Mark Scheme (Results)

Summer 2018

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM0)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks
    - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
    - cao – correct answer only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case
    - oe – or equivalent (and appropriate)
    - dep – dependent
    - indep – independent
    - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers may score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**
  Always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  Any case of suspected misread loses 2A (or B) marks on that part, but can gain the M marks.
  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right), \text{ where } |pq|=|c| \quad \text{leading to } x=....$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right) \text{ where } |pq|=|c| \text{ and } |mn|=|a| \quad \text{leading to } x=....$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$$x^2+bx+c=0: \ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0, \quad q\neq 0 \qquad \text{leading to } x=....$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n \to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n \to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is <u>not</u> quoted, the method mark can be gained by implication from the substitution of <u>correct</u> values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers <u>may</u> be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

June 2018
4PM0 Further Pure Mathematics Paper 1

<table>
<tr><th>Question number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td>1 (a)</td><td colspan="2">$\frac{1}{2}\times 10^2\theta = 25$
$\theta = \frac{1}{2}$</td><td>M1
A1
[2]</td></tr>
<tr><td>(b)</td><td colspan="2">arc length $= r\theta = 10\times\frac{1}{2} = 5$ (cm)</td><td>M1A1
[2]</td></tr>
<tr><td colspan="4" align="right">Total 4 marks</td></tr>
<tr><td colspan="4" align="center">Notes</td></tr>
<tr><td>(a)</td><td>M1</td><td colspan="2">Uses correct formula for area of a sector $A = \frac{1}{2}r^2\theta$ or rearranged to give $\theta = \frac{2A}{r^2}$ with fully correct substitution to obtain a value for θ</td></tr>
<tr><td></td><td>A1</td><td colspan="2">$\theta = \frac{1}{2}$ or 0.5 in radians accept any equivalent fraction e.g. $\frac{50}{100}$</td></tr>
<tr><td>(b)</td><td>M1</td><td colspan="2">1st Method Uses their value for θ in the correct formula $l = r\theta$ to achieve a value for l. Accept only r = 10 cm and their value for θ

2nd Method Uses the formula $l = \frac{2A}{r} \Rightarrow l = \frac{2\times 25}{10} = (5)$ Accept only correct values for r and A.</td></tr>
<tr><td></td><td>A1</td><td colspan="2">$l = 5$ (cm)</td></tr>
<tr><td colspan="4">ALT – Works in degrees</td></tr>
<tr><td>(a)</td><td>M1</td><td colspan="2">Uses correct formula for area of a sector $A = \pi r^2\frac{\theta^\circ}{360^\circ}$ AND attempts to convert their angle (28.647... °) correctly into radians $\frac{'28.647...^\circ'\times\pi}{180}$ (0.16π)</td></tr>
<tr><td></td><td>A1</td><td colspan="2">$\theta = \frac{1}{2}$ or 0.5 accept any equivalent fraction e.g. $\frac{50}{100}$
Accept 0.499 or better</td></tr>
<tr><td>(b)</td><td>M1</td><td colspan="2">Use the correct formula $l = 2\pi r\frac{\theta^\circ}{360^\circ}$ with their angle in degrees to find a value for l</td></tr>
<tr><td></td><td>A1</td><td colspan="2">$l = 5$ (cm) Accept 4.99 (cm) or better.</td></tr>
<tr><td colspan="4" align="right">Total 4 marks</td></tr>
</table>

Question number	Scheme	Marks
2	$\alpha+\beta=\frac{5}{3},\ \alpha\beta=\frac{4}{3}$	B1
	$\alpha+\frac{1}{2\beta}+\beta+\frac{1}{2\alpha}=\alpha+\beta+\frac{\alpha+\beta}{2\alpha\beta},=\frac{5}{3}+\frac{\frac{5}{3}}{\frac{8}{3}}=\frac{55}{24}$	M1,A1
	$\left(\alpha+\frac{1}{2\beta}\right)\left(\beta+\frac{1}{2\alpha}\right)=\alpha\beta+\frac{1}{2}+\frac{1}{2}+\frac{1}{4\alpha\beta},=\frac{4}{3}+1+\frac{3}{16}=\frac{121}{48}$	M1,A1
	$x^2-\frac{55}{24}x+\frac{121}{48}\ (=0)$	M1
	$48x^2-110x+121=0$	A1 [7]
		Total 7 marks

Notes

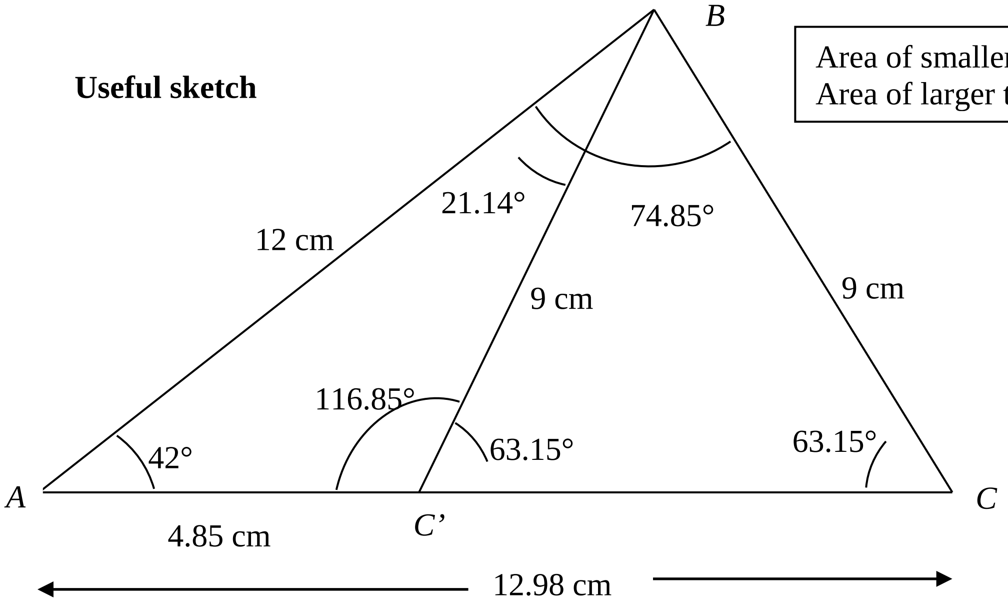
B1	For both correct values of the sum and product.
M1	For the **correct** algebra for the SUM. They must reach $\alpha+\beta+\frac{(\alpha+\beta)}{2\alpha\beta}$ or $\left[\alpha+\beta+\frac{2(\alpha+\beta)}{4\alpha\beta}\right]$ or $\frac{2\alpha\beta(\alpha+\beta)+(\alpha+\beta)}{2\alpha\beta}$ or $\frac{(\alpha+\beta)(2\alpha\beta+1)}{2\alpha\beta}$ Their **correct** expression for the sum must be such as to substitute $\alpha+\beta$ and $\alpha\beta$ directly in.
A1	**S**ubstitute in their values for $\alpha+\beta$ and $\alpha\beta$. Sum = $\frac{55}{24}$
M1	For the **correct** algebra for the PRODUCT. They must reach $\alpha\beta+\frac{1}{2}+\frac{1}{2}+\frac{1}{4\alpha\beta}$ or $\frac{(2\alpha\beta+1)^2}{4\alpha\beta}$ Their **correct** expression for the product must be such as to substitute $\alpha\beta$ directly in
A1	**S**ubstitute in their values for $\alpha\beta$. Product = $\frac{121}{48}$
M1	Use their SUM and PRODUCT correctly in a quadratic equation. $x^2+(-\text{ their sum})x+(\text{their product})\quad(=0)$
A1	$48x^2-110x+121=0$ oe for example $96x^2-220x+242=0$ Must be integer values only.

Question number	Scheme	Marks
3 (a)	$\dfrac{\sin C}{12}=\dfrac{\sin 42°}{9}$	M1A1
	$C=63.14...°\quad 116.85°$	A1
	$\angle ABC=180-("C"+42)$ $B=180-("C"+42)$, $\quad B=74.9°\quad 21.1°$ (Accept 21.2°)	M1A1 [5]
(b)	Area $=\dfrac{1}{2}\times 12\times 9\sin"B"$, $=\dfrac{1}{2}\times 12\times 9\sin 21.1°$ $=19$ or 20 (cm^2)	M1,A1 (smaller angle) A1
		Total 8 marks

Notes

(a)	M1	Uses Sine Rule either way around with correct values and achieves a value for an angle **in degrees**. (**Not just the sine of the angle**)
	A1	For either C = 63.1° - 63.2 ° **OR** C = 116.8° - 116.9°
	A1	For C = 63.1° **-** 61.2 ° **AND** C = 116.8° **-** 116.9°
	M1	For $\angle ABC=180-("C"+42)$ to achieve at least one value for $\angle ABC$
	A1	$\angle ABC=74.9°$ **AND** $21.1°$ both required rounded correctly (Accept 21.2°)
(b)	M1	For a correct expression for the area. They must use the appropriate angle with the correct lengths. For example; 9cm, 12 cm with their angle *B* (even if it is incorrect but identified as their angle *B*). If they do not have an angle *B* and use lengths 9 cm and 12cm, award M0. If they only have one value for angle *B*, allow this mark. isw extra attempts after a correct method seen.
	A1	For using 21.1° or 21.2° only
	A1	Area = 19 or 20 (cm^2) accept this for full marks even if area of 52 (cm^2) is seen as well.

Useful sketch



Area of smaller triangle *ABC* = 19 (cm^2)
Area of larger triangle *ABC* = 52 (cm^2)

Rounding: Please read the notes carefully on rounding in General Guidance

Question number	Scheme	Marks
4 (a) (i)	$x-\dfrac{1}{2x^2}=\dfrac{2x^3-1}{2x^2}$	B1
(ii)	$x=\sqrt[3]{0.5}\ \Rightarrow 2x^3=1\Rightarrow y=0,\quad x\approx 0.8$	M1,A1 [3]
(b)	$4-2x+\dfrac{1}{2x^2}=0 \quad x-\dfrac{1}{2x^2}=4-x$ Draw $y=4-x,\quad x=2.1$ or 2.0	M1 dM1,A1 [3]
		Total 6 marks

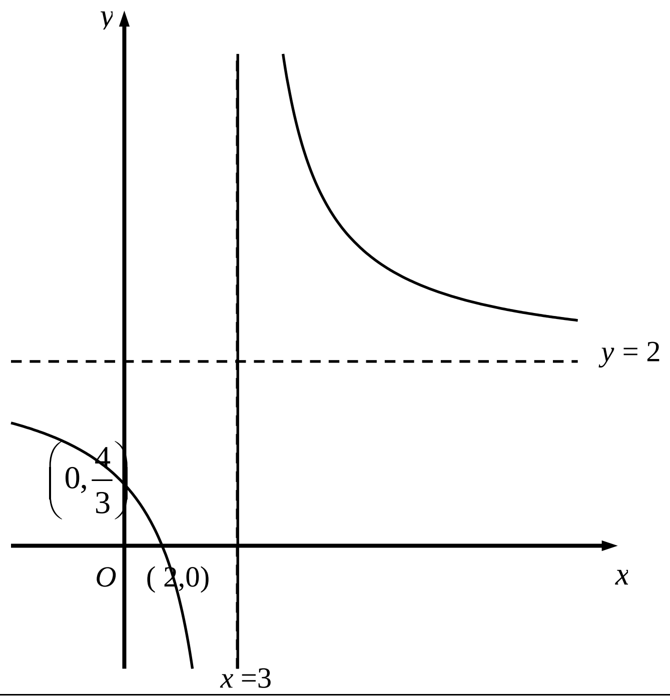
Notes

(a) (i)	B1	Correct fraction only $\dfrac{2x^3-1}{2x^2}$ Award when seen, and isw any attempts to simplify.
(ii)	M1	Substitutes $x=\sqrt[3]{0.5}$ into $y=\dfrac{2x^3-1}{2x^2}\Rightarrow y=\dfrac{2\left(\sqrt[3]{0.5}\right)^3-1}{2\left(\sqrt[3]{0.5}\right)^2}=\dfrac{'1'-1}{2\left(\sqrt[3]{0.5}\right)^2}\quad (=0)$ **and** uses the graph to write a value for *x* for their value of *y*. If there is no working with just an answer given here - M0 Minimum working we need to see; $y=0\Rightarrow x\approx 0.8$ or $y=0\Rightarrow x=0.8$
	A1	$x = 0.8$ only. More digits implies a calculator answer so is A0.
(b)	M1	For attempting to achieve a minimum of $x-\dfrac{1}{2x^2}=\pm 4\pm x$
	dM1	Draws their line correctly. Coordinates of the correct line are (0, 4) (1, 3) (2,2) (3, 1), (4, 0) **and** identifies a value of *x* for their intersection.
	A1	For either $x=2.1$ or 2.0 only

Question number	Scheme	Marks
5 (a) (i)	$\int\left(3-x+\frac{1}{x^3}\right)\mathrm{d}x=3x-\frac{1}{2}x^2-\frac{1}{2x^2}\ (+c)$ $\quad\left(\frac{1}{2}x^{-2}\text{ or }\frac{1}{2x^2}\right)$	M1A1
(ii)	$\left[3x-\frac{1}{2}x^2-\frac{1}{2x^2}\right]_1^2=6-2-\frac{1}{8}-\left(3-\frac{1}{2}-\frac{1}{2}\right)=1\frac{7}{8}$ (or 1.875)	M1A1 [4]
(b) (i)	$\int 6\sin 3x\,\mathrm{d}x=-2\cos 3x\ \ (+c)$	M1A1
(ii)	$\left[-2\cos 3x\right]_{\frac{\pi}{9}}^{\frac{\pi}{6}}=-2\cos\frac{\pi}{2}+2\cos\frac{\pi}{3},=1$	dM1,A1 [4]
		Total 8 marks

Notes

(a)(i)	M1	$x^{-3}\Rightarrow kx^{-2}$ **and** one of $3\Rightarrow 3x$ or $-x\Rightarrow -kx^2$
	A1	Fully correct integrated expression. Condone a missing (+*c*) Ignore spurious integral signs.
(ii)	M1	Substitutes 2 and 1 into their **integrated** expression (at least one term integrated) and subtracts the correct way around and reaches a value for the integrated expression.
	A1	For $1\frac{7}{8}$ or equivalent fraction or 1.875
(b)(i)	M1	Attempts to integrate the expression. Accept as a minimum $\sin 3x\Rightarrow -k\cos 3x\quad k\neq\pm1,\ \pm3$ or 0 $\pm18\cos 3x$ or $\pm6\cos 3x$ are M0
	A1	For $-2\cos 3x\ (+c)$ Condone a missing (+*c*) Ignore spurious integral signs.
(ii)	dM1	Substitutes $\frac{\pi}{6}$ and $\frac{\pi}{9}$ into their **integrated** expression and subtracts the correct way around and reaches a value for the integrated expression.
	A1	For 1

Question number	Scheme	Marks
6 (a)	(i) $y=2$ (ii) $x=3$	B1 B1 [2]
(b)	(i) $(2,0)$ Accept $x=2$ (ii) $\left(0,\frac{4}{3}\right)$ Accept $y=\frac{4}{3}$	B1 B1 [2]
(c)		B1 Shape B1 Asymptotes B1 Crossing pts (Non-zero coord needed only)
		Total 7 marks

Notes

(a) (i)	B1	$y=2$ only. Do not accept just ‘2’. This must be an **equation** of a line
(ii)	B1	$x=3$ only. Do not accept just ‘3’. This must be an **equation** of a line

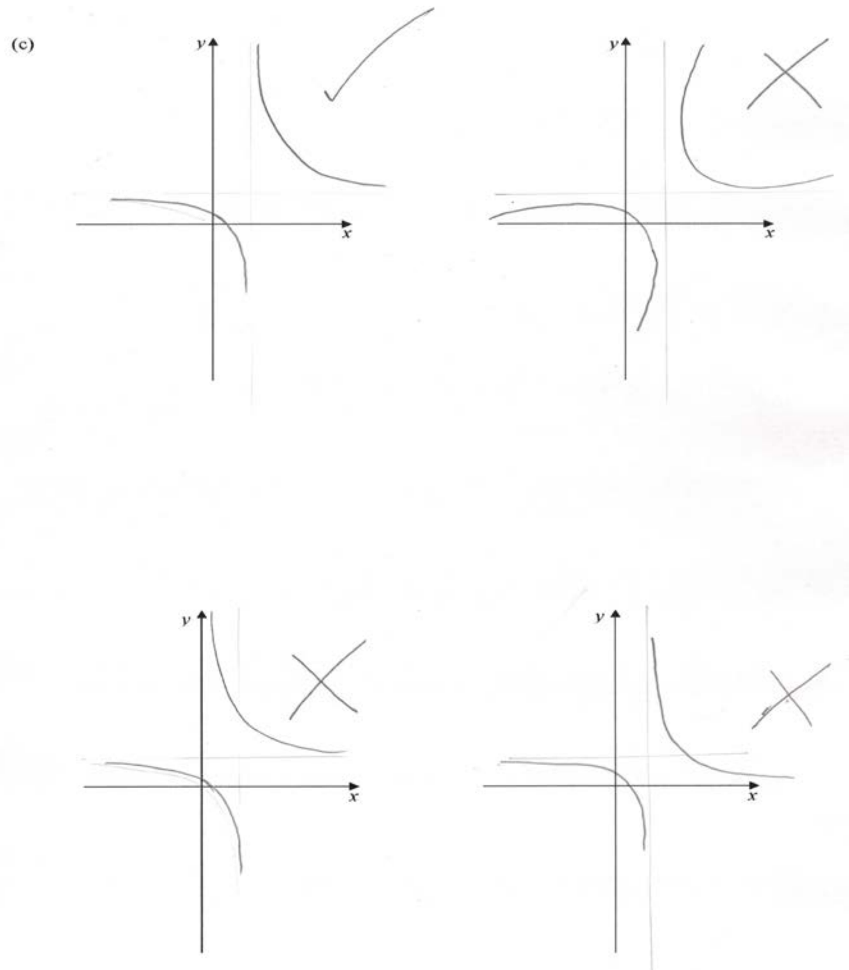
If there is only one answer or they are not marked (i) and (ii) given, mark them in the order written and award accordingly

(b) (i)	B1	$(2,0)$ Accept $x=2$
(ii)	B1	$\left(0,\frac{4}{3}\right)$ Accept $y=\frac{4}{3}$

If there is only one answer or they are not marked (i) and (ii) given, mark them in the order written and award accordingly

(c)	B1	Shape: One branch must be in the first quadrant as shown, and the second branch in the 1st, 2nd and 4th quadrants as shown. Do not accept curves that come back on themselves or overlap. See below for samples of error types.

	B1	**Both** of their asymptotes drawn and labelled correctly. Accept a vertical line drawn with 3 written on the *x*-axis, and a horizontal line drawn with 2 written on the *y* axis. **There must be at least one branch of the curve drawn for the award of this mark.**
	B1	**Both** intersections with the axes shown. 0 not required as long as values are clear. Ft their answers from (b)



Question number	Scheme	Marks
7 (a)	$v = 0 \Rightarrow 5\cos 2t = 0$ and solve to $t = \dots$	M1
	$t = \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi}{4}$ or 0.7853.... (accept 0.785 or better)	A1 [2]
(b)	$a = -10\sin 2t$	M1A1
	$\lvert a_{\max} \rvert = 10\left(\text{ m/s}^2\right)$	A1 [3]
(c)	$s = \int 5\cos 2t \, \mathrm{d}t = \frac{5}{2}\sin 2t \;(+c)$	M1A1
	$t = 0 \;\; s = 0.2 \Rightarrow c = 0.2$	dM1
	$t = \frac{\pi}{4} \quad s = \frac{5}{2}\sin\frac{\pi}{2} + 0.2 = 2.7$ oe (m)	A1 [4]
	ALT	
	$s - 0.2 = \int_0^{\frac{\pi}{4}} 5\cos 2t \, \mathrm{d}t = \left[\frac{5}{2}\sin 2t\right]_0^{\frac{\pi}{4}}$	{dM1A1}
	Substitute limits M1 Correct answer A1	
		Total 8 marks

Notes		
(a)	M1	Sets 5cos 2t = 0 and finds a value for t. Allow work in degrees for this mark.
	A1	$t = \dfrac{\pi}{4}$ (accept 0.785 or better)
(b)	M1	Attempts to differentiate the given v to achieve as a minimum $-k\sin 2t \quad k \neq 0$
	A1	For $a = -10\sin 2t$
	A1	For 10 (m/s^2) do not accept -10 for this mark
(c)	M1	For an attempt to integrate the given v to achieve as a minimum $\dfrac{k\sin 2t}{2}, k \neq 2$
	A1	For the correct integrated expression for s, $+c$ not required for this mark.
	dM1	For an attempt to find c when t = 0 and uses $t = \dfrac{\pi}{4}$ (allow 45°) to find a value for s. Some are adding 0.2 at the end of their calculation which is fine for this mark.
	A1	s = 2.7
ALT		
(c)	M1	For an attempt to integrate the given v to achieve as a minimum $\dfrac{k\sin 2t}{2}, \; k \neq 2$
	A1	For the correct integrated expression for s
	dM1	Substitutes in **both** limits of $\dfrac{\pi}{4}$ and 0 (allow 45°) into a changed expression, and adds 0.2 to find a value of s
	A1	For s = 2.7

Question number	Scheme	Marks
8 (a)	$y=15-7x,=x^2-6x+9$	M1
	$x^2+x-6=0$	A1
	$(x+3)(x-2)=0$	dM1
	$x=-3 \quad y=36 \qquad (-3,36)$	A1
	$x=2 \quad y=1 \qquad (2,1)$	A1 [5]
(b)	Area $=\int_{-3}^{2}\left((15-7x)-\left(x^2-6x+9\right)\right)\mathrm{d}x=\int_{-3}^{2}\left(-x^2-x+6\right)\mathrm{d}x$	M1
	$=\left[-\frac{1}{3}x^3-\frac{1}{2}x^2+6x\right]_{-3}^{2}$	M1A1
	$=-\frac{1}{3}\times2^3-\frac{1}{2}\times2^2+12-\left(\frac{27}{3}-\frac{1}{2}\times9-18\right)$	M1
	$=20\frac{5}{6}$	A1 [5]
		Total 10 marks

Notes			
(a)	M1	Sets the equation of $l =$ the equation of C and attempts to form a 3TQ	
	A1	Correct 3TQ $x^2 + x - 6 = 0$	
	dM1	For any acceptable attempt to solve their 3TQ (please see general guidance for the definition of an attempt using either factorisation, formula or completing the square. This mark is dependent on the first M mark. Their 3TQ must have come from an attempt to equate and rearrange the equations of the line and the curve.	
	A1	For either $(-3, 36)$ or $(2, 1)$ Accept $x = 1, y = 1$ or $x = -3, y = 36$	
	A1	For **both** $(-3, 36)$ and $(2, 1)$ Accept $x = 1, y = 1$ and $x = -3, y = 36$	
Method 1 Line – Curve combined			
(b)	M1	For the correct method to find the area using integration. ft their limits which must be the correct way around for this mark. Follow through their combined equation from part (a) $\text{Area} = \int_{'-3'}^{'2'} (\text{equation of line} - \text{equation of curve})\ \mathrm{d}x$ Accept $\text{Area} = \int_{'-3'}^{'2'} (\text{equation of curve} - \text{equation of line})\ \mathrm{d}x$	
	M1	For an attempt to integrate the combined expression even if there are errors when combined. Ignore limits for this mark. Please see General Guidance for the definition of an attempt.	
	A1	For the correct integrated expression either way (line – curve or curve – line) Ignore limits for this mark. $I = \pm\left(-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 6x\right)$	
	M1	For substituting in the limits '2 'and '−3' the correct way around into their combined integrated expression. ft through their values given in their initial statement.	
	A1	For $20\frac{5}{6}$ or $\frac{125}{6}$ only.	If they find a negative value (from curve – line) allow only of they give a final positive value for the area.
Method 2 Line – Curve separately			
(b)	M1	For the correct method to find the area using integration. ft their limits which must be the correct way around for this mark and must be the same for the curve and the line. $\text{Area} = \int_{'-3'}^{'2'} \text{equation of line}\ \mathrm{d}x - \int_{'-3'}^{'2'} \text{equation of curve}\ \mathrm{d}x$ Accept for this mark $\text{Area} = \int_{'-3'}^{'2'} \text{equation of curve}\ \mathrm{d}x - \int_{'-3'}^{'2'} \text{equation of line}\ \mathrm{d}x$	
	M1	For an attempt to integrate the expression for the curve **and** the line Ignore limits for this mark. Please see General Guidance for the definition of an attempt.	
	A1	For correct integrals of the line **and** the curve. $\mathrm{I}_l = \int 15 - 7x\ \mathrm{d}x = 15x - \frac{7}{2}x^2, \quad \mathrm{I}_C = \int x^2 - 6x + 9\ \mathrm{d}x = \frac{x^3}{3} - 3x^2 + 9x$	
	M1	For substituting in the limits '2' and '−3' the correct way around individually into **both**, and attempting to evaluate the area. They must find a value for the area for this mark. ft through their values given in their initial statement. If they only substitute limits into the equation of a curve OR a line, withhold this mark.	
	A1	For $20\frac{5}{6}$ or $\frac{125}{6}$ only.	If they find a negative value (from curve – line) allow only if they give a final positive value for the area.
Method 3 Trapezium – curve			

(b)	M1	For the correct method to find the area using a trapezium and curve. Area = $\frac{'5'}{2}('36'+'1')-\int_{'-3'}^{'2'}\text{curve } dx$ or accept Area = $\int_{'-3'}^{'2'}\text{curve } dx-\frac{'5'}{2}('36'+'1')$	
	M1	For an attempt to integrate the given expression for the area under the curve. Ignore limits for this mark Please see General Guidance for the definition of an attempt. **Integrating the curve only without evidence of an attempt to find the area of the trapezium (or equivalent) is M0.**	
	A1	For Area of curve = $\left[\frac{x^3}{3}-3x^2+9x\right]_{-3}^{2}$ **and** area of trapezium (or equivalent) of $92\frac{1}{2}$ Ignore limits for this mark.	
	M1	For substituting their limits '2' and '-3' the correct way around and combining this either way round with their trapezium to evaluate the area. They must find a value for the area for this mark. ft through their values given in their initial statement.	
	A1	For $20\frac{5}{6}$ or $\frac{125}{6}$ only.	If they find a negative value (from trapezium – line) allow only if they give a final positive value for the area.

Question number	Scheme	Marks
9 (a)	$a+3d=108$ $a+10d=80$ (i) $7d=-28,\quad d=-4$ (ii) $a=120$	M1 A1 M1 A1 [4]
ALT	(7 terms dec by 28) $\therefore d=-\frac{28}{7},=-4$ $80=a+10(-4)$ or $108=a+3(-4)$ $a=120$	M1A1 M1 A1 [4]
(b)	$S_n=\frac{n}{2}\left(2\times120-4(n-1)\right)$ $S_n=\frac{4n}{2}(60-n+1)=2n(61-n)$ *	M1A1 A1 cso [3]
(c)	$2n(61-n)=1100$ $2n^2-122n+1100=0$ $(n-11)(n-50)=0\qquad n=11,\ 50$	M1 A1 dM1A1 [4]
		Total 11 marks

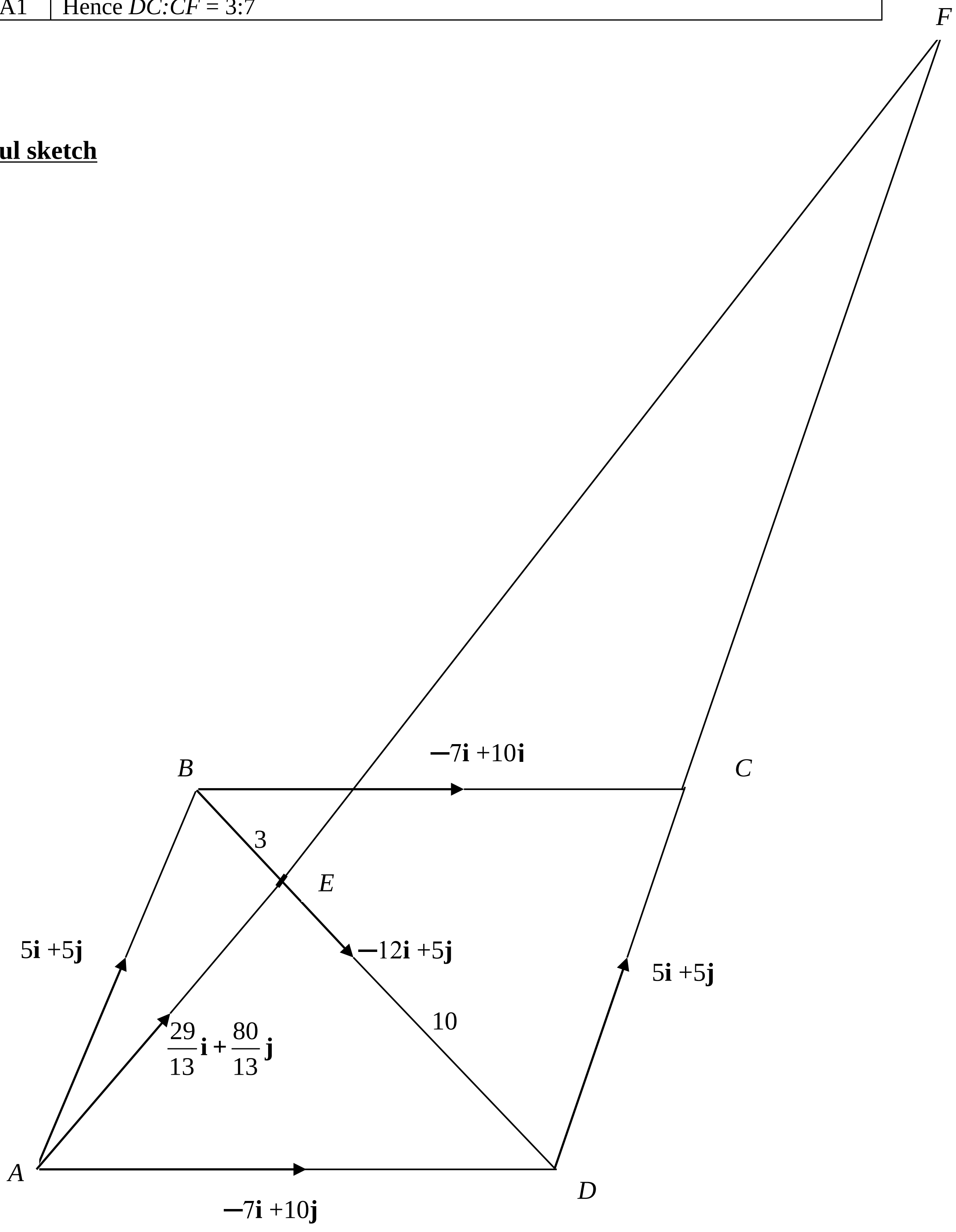
Notes		
(a)	M1	For forming either $a+3d=108$ OR $a+10d=80$
	A1	For forming both $a+3d=108$ AND $a+10d=80$
	M1	For attempting to solve their simultaneous equations by any valid method.
	A1	Both $d = -4$ and $a = 120$
ALT 1		
(a)	M1	$d=\dfrac{80-108}{7}=...$
	A1	$d=-4$
	M1	Uses either $a+3d=108$ or $a+10d=80$ and substitutes their *d* to find a value for *a*.
	A1	$a = 120$
No working: Award M1A1 for $d = -4$ with no working Award M1A1 for $a = 120$ with no working		
(b)	M1	Uses the **correct** summation formula with their *a* and their *d*.
	A1	A fully correct un-simplified summation formula with both values of *a* and *d* correct.
	A1 cso	Achieves the given answer of $S_n = 2n(61-n)$ Note this is a show question, every step must be seen for the award of this mark.
(c)	M1	States $2n(61-n)=1100$ or $1100=\dfrac{n}{2}(2\times'120'+(n-1)\times-4)$
	A1	Forms the correct 3TQ Note: The 3TQ does not need to = 0. For example, $1100=122n-2n^2$ is acceptable for this mark.
	dM1	Solves their 3TQ by any method
	A1	$n=11,\ 50$

Question number	Scheme	Marks
10 (a) (i)	$\overrightarrow{DC} = \overrightarrow{DA} + \overrightarrow{AC}, = 7\mathbf{i} - 10\mathbf{j} - 2\mathbf{i} + 15\mathbf{j} = 5\mathbf{i} + 5\mathbf{j}$	M1,A1
(ii)	$\overrightarrow{DC} = \overrightarrow{AB}$	M1
	$\therefore$ *ABCD* is a parallelogram	A1 [4]
(b)	$\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD} = -5\mathbf{i} - 5\mathbf{j} - 7\mathbf{i} + 10\mathbf{j} = -12\mathbf{i} + 5\mathbf{j}$	M1A1
	unit vector $= (\pm)\dfrac{1}{13}, (-12\mathbf{i} + 5\mathbf{j})$	B1ft,B1 [4]
(c)	$\overrightarrow{BE} = \dfrac{3}{13}(-12\mathbf{i} + 5\mathbf{j})$	
	$\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE} = \dfrac{29}{13}\mathbf{i} + \dfrac{80}{13}\mathbf{j}$	M1A1 [2]
(d)	$\overrightarrow{AF} = \lambda\overrightarrow{AE} = \lambda\left(\dfrac{29}{13}\mathbf{i} + \dfrac{80}{13}\mathbf{j}\right)$ or $\lambda'(29\mathbf{i} + 80\mathbf{j})$	B1
	$\overrightarrow{AF} = \overrightarrow{AC} + \mu\overrightarrow{DC} = -2\mathbf{i} + 15\mathbf{j} + \mu(5\mathbf{i} + 5\mathbf{j})$	M1A1
	$\dfrac{29}{13}\lambda = -2 + 5\mu \qquad \dfrac{80}{13}\lambda = 15 + 5\mu$	
	$\mu = \dfrac{7}{3} \qquad \left(\lambda = \dfrac{13}{3}, \quad \lambda' = \dfrac{1}{3}\right)$	M1A1
	$DC : CF = 1 : \dfrac{7}{3} \qquad (= 3 : 7)$	A1 [6]
		Total 16 marks

		Notes
(a)	M1	For the correct vector statement for $\overrightarrow{DC}$ so $\overrightarrow{DC}=\overrightarrow{DA}+\overrightarrow{AC}$
	A1	For the correct simplified expression $\overrightarrow{DC}=5\mathbf{i}+5\mathbf{j}$
	M1	States $\overrightarrow{DC}=\overrightarrow{AB}$ Condone lack of arrows on vectors if they are clearly using vectors. i.e accept $DC=AB$
	A1	Conclusion required, therefore *ABCD* is a parallelogram. Accept; a labelled diagram, shown, QED, or even a tick or # etc.
(b)	M1	For the correct vector statement for $\overrightarrow{BD}=\overrightarrow{BA}+\overrightarrow{AD}$ or $\overrightarrow{BD}=\overrightarrow{BC}+\overrightarrow{CD}$
	A1	For the correct simplified expression $\overrightarrow{BD}=-12\mathbf{i}+5\mathbf{j}$
	B1ft	For the correct magnitude of their vector. So that for $\overrightarrow{BD}=a\mathbf{i}+b\mathbf{j}\ \Rightarrow\ \lvert\overrightarrow{BD}\rvert=\sqrt{a^2+b^2}$, $\lvert\overrightarrow{BD}\rvert=13$
	B1ft	Writes $'\frac{1}{13}'(a\mathbf{i}+b\mathbf{j})$ for their $\overrightarrow{BD}$
(c)	M1	For any correct path for $\overrightarrow{AE}$ with correct use of the ratio for $\overrightarrow{ED}$ or $\overrightarrow{BE}$
	A1	For $\overrightarrow{AE}=\frac{29}{13}\mathbf{i}+\frac{80}{13}\mathbf{j}$ oe
Method 1 using triangle *ACF* (they may use any letters for μ and λ)		
(d)	B1	States $\overrightarrow{AF}=\lambda\overrightarrow{AE}$ or $\overrightarrow{AF}=\lambda'\left(\frac{29}{13}\mathbf{i}+\frac{80}{13}\mathbf{j}\right)'$
	M1	$\overrightarrow{AF}=\overrightarrow{AC}+\mu\overrightarrow{DC}$
	A1	For the fully correct expression which need not be simplified $\overrightarrow{AF}=-2\mathbf{i}+15\mathbf{j}+\mu'(5\mathbf{i}+5\mathbf{j})'$
	M1	Sets $\lambda'\left(\frac{29}{13}\mathbf{i}+\frac{80}{13}\mathbf{j}\right)'=-2\mathbf{i}+15\mathbf{j}+\mu'(5\mathbf{i}+5\mathbf{j})'$ and equates coefficients of $\mathbf{i}$ and $\mathbf{j}$ to form two equations in λ and μ. Condone **i** and **j** in their equations.
	A1	For finding $\mu=\frac{7}{3}$
	A1	$DC:CF=1:\frac{7}{3}$ or $DC:CF=3:7$
Method 2 using triangle *ADF* (they may use any letters for μ and λ)		
	B1	States $\overrightarrow{AF}=\lambda\overrightarrow{AE}$ or $\overrightarrow{AF}=\lambda'\left(\frac{29}{13}\mathbf{i}+\frac{80}{13}\mathbf{j}\right)'$
	M1	$\overrightarrow{AF}=\overrightarrow{AD}+\mu\overrightarrow{DC}$
	A1	For a fully correct expression which need not be simplified $\overrightarrow{AF}=-7\mathbf{i}+10\mathbf{j}+\mu'(5\mathbf{i}+5\mathbf{j})'$
	M1	Sets $\lambda'\left(\frac{29}{13}\mathbf{i}+\frac{80}{13}\mathbf{j}\right)'=-7\mathbf{i}+10\mathbf{j}+\mu'(5\mathbf{i}+5\mathbf{j})'$ and equates coefficients of $\mathbf{i}$ and $\mathbf{j}$ to form two equations in μ and λ. Condone **i** and **j** in their equations.
	A1	For finding $\mu=\frac{10}{3}$

	A1	$DC : DF = 1 : \frac{10}{3} \quad \Rightarrow \quad DC : CF = 1 : \frac{7}{3}$ or $DC : CF = 3 : 7$
Method 3 using similar triangles		
(d)	B1	States triangles *AEB* and *DEF* are similar. Can be implied from correct work.
	M1	$BE : ED = 3 : 10$ (given)
	A1	So therefore correspondingly $AB : DF = 3 : 10$
	M1	*AB* = *DC* parallelogram
	A1	So *DC:DF* = 3:10
	A1	Hence *DC:CF* = 3:7

Useful sketch



Question number	Scheme	Marks
11 (a)	$AC = 20x$ or $AX = 10x$	B1
	$EX = AX \tan 45° = 10x$	M1A1 [3]
(b)	$EA = \dfrac{EX}{\sin 45°} = \dfrac{10x}{\sin 45°}, = \sqrt{200}x = \left(10\sqrt{2}x\right)$	M1A1
	Or use ΔEAC which is right-angled and isosceles	[2]
(c)	$\tan\theta = \dfrac{EX}{\frac{1}{2}AD} = \dfrac{10}{6}$	M1A1ft
	$\theta = 59.0°$	A1 [3]
(d)	Reqd angle is $AXD\left(=\phi\right)$	
	$\tan\frac{1}{2}\phi = \dfrac{6}{8}$	M1A1
	$\phi = 73.7°$ must be acute	A1 [3]
	ALT: Use cosine rule in triangle *AXD*	
(e)	*Y* is midpoint of *AD*	
	$EY = x\sqrt{\left(10\sqrt{2}\right)^2 - 6^2}\ \left(= x\sqrt{164} \text{ or } 2x\sqrt{41}\right)$	M1
	Area $\Delta AED = 6x^2\sqrt{164} = 250$	M1
	$x = 1.8037... = 1.804$	A1 [3]
		Total 14 marks

In this question penalise ROUNDING of angles only once in parts (c) and (d).

This applies only if both answers are correct but over-accurate, i.e. they would both round to the correct angle to 1 decimal place. For example; for an angle in (c) given as 59.04° award M1A1A0, but if the angle in (d) is then given as 73.74° do not penalise the angle in (d), and award M1A1A1.

If however the answer given in (c) is 59° without 59.04° seen, this is M1A1A0, and because it is under-accurate, if they then give the angle in (d) as 73.74°, then this is awarded M1A1A0 as well.

<table>
<tr><th colspan="3">Notes</th></tr>
<tr><td rowspan="3">(a)</td><td>B1</td><td>For using Pythagoras theorem to find either $AC = 20x$ or $AX = 10x$
Do not accept $AC = 20$ or $AX = 10$</td></tr>
<tr><td>M1</td><td>For $EX = AX \tan 45°$ = ($10x$)
Ft their AX but not if they use their AC
If there is no x in their working it is M0 UNLESS they put x into their final answer.</td></tr>
<tr><td>A1</td><td>For $10x$</td></tr>
<tr><td colspan="3">ALT</td></tr>
<tr><td rowspan="2"></td><td>M1</td><td>Triangle AEC is isosceles with $\angle EAX = \angle ECX = 45^{\circ}$
Hence triangle AEX is also isosceles with $\angle EAX = \angle AEX = 45^{\circ}$

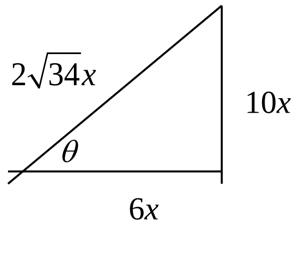
If there is no x in their working it is M0 UNLESS they put x into their final answer.</td></tr>
<tr><td>A1</td><td>Hence $AX = EX$ so $EX = 10x$</td></tr>
<tr><td colspan="3">Accept $EX = 10x$ just stated without working.</td></tr>
<tr><td rowspan="2">(b)</td><td>M1</td><td>Uses Pythagoras theorem or any acceptable trigonometry to find length AE

$AE = \sqrt{(10x)^2 + (10x)^2} = \left(\sqrt{200}x\right)$

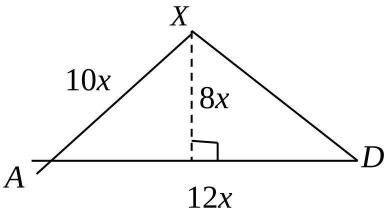
$AE = \dfrac{10x}{\sin 45^{\circ}} = \left(\sqrt{200}x\right)$ or $AE = \dfrac{10}{\cos 45^{\circ}} = \left(\sqrt{200}x\right)$

If there is no x in their working it is M0 UNLESS they put x into their final answer.</td></tr>
<tr><td>A1</td><td>$AE = 10x\sqrt{2}$ or $\sqrt{200}x$ Also accept $14.1x$ or better
Accept unsimplified answers. Even accept answers given as $AE = \dfrac{10x}{\sqrt{2}/2}$</td></tr>
<tr><td rowspan="3">(c)</td><td>M1</td><td>Uses their EX to find $\tan\theta = \dfrac{'EX'}{\frac{1}{2}AD} = \dfrac{'10x'}{6x} \Rightarrow \theta = ...$

(Or any other complete method for the required angle)



Accept working without x’s as this is a ratio, but do not accept an x in the numerator or denominator only.</td></tr>
<tr><td>A1ft</td><td>A correct value of θ for their EX</td></tr>
<tr><td>A1</td><td>$\theta = 59.0°$ rounded correctly</td></tr>
</table>

(d)	M1	 Angle required is AXD $\tan\left(\frac{1}{2}\angle AXD\right)=\dfrac{\frac{1}{2}\times AD}{\frac{1}{2}\times CD}=\dfrac{6x}{8x}\Rightarrow \angle AXD=.....$ $\sin\left(\frac{1}{2}\angle AXD\right)=\dfrac{\frac{1}{2}\times AD}{'10x'}=\dfrac{6x}{10x}\Rightarrow \angle AXD=.....$ $\cos\left(\frac{1}{2}\angle AXD\right)=\dfrac{\frac{1}{2}\times CD}{'10x'}=\dfrac{8x}{10x}\Rightarrow \angle AXD=.....$ Accept working without x's as this is a ratio, but do not accept x in the numerator or denominator only.
	A1	Uses correct values for their method
	A1	$\angle AXD=73.7°$ (must be acute) **Note: Do not isw if both angles are given and the acute angle not identified.**
ALT (using cosine rule)		
(d)	M1	$\angle AXD=\cos^{-1}\left(\dfrac{(10x)^2+(10x)^2-(12x)^2}{2\times 10x\times 10x}\right)=\dfrac{56x^2}{200x^2}=....$ Accept working without x's as this is a ratio, but do not accept x^2 in the numerator or denominator only.
	A1	Uses the correct value for AX
	A1	$\angle AXD=73.7°$ (must be acute) **Note: Do not isw if both angles are given and the acute angle not identified.**
(e)	M1	Finds the length of E to the midpoint of AD (Y) $EY=x\sqrt{('10\sqrt{2}')^2-6^2}\ \left(=x\sqrt{164}\text{ or }2x\sqrt{41}\right)$ ft their AE Any working without x is M0.
	M1	Equates area of 250 cm^2 to $\frac{1}{2}\times AD\times EY=\frac{1}{2}\times 12x\times x\sqrt{164}\Rightarrow x=...$
	A1	$x=1.804$ rounded correctly
ALT cosine rule and $\frac{1}{2}ab\sin C$		
(e)	M1	Finds angle AED $\angle AED=\cos^{-1}\left(\dfrac{('10\sqrt{2}x')^2+('10\sqrt{2}x')^2-(12x)^2}{2\times('10\sqrt{2}x')\times('10\sqrt{2}x')}\right)=50.208...^{o}$ Any working without x is M0.
	M1	Equates area of 250 cm^2 to expression for area of triangle: $250=\frac{1}{2}\times AE\times ED\sin\angle AED=\frac{1}{2}\times 10\sqrt{2}x\times 10\sqrt{2}x\sin 50.208^{o}\Rightarrow x^2=3.2536...^{o}$ $\Rightarrow x=1.8037...$
	A1	x = 1.804 rounded correctly