Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. Given that

$$y = 3x \arcsin 2x \qquad 0 \leqslant x \leqslant \frac{1}{2}$$

(a) determine an expression for $\frac{dy}{dx}$

(2)

(b) Hence determine the exact value of $\frac{dy}{dx}$ when $x = \frac{1}{4}$, giving your answer in the form $a\pi + b$ where a and b are fully simplified constants to be found.

(1)

Question 1 continued

(Total for Question 1 is 3 marks)

2. A hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{5} = 1 \quad \text{where } a \text{ is a positive constant}$$

The line with equation $x = \frac{4}{3}$ is a directrix of H

(a) Write down an equation of the other directrix.

(1)

(b) Determine

(i) the value of a

(ii) the coordinates of each of the foci of H

(5)

Question 2 continued

(Total for Question 2 is 6 marks)

3. Solve the equation

$$4\tanh x - \operatorname{sech} x = 1$$

giving your answer in the form $x = \ln k$ where k is a fully simplified rational number.

(6)

Question 3 continued

(Total for Question 3 is 6 marks)

4. (a) Determine

$$\int \frac{1}{\sqrt{9x^2 + 16}} \, dx$$

(2)

(b) Hence determine the exact value of

$$\int_{-2}^{2} \frac{1}{\sqrt{9x^2 + 16}} \, dx$$

Give your answer in the form $a \ln(b + c\sqrt{13})$, where a, b and c are rational numbers.

(3)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 5 marks)

5.

$$\mathbf{A} = \begin{pmatrix} a & a & 1 \\ -a & 4 & 0 \\ 4 & a & 5 \end{pmatrix}$$ where a is a positive constant

(a) Determine the exact value of a for which the matrix $\mathbf{A}$ is singular.

(2)

Given that 2 is an eigenvalue of $\mathbf{A}$

(b) determine

(i) the value of a

(ii) the other two eigenvalues of $\mathbf{A}$

(5)

A normalised eigenvector for the eigenvalue 2 is $\begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \end{pmatrix}$

(c) Determine a normalised eigenvector for each of the other eigenvalues of $\mathbf{A}$

(5)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

6. A curve has parametric equations

$$x = a(\theta - \sin\theta)$$
$$y = a(1 - \cos\theta)$$

where a is a positive constant.

(a) Show that

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = ka^2 \sin^2 \frac{\theta}{2}$$

where k is a constant to be determined.

(4)

The part of the curve from $\theta = 0$ to $\theta = 2\pi$ is rotated through 2π radians about the x-axis.

(b) Determine the area of the surface generated, giving your answer in terms of π and a.

[*Solutions relying on calculator technology are not acceptable.*]

(5)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. The plane Π has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Determine a vector perpendicular to Π

(2)

The line l meets Π at the point (1, 2, 3) and passes through the point (1, 0, 1)

(b) Determine the size of the acute angle between Π and l

Give your answer to the nearest degree.

(4)

(c) Determine the shortest distance between Π and the point (6, −3, −6)

(4)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)

8.

$$I_n = \int \cos^n x \, dx \qquad n \geqslant 0$$

(a) Prove that for $n \geqslant 2$

$$I_n = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}$$

(4)

(b) Show that for positive even integers n

$$\int_0^{\frac{\pi}{2}} \cos^n x \, dx = \frac{(n-1)(n-3)...5 \times 3 \times 1}{n(n-2)(n-4)...6 \times 4 \times 2} \times \frac{\pi}{2}$$

(4)

(c) Hence determine the exact value of

$$\int_0^{\frac{\pi}{2}} \cos^6 x \sin^2 x \, dx$$

(3)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 11 marks)

9. The ellipse E has equation

$$x^2 + 9y^2 = 9$$

The foci of E are F_1 and F_2

(a) (i) Determine the coordinates of F_1 and the coordinates of F_2

(ii) Write down the equation of each of the directrices of E

(4)

The point P lies on the ellipse.

(b) Show that $|PF_1| + |PF_2| = 6$

(3)

The straight line through P with equation $y = 2x + c$ meets E again at the point Q

The point M is the midpoint of PQ

(c) Show that as P varies the locus of M is a straight line passing through the origin.

(6)

Question 9 continued

Question 9 continued

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Question 9 continued

Question 9 continued

(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

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