ solved by calculator.	
	All 3 solutions needed (and correct) $=0$ not needed M2	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=3x^2-2x-4$ differentiate	M1
	$x=2\quad \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=12-4-4>0\quad \therefore$ min at $x=2$	A1cso
	$x=1\quad \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=3-2-4<0\quad \therefore$ max at $x=1$	A1cso
(ii)	$x=1\Rightarrow y=1\frac{11}{12}\qquad x=2\Rightarrow y=1\frac{1}{3}$	B1B1 (7)
(c)	$y'=(x-1)(x-2)(x+2)$	M1
(i)	$x=-2,\ y=-\frac{28}{3}$ or $\left(-2,-\frac{28}{3}\right)$	A1
(ii)	$x=-2\quad \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=12+4-4>0\quad \therefore$ min point	A1cso (3)
		[14]

Question Number	Scheme	Marks
(a)M1	Attempt to integrate f'(x). The power of at least one x term must increase and none should decrease. c not needed	
A1	Correct integration, c not needed	
M1	Substitute the given coordinates to show $c = 0$. If c is not included (or assumed to be 0), then showing that substitution of $x = -2$ gives $y = -28/3$ is acceptable. Substitutions must be shown.	
A1cso	Correct conclusion from fully correct work. Accept eg $\text{f}(0) = 0 \therefore$ shown	
(b)	Ignore labels (i) and (ii) when marking (b)	
(i)M1	Substitute $x = 2$ in the expression for f'(x) to show f'(x) = 0. Substitution must be shown	
M1	Differentiate the expression for f'(x). At least one power must decrease and none increase.	
A1cso	Show second derivative is > 0 at $x = 2$ and give the conclusion. No errors or omissions in the working.	
M1	Substitute $x = 1$ in the expression for f'(x) to show f'(x) = 0. Substitution must be shown	
A1cso	Show second derivative is < 0 at $x = 1$ and give the conclusion. No errors or omissions in the working.	
(ii)B1	For either y coordinate correct (and x coordinate correctly indicated; substitution shown indicates this)	
B1	For the second y coordinate correct	
(c)	(May have been seen in (b))	
M1	Factorise f'(x) completely – any valid method OR use the factor theorem to find $x = -2$	
(i)A1	Extract the x coordinate of the third turning point and obtain the corresponding y coordinate. May quote y coordinate from the question	
(ii)A1cso	Test the sign of the second derivative at this point and make the conclusion. All work in (c) and $\frac{\text{d}^2y}{\text{d}x^2}$ (from (b)) must be completely correct for this mark to be awarded.	
	Alternative ways to determine the nature of the turning points:	
1.	If the change of sign of f'(x) is used then values of f'(x) either side of 1 and 2 must be calculated to provide evidence.	
2.	The continuity of a cubic function can be used to establish the nature of the turning points. If in doubt send to review.	

Question Number	Scheme	Marks
10(a)	$\alpha+\beta=-3 \quad \alpha\beta=-5$	B1
(i)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta, \ =19$	M1,A1
(ii)	$\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2\alpha^2\beta^2, \ =19^2-50=311$	M1,A1(5)
	OR: $\alpha^4+\beta^4=(\alpha+\beta)^4-4\alpha\beta(\alpha^2+\beta^2)-6\alpha^2\beta^2, \ =19^2-50=311$	
(b)	$(\alpha-\beta)^2=\alpha^2-2\alpha\beta+\beta^2=19+10$ OR $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta=9-(-20)$	M1
	$\alpha-\beta=\sqrt{29}$ *	A1 cso (2)
(c)	$\alpha^4-\beta^4=(\alpha^2-\beta^2)(\alpha^2+\beta^2)=(\alpha-\beta)(\alpha+\beta)(\alpha^2+\beta^2)$	M1A1A1 (3)
(d)	$\alpha^4-\beta^4=\sqrt{29}\times(-3)\times19=-57\sqrt{29} \ \left(-\sqrt{94221}\right)$	M1A1 (2)
(e)	$2\beta^4=\alpha^4+\beta^4-(\alpha^4-\beta^4)$	M1
	$\beta^4=\frac{1}{2}\left(311+57\sqrt{29}\right), =\frac{311}{2}+\frac{57}{2}\sqrt{29}$	A1,A1 (3)
	$p=\frac{311}{2} \qquad q=\frac{57}{2}$	[15]
ALT	$\beta^4=\left(\frac{-3-\sqrt{29}}{2}\right)^4$ **and** use a correct binomial expansion M1A1	
	Correct final answer A1	
(a)B1	Correct sum **and** product of roots, seen explicitly or used (in (a)). Must be clear that sum is negative	
(i)M1	Correct algebra, ready for substitution of sum and product	
A1	Correct answer, condone use of $\alpha+\beta=3$.	
(ii)M1	Correct algebra, ready for substitution	
A1	Correct answer, condone use of $\alpha+\beta=3$.	
(b)M1	Correct algebra and substitution of their values	
A1cso	Correct answer from correct working. Must have seen sum = -3 here if not shown in (a)	
(c)		
M1	Factorise to 2 quadratic brackets or 2 linear and one quadratic bracket	
A1	Obtain 2 linear and 1 quadratic brackets with 2 of the 3 brackets correct	
A1	Third correct bracket Accept $(\alpha^2+\beta^2)$ or $\left((\alpha+\beta)^2-2\alpha\beta\right)$	
(d)M1	Substitute their values for each of the 3 brackets obtained in (c)	
A1	Correct answer as shown or equivalent **exact** value	
(e)		
M1	Correct expression for $2\beta^4$ or β^4	
A1ft	Substitute their numbers to obtain a numerical expression for β^4 The expression must be exact but need not be simplified **NB** A correct numerical expression for their values implies M1	
A1	Correct answer in the required form. *p* and *q* need not be shown explicitly.	

Question Number	Scheme	Marks
11(a)	$AC=\sqrt{(16^2+16^2)}=16\sqrt{2}$	M1A1
ALT:	$AP^2+PD^2=16^2 \Rightarrow AP=8\sqrt{2}$	
	$VP=8\sqrt{2}\tan 45=8\sqrt{2}$ (where P is the centre of the base) *	M1A1cso (4)
(b)	$VA^2=\left(8\sqrt{2}\right)^2+\left(8\sqrt{2}\right)^2\ (=256)$ or $VA=\dfrac{8\sqrt{2}}{\sin 45°}$	M1A1
	$VA=16$ cm	A1 (3)
(c)	$DX^2=16^2-8^2$ where X is the foot of the perpendicular from D to VA	M1A1
	$DX=8\sqrt{3}$	A1 (3)
(d)	$\tan\theta=\dfrac{8\sqrt{2}}{8}, \sin\theta=\dfrac{8\sqrt{2}}{8\sqrt{3}}, \cos\theta=\dfrac{8}{8\sqrt{3}}$	M1A1
	(or unsimplified if cosine or sine rule used) $\theta=54.7°$	A1 (3)
(e)	$\cos\phi=\dfrac{\left(8\sqrt{3}\right)^2+\left(8\sqrt{3}\right)^2-\left(16\sqrt{2}\right)^2}{2\times8\sqrt{3}\times8\sqrt{3}}\ \left(=-\dfrac{1}{3}\right)$	M1A1
	$\phi=109.5°$	A1 (3) [16]

(a)	
M1	Use Pythagoras (with a + sign) to obtain the length of the diagonal of the base. Or use Pythagoras with correct sign to obtain the half diagonal
A1	Correct length for the diagonal or half diagonal
M1	Use tan in ΔAPV, their AP and angle of 45° to obtain the height
A1cso	Correct answer with no errors in the working
	OR: State ΔAVP is isosceles, or shown the 2 correct angles – can be on a diagram M1
	$VP(=AP)=8\sqrt{2}$ A1
	OR use any other complete valid method M1 Correct result A1
(b)	
M1	Use Pythagoras or trigonometry in ΔAPV or ΔAVC (or any other complete, valid method) to obtain a numerical expression for VA.
A1	Correct numbers in their choice of method.
A1	Correct length obtained.
	AV = 16 w/o working scores M1A1A1

Question Number	Scheme	Marks
(c)		
M1	Use Pythagoras with a minus sign (seen or implied) or trigonometry in ΔADX OR any other complete valid method **NB**: triangle ΔADV is equilateral.	
A1	Correct numbers in their choice of method.	
A1	Correct **exact** length for the perpendicular.	
(d)		
M1	Identify the correct triangle needed with the required angle marked (may be on Figure 1). This may be shown explicitly or implied by their work that follows.	
A1	Reach one of $\tan\theta = \frac{8\sqrt{2}}{8}$, $\sin\theta = \frac{8\sqrt{2}}{8\sqrt{3}}$, $\cos\theta = \frac{8}{8\sqrt{3}}$ oe	
A1	Correct answer, **must be 1 dp.**	
(e)		
M1	Use cosine rule: $\cos\phi = \frac{"DX"^2 + "XB"^2 - "BD"^2}{2\times "DX" \times "XB"}$ (their values)	
A1	Correct numbers substituted, follow through their previous answers	
A1	Correct answer, **must be 1 dp unless already penalised in (d)**	
	Any other routes should be marked: M1 Correct, complete method (ie it must be possible to reach a value for the required angle) A1 Correct numbers substituted A1 Correct answer, **must be 1 dp unless already penalised in (d)**	