Pearson Edexcel

Mark Scheme (Results)

October 2023

Pearson Edexcel International Advanced Level in Pure Mathematics (WMA13) Paper 01

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October 2023
Question Paper Log Number P74328A
Publications Code WMA13_01_ rms_20240118

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

The total number of marks for the paper is 75.

Edexcel Mathematics mark schemes use the following types of marks:

'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation, e.g. resolving in a particular direction; taking moments about a point; applying a suvat equation; applying the conservation of momentum principle; etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) each term needs to be dimensionally correct

For example, in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

'M' marks are sometimes dependent (DM) on previous M marks having been earned, e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. e.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph).

A and B marks may be f.t. – follow through – marks.

General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod means benefit of doubt
- ft means follow through
    - the symbol $\sqrt{}$ will be used for correct ft
- cao means correct answer only
- cso means correct solution only, i.e. there must be no errors in this part of the question to obtain this mark
- isw means ignore subsequent working
- awrt means answers which round to
- SC means special case
- oe means or equivalent (and appropriate)

- dep means dependent
- indep means independent
- dp means decimal places
- sf means significant figures
- ***** means the answer is printed on the question paper
- ⎣ means the second mark is dependent on gaining the first mark

All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

If a candidate makes more than one attempt at any question:
- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

- Factorisation
  - $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$
  - $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$
- Formula
  - Attempt to use the correct formula (with values for *a*, *b*and *c*).
- Completing the square
  - Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

- Differentiation
  - Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)
- Integration
  - Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first. Normal marking procedure is as follows:

- Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.
- Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1(a)	$(\mathrm{f}(1)=)1-5+\mathrm{e}=-1.281...<0$ $(\mathrm{f}(2)=)2^2-5\times 2+\mathrm{e}^2=1.389...>0$	M1
	As there is a <u>change of sign</u> and $\mathrm{f}(x)$ is <u>continuous</u> over the interval $[1,\ 2]$ then <u>there is a root</u> *	A1*
		(2)
(b)(i)	$x_2=\sqrt{5\times 1-\mathrm{e}^1}=\text{awrt}\,1.5105$	M1A1
(ii)	$\alpha=\text{awrt }1.7340$	A1
		(3)
		(5 marks)

Notes

(a)

M1: Attempts $\mathrm{f}(1)$ and $\mathrm{f}(2)$ with substitution seen or at least one correct to 1 d.p. rounded or truncated **and considers their signs**. Note showing f(1)f(2) < 0 is a consideration of signs and is sufficient for the "sign change" part of reasoning for the A1.

A1*: Must have

- both $\mathrm{f}(1)$ and $\mathrm{f}(2)$ correct (as expressions or correct values imply these, need not be labelled)
- reason, which must mention continuity and state or indicate sign change in some way
- conclusion, "hence root", or accept e.g. hence "f(x)=0 between x =1 and x = 2"

(b)

(i)

M1: Attempts to find x_2 using the iteration formula. Implied by sight of 1 embedded in the formula or awrt 1.5

A1: awrt 1.5105

(ii)

A1: awrt 1.7340 provided M1 has been scored. Allow 1.734 with trailing zero omitted.

Note 1.7340 only may be from a graphical calculator, so scores M0A0A0. There must be evidence (i.e. the M is scored) of an attempt at least one iteration first.

Question Number	Scheme	Marks
2(a)	$\text{ff}(6)=\text{f}\left(\text{"}\frac{9}{2}\text{"}\right)=\frac{\text{"}9/2\text{"}+3}{\text{"}9/2\text{"}-4}=\ldots;\ =15$	M1; A1
		(2)
(b)	$\text{f}^{-1}(x)=\frac{4x+3}{x-1}$	M1A1
	$x\in\mathbb{R},\ x\neq 1$	B1
		(3)
(c)	E.g. $\left(\frac{x+3}{x-4}\right)^2+5=7$ or $\frac{a+3}{a-4}=(\pm)\sqrt{7-5}$	M1
	$\Rightarrow x^2-22x+23=0\Rightarrow x=\ldots$ or $(a+3)=(\pm)\sqrt{2}(a-4)\Rightarrow a=\ldots$	dM1
	$(a=)11+7\sqrt{2}$ oe	A1
		(3)
		(8 marks)

Notes

(a)

M1: Substitutes $x=6$ into f and substitutes the result back into f to find a value for $\text{ff}(6)$

Alternatively substitutes $x=6$ into $\text{ff}(x)=\dfrac{\left(\frac{x+3}{x-4}\right)+3}{\left(\frac{x+3}{x-4}\right)-4}$ condoning slips.

A1: 15

(b)

M1: Changes the subject of $y=\frac{x+3}{x-4}$ or $x=\frac{y+3}{y-4}$ to $y=\frac{\ldots x\pm 3}{\ldots x\pm 1}$ or $x=\frac{\ldots y\pm 3}{\ldots y\pm 1}$

A1: Achieves $\text{f}^{-1}(x)=\frac{4x+3}{x-1}$. Accept $\text{f}^{-1}=\ldots$ or $y=\ldots$ instead of $\text{f}^{-1}(x)=\ldots$.

B1: $x\neq 1$. The omission of $x\in\mathbb{R}$ is condoned. Accept alternative notations.

(c) Note this is M1A1A1 on epen but is being marked as M1dM1A1.

M1: Correct attempt to set up an equation in *x* or *a*. This will usually be $\left(\frac{x+3}{x-4}\right)^2+5=7$ but $\text{f}(a)=\text{g}^{-1}(7)$ may be used first. Score when a suitable equation is set up and allow if a minor slip or miscopy is made if the intention is clear.

dM1: Proceeds to solve for *x* or *a*, e.g. forms a 3TQ and solves, or take the 5 across, square roots, cross multiplies and makes *a* the subject, condoning e.g. sign slips when rearranging.
Alternatively, uses $(\text{f}(x))^2+5=7\Rightarrow \text{f}(x)=(\pm)\sqrt{2}$ and their part (b) leading to $a=\text{f}^{-1}\left(\text{"}\sqrt{2}\text{"}\right)=\frac{\text{"}4\sqrt{2}+3\text{"}}{\text{"}\sqrt{2}-1\text{"}}$. In this method both M's may be scored together.

A1 $11+7\sqrt{2}$ or $\frac{4\sqrt{2}+3}{\sqrt{2}-1}$ Accept $11+\sqrt{98}$ or $\frac{-4\sqrt{2}-3}{1-\sqrt{2}}$. Common factors should be cancelled.
Ignore labelling of *x* or *a*. Condone e.g. if $11-7\sqrt{2}$ is also given as a solution but A0 if any other extra solutions are included.

Question Number	Scheme	Marks
3(a)	$(\cos 2A \equiv)\cos A\cos A - \sin A\sin A \equiv \cos^2 A - (1-\cos^2 A)$	M1
	$\Rightarrow \cos 2A \equiv 2\cos^2 A - 1 \quad *$	A1*
		(2)
(b)	$\int (3-2\cos 6x)\,\mathrm{d}x = 3x - \dfrac{\sin 6x}{3} \quad (+c)$	M1A1
	$\left[3x - \dfrac{\sin 6x}{3}\right]_{\frac{\pi}{12}}^{\frac{\pi}{8}} = \left(3\left(\dfrac{\pi}{8}\right) - \dfrac{\sin\left(6\times\frac{\pi}{8}\right)}{3}\right) - \left(3\left(\dfrac{\pi}{12}\right) - \dfrac{\sin\left(6\times\frac{\pi}{12}\right)}{3}\right) = \dfrac{1}{8}\pi + \dfrac{2-\sqrt{2}}{6}$	dM1A1
		(4)
		(6 marks)

Notes

(a)

M1: Writes $(\cos 2A \equiv)\cos A\cos A - \sin A\sin A$ and attempts to **use** (not just stated separately) the identity $\pm\sin^2 A \pm \cos^2 A = \pm 1$ Note going directly to $\cos 2A \equiv \cos^2 A - \sin^2 A$ is allowed M1 as long as a use of the Pythagorean identity is clear. NB Watch out for answers which use double angle identities in the proof (circular reasoning) and score M0 if no Pythagorean identity is used.

A1*: Achieves the given answer with no errors seen (and no circular reasoning). Must see $\cos 2A \equiv \cos A\cos A - \sin A\sin A$ and clear attempt at substitution of $\sin^2 A + \cos^2 A = 1$ but allow "LHS" for "cos 2*A*". Going directly to $\cos 2A \equiv \cos^2 A - \sin^2 A$ in the first step would score A0. Correct notation must used throughout but condone a missing closing bracket.

Note : $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1$ with no further evidence scores M0A0 as they are just writing out known formulae.

(b)

M1: Attempts to substitute in the given result in part (a) and integrates to the form $...x \pm ...\sin 6x$ Must be a full substitution in terms of *x*, $\cos 2A \to k\sin 2A$ is M0 unless recovered, e.g by later substitution may be implied by the limits substituted).

A1: $3x - \dfrac{\sin 6x}{3}$

dM1: Shows evidence of substituting in both limits into an expression of the form $...x \pm ...\sin 6x$ and subtracts either way round to achieve an exact (not necessarily simplified but trig must be evaluated) answer.

A1: Correct answer e.g as scheme or $\dfrac{1}{8}\pi + \dfrac{1}{3} - \dfrac{\sqrt{2}}{6}$ or exact equivalent (and isw after a correct answer).

Question Number	Scheme	Marks
4(a)	$A = 93$	B1
		(1)
(b)	$100 = 125 - "93"\mathrm{e}^{-0.109T} \Rightarrow A\mathrm{e}^{-0.109T} = ...$	M1
	$A\mathrm{e}^{kT} = B \Rightarrow T = \dfrac{\ln\left(\dfrac{B}{A}\right)}{k}$	dM1
	$T = 12.05$	A1
		(3)
(c)	$\dfrac{\mathrm{d}N}{\mathrm{d}t} = 0.109 \times "93"\mathrm{e}^{-0.109\times 7}$	M1
	4 730 (total sales per month)	A1
		(2)
(d)	The limit is 125 000 / the model has a limit below 150 000	B1
		(1)
		(7 marks)

Notes

(a)

B1: 93

(b)

M1: Sets $100 = 125 - "93"\mathrm{e}^{-0.109T}$ and proceeds to $A\mathrm{e}^{-0.109T} = ...$ May use *t* instead of *T*. Condone slips.

dM1: Uses correct order of operations and correct log work from an equation of form $A\mathrm{e}^{kT} = B,\ AB > 0$ to proceed to a value for *T* (must be a solvable equation). Implied by a correct answer for their equation.

A1: 12.05 cao

Note: Candidates who use 100 000 in the equation will score no marks in this part.

Note: Answer only with no working send to review, but 12.05 following a correct equation seen score M1dM1A1.

(c)

M1: Attempts to differentiate to form $\lambda\mathrm{e}^{-0.109\times t}$, λ constant, and substitutes in $t = 7$. Award for an expression of the form $...\mathrm{e}^{-0.109\times 7}$ if no incorrect working is seen. The substitution of $t = 7$ may be implied by a correct value for their derivative of correct form.

A1: awrt 4 730 (total sales per month). Accept equivalent forms for the answer, e.g. 4.73 thousand, but just 4.73 is A0. "Sales per month" may be omitted, but score A0 if an incorrect unit is given.

Note: Answer only scores no marks.

(d)

B1: A **correct** reason given – any factual statements must be correct. E.g The limit is 125 000 / the model has a limit below 150 000 oe. Accept maximum value of *N* is 125, or *N* cannot reach 150 as asymptote at 125. Also allow attempts to substitute in 150 and find that they cannot take the log a negative number, but score B0 if a "negative time" is reached and used as reason.

Attempts using e.g 150 000 in the equation score B0.

There must be a reference to the information given in the answer, just "number is too large" or similar is B0.

Question Number	Scheme	Marks
5(a)	$\dfrac{dy}{dx}=\dfrac{(x^2+k)\times\dfrac{2x}{x^2+k}-2x\ln(x^2+k)}{(x^2+k)^2}$	M1
	$\dfrac{dy}{dx}=\dfrac{2x-2x\ln(x^2+k)}{(x^2+k)^2}=\dfrac{2x(1-\ln(x^2+k))}{(x^2+k)^2}$	M1A1
		(3)
(b)	$x=0$	B1
	$"1"-\ln(x^2+k)=0\Rightarrow x^2=e^{"1"}\pm k$	M1
	$x=\pm\sqrt{e-k}$	A1ft
		(3)
(c)	Upper limit is e or $k<e$	B1ft
		(1)
		(7 marks)

Notes

(a)

M1: Attempts to apply the quotient rule. Score for an expression of the form

$$\frac{dy}{dx}=\frac{(x^2+k)\times\dfrac{...}{x^2+k}-Qx\ln(x^2+k)}{(x^2+k)^2}$$ oe (may be simplified) where … is P or Px and P,$Q>0$

May also attempt the product rule, in which case look for

$-Px\left(x^2+k\right)^{-2}\ln(x^2+k)+\left(x^2+k\right)^{-1}\times\dfrac{...}{x^2+k}$ where … is Q or Qx and $P,Q>0$

M1: Cancels x^2+k and correctly takes a factor of (2)x from the numerator to achieve "$B-\ln(x^2+k)$" (may be done in one step). They must have a factor x in both terms. Note this may be scored following M0 e.g. if a numerator $uv'-vu'$ is used, or if the denominator misses the square.

A1: $\left(\dfrac{dy}{dx}=\right)\dfrac{2x(1-\ln(x^2+k))}{(x^2+k)^2}$ from fully correct work. Both M's must have been scored.

(b)

B1: $x=0$

M1: Sets $"1"-\ln(x^2+k)=0$ and proceeds to $x^2=...$ with correct undoing of ln.

A1ft: $x=\pm\sqrt{e-k}$ or follow through $x=\pm\sqrt{e^B-k}$ for a numerical B in their $B-\ln\left(x^2+k\right)$ factor

(c)

B1ft: Upper limit is e or $k<e$ stated. Follow on their (b) of form $x=(\pm)\sqrt{e^B\pm k}$ B0 if $k=e$ or $k\leqslant e$ is given as the answer without statement about upper limit being e.

Question Number	Scheme	Marks
6(a)	$\log_{10} S = 4.5 - 0.006\times 2 \Rightarrow S = 10^{4.5-0.006\times 2} = 30800\,\text{km}^2$	M1A1
		(2)
(b)	$\log_{10} S = 4.5 - 0.006t \Rightarrow S = 10^{4.5-0.006t}$ (or $p = 10^{4.5}$ or $q = 10^{-0.006}$)	M1
	$S = 10^{4.5-0.006t} = 10^{4.5}\times\left(10^{-0.006}\right)^t$ (or $p = 10^{4.5}$ and $q = 10^{-0.006}$)	dM1
	$S = 31600\times(0.986)^t$	A1
		(3)
(c)	E.g. The proportion of area covered by coral reefs retained from year to year.	B1
		(1)
		(6 marks)

Notes

(a)

M1 Substitutes in $t = 2$ and proceeds to find **a value** for S via $10^{\cdots}$ May be implied by a correct value for their equation. Note awrt 24000 may arise from a slip with 0.006×2 = 0.12, and is acceptable for M1.

A1: awrt $30800\,\text{km}^2$ including units, from correct work and isw as long as units are included with a correct value. But e.g. 30760 followed by 307 km^2 is A0.

(b)

M1: Writes $S = 10^{4.5-0.006t}$ or award for either $(p =)10^{4.5}$ or $(q =)10^{-0.006}$ even if from an incorrect step. Essentially for knowing they need to raise to base 10. May arise from attempts at solving simultaneous equations using the answer to (a) (allowing for rounding errors).

dM1: Writes $S = 10^{4.5}\times\left(10^{-0.006}\right)^t$ or award for both $p = 10^{4.5}$ and $q = 10^{-0.006}$ (oe forms e.g accept $\left(\frac{1}{10^{0,006}}\right)$ or $\left(\frac{1}{1.01}\right)$ for q) Must be fully correct method to this point but allow if all that is wrong is rounding errors from other methods such solving simultaneous equations (e.g. 0.990^t).

A1cso: $S = 31600\times(0.986)^t$ Must be awrt 3 s.f. values. Must be an equation including the S, not just p and q values, but accept if seen in (c).

(c)

B1: E.g. accept the following or variations of them

- The proportion of area covered by coral reefs retained from year to year.
- The seafloor covered by coral reduces by 1.4% each year.
- The proportional change/decrease in sea floor area covered by coral reefs per year

Do not accept "rate of change" or "decrease in area of reefs per year" or "q is the gradient" as the answer as it is an incorrect interpretation. The proportional change should be conveyed.

There must be reference to, or equivalents of, the following:

- sea floor area or sea floor or area or seabed
- coral reefs or coral or reef,
- each year or year to year or per year
- (q or its value is) a percentage/proportional decrease/remaining

Some acceptable answers:

- After 1 year the area will reduce to S last year times q. It shows the speed that coral reefs seafloor reduce.
- The percentage of remaining area of sea floor covered by coral reefs to the area of that covered last year.
- q represents the proportion of decreasing of the area of sea floor covered by coral reefs every year.

Some unacceptable answers:

- When there is extra t, the S will increase one more unit of 0.986
- q means the percentage of the part of sea floor covered by coral reefs after t years.
- q is the amount of coral reefs die/shrink in accordance to time.
- The area of the sea floor is decreasing at almost 0.986 times before.
- For every increase in the value of t, the total area of coral reefs will decrease in a rate of 0.986^t

Question Number	Scheme	Marks
7(a)	$\mathrm{f}(0)=(0-3)^2=9$	M1
	$0 \leqslant \mathrm{f}(x) \leqslant 9$	A1
		(2)
(b)	$\mathrm{f}'(x)=-2x\mathrm{e}^{-x^2}(2x^2-3)^2+\mathrm{e}^{-x^2}\times 8x(2x^2-3)$	M1A1
	$=2x(2x^2-3)\mathrm{e}^{-x^2}\left(-(2x^2-3)+4\right)=2x\mathrm{e}^{-x^2}(2x^2-3)\left(7-2x^2\right)$	dM1A1
		(4)
(c)	$x^2=\frac{3}{2},\ \frac{7}{2}\Rightarrow \mathrm{f}\left(\sqrt{\frac{7}{2}}\right)=\mathrm{e}^{-\frac{7}{2}}\left(2\times\frac{7}{2}-3\right)^2=16\mathrm{e}^{-\frac{7}{2}}$	M1A1
	$16\mathrm{e}^{-\frac{7}{2}}<k<9$	dM1A1
		(4)
		(10 marks)

Notes

(a) Work for part (a) must be seen in part (a) not recovered in part (c).

M1: Substitutes $x=0$ and proceeds to find a value for *y*. Implied by sight of 9.

A1: $0 \leqslant \mathrm{f}(x) \leqslant 9$ Accept with *y* or just f, but not with *x*. Accept interval, [0, 9], or set notation.

(b)

M1: Attempts the product rule and chain rule achieving $\pm Px\mathrm{e}^{-x^2}(2x^2\pm 3)^2+\mathrm{e}^{-x^2}\times Qx(2x^2\pm 3)$ with *P*, *Q* > 0. Alternatively, attempts the quotient rule on $\frac{(2x^2-3)^2}{\mathrm{e}^{x^2}}$ achieving $\frac{Px(2x^2\pm 3)\mathrm{e}^{x^2}-Qx\mathrm{e}^{x^2}(2x^2\pm 3)^2}{\left(\mathrm{e}^{x^2}\right)^2}$ *P*, *Q* > 0

A1: $-2x\mathrm{e}^{-x^2}(2x^2-3)^2+\mathrm{e}^{-x^2}\times 8x(2x^2-3)$ oe (need not be simplified)

dM1: Must have scored previous M. Attempts to takes out a factor of $2x\mathrm{e}^{-x^2}(2x^2-3)$ to obtain a factor $\left(\pm C\pm Dx^2\right)$ which may be unsimplified. Allow if e.g. the *x* or e^{-x^2} is dropped when taking out the factor.

A1: Achieves $2x\mathrm{e}^{-x^2}(2x^2-3)(7-2x^2)$ with no errors seen.

(c) **Note : allow all the marks in (c) from answers which were a constant multiple out in (b) or missing the factor *x*, which lead to the correct answers here.**

M1: Attempts to find a *y* value (which may be zero) for at least one of the valid non-zero roots for f'(*x*)=0. Allow for an attempt at either $\mathrm{f}\left(\pm\sqrt{\frac{3}{2}}\right)$ or $\mathrm{f}\left(\pm\sqrt{\frac{A}{B}}\right)$ leading to a value, where *A* and *B* are their values from (b) with *AB* > 0. Note e.g. $\mathrm{f}\left(\frac{A}{B}\right)$ attempted is M0.

A1: For obtaining $16\mathrm{e}^{-\frac{7}{2}}$ Accept awrt 0.483 for this mark following the award of M.

dM1: Attempts the inside region, allowing $\leqslant$, using their *y* intercept from (a) and their **positive** *y* value, less than their intercept, from an attempt at the at the *y* value of the maxima. Allow a positive *y* value from an attempt at any non-zero root of f'(*x*) as such an attempt. It is dependent on the previous method mark.

A1: $16\mathrm{e}^{-\frac{7}{2}}<k<9$ or in any equivalent form e.g. interval notation $k\in\left(\frac{16}{\mathrm{e}^{7/2}},9\right)$ Allow *y* instead of *k*

Ignore references to "*k* = 0"

Question Number	Scheme	Marks
8(a)	Starting with the LHS: $2\text{cosec}^2 2\theta(1-\cos 2\theta)=\dfrac{2-2\cos 2\theta}{\sin^2 2\theta}$	M1
	$=\dfrac{2-2(1-2\sin^2\theta)}{4\sin^2\theta\cos^2\theta}$	M1dM1
	$=\sec^2\theta = 1+\tan^2\theta \equiv \text{RHS}$ *	A1*
		(4)
(b)	$\sec^2 x - 3\sec x - 4 = 0 \Rightarrow \sec x = ...$	M1
	$\cos x = \dfrac{1}{4}$ (ignore -1)	A1
	$\cos x = \dfrac{1}{4} \Rightarrow x = ...$	dM1
	$x = 75.5°,\ 284.5°$	A1
		(4)
		(8 marks)

Notes

(a) The most common method and where to award marks is as follows:

M1: Uses $\text{cosec}\,2\theta = \dfrac{1}{\sin 2\theta}$ (oe) at some stage in the proof to convert the cosec into sine.

M1 Attempts to use one of $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ or $\sin 2\theta = 2\sin\theta\cos\theta$ but condone $\sin^2 2\theta = 2\sin^2\theta\cos^2\theta$ as an attempt at squaring.

dM1: Depends on previous M. Attempts to use both $\cos 2\theta = 1-2\sin^2\theta$ oe and $\sin 2\theta = 2\sin\theta\cos\theta$

A1*: Achieves the RHS with no mathematical errors seen and all stages of working shown. Must see the $\sec^2\theta$ before the final answer. Condone minor notational errors (e.g. a missing θ) but not consistent poor notation throughout. There will be other solutions but if they work from both sides then they need a conclusion at the end.

For other methods or variations apply scheme as follows:

M1: Uses a correct identity for either cosec 2θ or $\text{cosec}^2 2\theta$ or $\cot^2 2\theta$ to get the equation in terms of sin and cosine only or to introduce cosec2θ if working in reverse. Condone the 2 becoming ½.

M1: Attempts to apply a double angle identity correct up to sign errors, e.g. $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$

dM1: Depends on previous M. Use of at least two **correct** double angle identities and/or Pythagorean identities to *progress sufficiently towards the goal* (e.g. reduces all terms to single argument form in cosine or sine only, or identify an appropriate factor to cancel to such, or to get all double arguments if working in reverse with the sin 2θ identified). There may be algebraic slips, but all trig identities used up to this point must be correct (e.g. see stag reach in main scheme).

A1*: Fully correct proof with sufficient stages shown including use of $\sec^2\theta = 1+\tan^2\theta$ or equivalent identity (e.g. other Pythagorean relation), and if necessary, a conclusion given.

For example $1+\tan^2\theta = \sec^2\theta = \dfrac{1}{\cos^2\theta} = \dfrac{2}{1+\cos 2\theta}$ 2nd M1, use of a suitable double angle identity

$=\dfrac{2(1-\cos 2\theta)}{(1+\cos 2\theta)(1-\cos 2\theta)} = \dfrac{2(1-\cos 2\theta)}{1-\cos^2 2\theta} = \dfrac{2(1-\cos 2\theta)}{\sin^2 2\theta}$ 3rd dM1, correct initial double angles identity and Pythagorean identity used to identify the sin in the denominator, all identities correct

$=2\text{cosec}^2 2\theta(1-\cos 2\theta)$ 1st M1, use of correct identity for $\text{cosec}^2 2\theta$. (A1 if all correct)

(b) Note: allow with x or θ and allow recovery of mixed variables in this part.

M1: Uses part (a) to forms a 3 term quadratic in $\sec x$ or cos x, solves their quadratic and finds cos x =… or sec x =… for at least one solution. For reference the quadratic in cos is $4\cos^2 x + 3\cos x - 1 = 0$

A1: $\cos x = \frac{1}{4}$ (ignore any reference to $\cos x = -1$) Must have a correct cosine or implied by a correct answer after reaching a secant if no incorrect working seen. May be implied if not seen explicitly.

dM1: Correct method to find a value for x from cos $x = k$ where $|k| < 1$ as a solution for their equation. May be implied by one correct angle from $\sec x = k$ or $\cos x = k$. Note radian answer awrt 1.3 implies dM1.

A1: awrt 75.5° and awrt 284.5° Allow with or without 180° included, but there must be no other angles in given interval. All previous marks must have been scored. Ignore answers outside the domain.

(b)Alt	$1+\tan^2 x = 4+3\sec x \Rightarrow 9\sec^2 x = \tan^4 x - 6\tan^2 x + 9 \Rightarrow \tan^4 x - 15\tan^2 x = 0$ $\Rightarrow \tan^2 x = ...(\neq 0)$	M1
	$\tan x = (\pm)\sqrt{15}$	A1
	$\tan x = k \neq 0 \Rightarrow x = ...$	dM1
	$x = 75.5°,\ 284.5°$	A1
		(4)

Notes

(b) Alt

M1: Makes sec x the subject, squares and solves the resulting quadratic in $\tan^2 x$ leading to a value for $\tan^2 x$. Note there may be variations on this approach, such as $1+\tan^2 x = 4+3\sqrt{1+\tan^2 x}$ being used before squaring, or solving for "$1+\tan^2 x$". Score for a correct approach leading to a value for $\tan^2 x$.

A1: Correct value for tan x. Need not give both values.

dM1: Solves from their tan $x = k$, $k \neq 0$ to find a value for x

A1: Both correct values obtained and no invalid solutions in the range (ignore 180° as main scheme). Must reject extra solutions.

Note: other methods or variations may be seen, which can be marked according to the same principles, first M for correct approach to find a value for a trig ratio, A1 correct non-trivial value, dM1 solves for x.

Question Number	Scheme	Marks
9(a)	$k=-1$	B1
		(1)
(b)(i)	$\text{f}(0)=\lvert 2-4\ln(0+1)\rvert=2-0=2$	B1
(ii)	$0=2-4\ln(x+1)\Rightarrow\ln(x+1)=\frac{1}{2}\Rightarrow x=\text{e}^{p}+q$	M1
	$x=\text{e}^{\frac{1}{2}}-1$	A1
		(3)
(c)	$2-4\ln(x+1)=3\Rightarrow\ln(x+1)=...$ or $-2+4\ln(x+1)=3\Rightarrow\ln(x+1)=...$	M1
	$2-4\ln(x+1)=3\Rightarrow x=...$ and $-2+4\ln(x+1)=3\Rightarrow x=...$	dM1
	CVs $\text{e}^{-\frac{1}{4}}-1,\ \text{e}^{\frac{5}{4}}-1$	A1
	$"-1"<x<\text{e}^{-\frac{1}{4}}-1$ or $x>\text{e}^{\frac{5}{4}}-1$	ddM1A1ft
		(5)
		(9 marks)

Notes

(a)

B1: For $(k=)-1$. Accept $x=-1$ or even $x>-1$

(b) (i)

B1: For 2 seen as the y coordinate. May be identified as y or f(0) or (0, 2).

(ii)

M1: Sets $0=2-4\ln(x+1)$, rearranges and proceeds to $x=...$ of the correct form $\text{e}^{p}+q,\ q\neq 0$. May be implied by awrt 0.65.

A1: $x=\text{e}^{\frac{1}{2}}-1$ or accept $\sqrt{\text{e}}-1$ or even $\text{e}^{\frac{2}{4}}-1$ May be seen as part of a coordinate pair.

(c) Note this is M1A1A1M1A1 on epen but is being marked as M1M1A1M1A1.

M1: Forms one valid equation with the modulus signs removed and attempts to solve at least as far as $\ln(x+1)$. Accept with > or < etc instead of = for the M.

dM1: Both equations attempted, allowing for "=" or any inequality used, to achieve values (may be decimals) for each via $\ln(x+1)=\ldots$ and suitable attempt to undo ln (raises to a base power, allow an error with the base). For reference the decimals are −0.221 and 2.49 to 3 s.f.

A1: Both $\text{e}^{-\frac{1}{4}}-1,\ \text{e}^{\frac{5}{4}}-1$ Must be exact.

ddM1: Chooses the outside region for their values. Allow with strict or non-strict inequalities. It is dependent on both previous method marks and requires two distinct critical values. The left hand bound −1 may be missing for this mark (i.e. allow for $x<\text{e}^{-\frac{1}{4}}-1$ or $x>\text{e}^{\frac{5}{4}}-1$) Allow with decimal values for this mark. ddM0 if the middle section is also included but isw if they choose the correct outside region but later reject the left hand portion due to confusion with $x>-1$ Watch out for $x>\ldots$ for both inequalities, which is ddM0.

A1ft: $"-1"<x<\text{e}^{-\frac{1}{4}}-1$ or $x>\text{e}^{\frac{5}{4}}-1$ or any equivalent form following through on their **negative** k from (a). Must be exact (not decimals). Allow "and" instead of "or" – ie both regions are identified as the solution. But use of $\cap$ in set notation is A0 as it is incorrect.

Question Number	Scheme	Marks
10(a)	$\frac{1}{4}=\sin^2 4y \Rightarrow y=\frac{\pi}{24}$	M1A1
		(2)
(b)	$\frac{dx}{dy}=8\sin 4y\cos 4y$	M1A1
		(2)
(c)	$\frac{dx}{dy}=8\sin 4y\cos 4y \to \frac{dy}{dx}=\frac{1}{8\sin 4y\cos 4y}$	M1
	$\frac{dy}{dx}=\frac{1}{8\sqrt{x(1-x)}}$	M1
	$\frac{dy}{dx}=\frac{1}{\sqrt{16-64\left(x-\frac{1}{2}\right)^2}}$	A1
		(3)
(d)(i)	$x=\frac{1}{2}$	B1ft
(ii)	$\frac{dy}{dx}=\frac{1}{4}$	B1ft
		(2)
		(9 marks)

Notes

(a)

M1: Attempts to substitute in $x=\frac{1}{4}$ and rearranges to find an exact value for y. Look for $y=M\arcsin\left(\pm\frac{1}{2}\right)\to k\pi$ or $k(30°)$ allowing errors in dividing by the 4, or equivalents criteria via a double angle identity, correct up to sign error.

A1: $\frac{\pi}{24}$ Ignore extra solutions outside the domain.

(b)

M1: Attempts to differentiate achieving a form $\left(\frac{dx}{dy}=\right)A\sin 4y\cos 4y$ or $A\sin 8y$ oe Accept alternative forms e.g. via implicit differentiation $1=A\sin 4y\cos 4y\frac{dy}{dx}$. The $\frac{dx}{dy}$ may be missing, or labelled $\frac{dy}{dx}$ for this mark.

A1: $\frac{dx}{dy}=8\sin 4y\cos 4y$ oe eg $\frac{dx}{dy}=4\sin 8y$ Coefficients must be simplified. Must include the $\frac{dx}{dy}$

(c)

M1: Uses $\frac{dy}{dx}=1\div\frac{dx}{dy}$ (allow for the reciprocal of their answer to (b) if left hand side is omitted). Variables must be consistent.

M1: Attempts to use $\sin 4y = \pm\sqrt{x}$ and $\cos 4y = \pm\sqrt{1-x}$ to write $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x only with no trig terms.

A1: $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{16-64\left(x-\frac{1}{2}\right)^2}}$ and isw after correct answer. Allow with terms under the square root reversed.

(d) Note to score marks in (d) they must have a derivative which does have a minimum and is of a completed square form (the *r* may be inside the bracket) or in some other way clearly uses their (c) (e.g. minimising their quadratic).

(i) B1ft: $(x=)\frac{1}{2}$ ft their $-s$ provided their $q>0$, their $r<0$ and their x is in the range $0 \leqslant x \leqslant 1$

(ii) B1ft: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{1}{4}$ ft their q provided it is positive and their r was negative.

(b) Alt	$x=\sin^2 4y \Rightarrow y=\frac{1}{4}\arcsin\sqrt{x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{4}\frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}}\times\frac{1}{2}x^{-\frac{1}{2}}$	M1
	$\frac{\mathrm{d}x}{\mathrm{d}y}=8\sqrt{x}\sqrt{1-x}$	A1
		(2)
(c)	$\frac{\mathrm{d}x}{\mathrm{d}y}=8\sin 4y\cos 4y \rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8\sin 4y\cos 4y}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8\sqrt{x(1-x)}}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\sqrt{16-64\left(x-\frac{1}{2}\right)^2}}$	A1
		(3)

Notes

Alt by finding *y* first. Mark (b) and (c) together via such approaches. But note that part (c) says "Hence" and the reciprocal law must have been used at some stage to score full marks.

(b)

M1: Makes y the subject and differentiates to reach form $\frac{\mathrm{d}y}{\mathrm{d}x}=A\frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}}\times...$ where … is a function of x

A1: For $\frac{\mathrm{d}x}{\mathrm{d}y}=8\sqrt{x}\sqrt{1-x}$ oe with square roots combined. Coefficients must have been gathered.

(c)

M1: For use of the reciprocal rule of derivatives evidenced in the working – allow for it being used in (b) to find $\frac{\mathrm{d}x}{\mathrm{d}y}$ from $\frac{\mathrm{d}y}{\mathrm{d}x}$.

M1: Award for the correct procedure of having made y the subject and differentiating to reach the correct form for the answer, ie $\frac{\mathrm{d}y}{\mathrm{d}x}=A\frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}}\times Bx^{-\frac{1}{2}}$

A1: As main scheme.

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