

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

## Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper  
reference

**WFM03/01**

### Mathematics

International Advanced Subsidiary/ Advanced Level  
Further Pure Mathematics F3

**You must have:**

Mathematical Formulae and Statistics Tables (Yellow), ca

**NOTE:**

This is the original version of WFM03\_01, sat by candidates. Question 5c cannot be answered, as  $A$  is not a symmetric matrix.

Appropriate actions have been taken to ensure candidates who sat this version have not been negatively impacted by this error.

Please do not use this version of the Question Paper for mocks and future student assessments. An updated version of the question paper will be released to be used for mocks and class assessments.

Candidates may use any calculator allowed by Pearson re  
must not have the facility for symbolic algebra manipul  
integration, or have retrievable mathematical formulae st

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of q
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

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2. A hyperbola  $H$  has equation

$$\frac{x^2}{a^2} - \frac{y^2}{5} = 1 \quad \text{where } a \text{ is a positive constant}$$

The line with equation  $x = \frac{4}{3}$  is a directrix of  $H$

(a) Write down an equation of the other directrix.

(1)

(b) Determine

(i) the value of  $a$

(ii) the coordinates of each of the foci of  $H$

(5)

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3. Solve the equation

$$4 \tanh x - \operatorname{sech} x = 1$$

giving your answer in the form  $x = \ln k$  where  $k$  is a fully simplified rational number.

(6)

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4. (a) Determine

$$\int \frac{1}{\sqrt{9x^2 + 16}} dx \quad (2)$$

(b) Hence determine the exact value of

$$\int_{-2}^2 \frac{1}{\sqrt{9x^2 + 16}} dx$$

Give your answer in the form  $a \ln(b + c\sqrt{13})$ , where  $a$ ,  $b$  and  $c$  are rational numbers.

(3)





**Question 4 continued**

Lined writing area for the answer.

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Question 4 continued

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5.

$$\mathbf{A} = \begin{pmatrix} a & a & 1 \\ -a & 4 & 0 \\ 4 & a & 5 \end{pmatrix} \quad \text{where } a \text{ is a positive constant}$$

Given that 2 is an eigenvalue of  $\mathbf{A}$ 

- (a) determine
- (i) the value of  $a$
  - (ii) the other two eigenvalues of  $\mathbf{A}$

(5)

A normalised eigenvector for the eigenvalue 2 is

$$\begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \end{pmatrix}$$

- (b) Determine a normalised eigenvector for each of the other eigenvalues of  $\mathbf{A}$

(5)

- (c) Hence determine a matrix  $\mathbf{P}$  and a diagonal matrix  $\mathbf{D}$  such that  $\mathbf{P}^T \mathbf{A} \mathbf{P} = \mathbf{D}$

(2)

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**Question 5 continued**

Handwriting practice area with horizontal lines.



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Question 5 continued

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**Question 5 continued**

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Handwriting practice area consisting of 25 horizontal lines.

**(Total for Question 5 is 12 marks)**



6. A curve has parametric equations

$$x = a(\theta - \sin \theta)$$

$$y = a(1 - \cos \theta)$$

where  $a$  is a positive constant.

(a) Show that

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = ka^2 \sin^2 \frac{\theta}{2}$$

where  $k$  is a constant to be determined.

(4)

The part of the curve from  $\theta = 0$  to  $\theta = 2\pi$  is rotated through  $2\pi$  radians about the  $x$ -axis.

(b) Determine the area of the surface generated, giving your answer in terms of  $\pi$  and  $a$ .

*[Solutions relying on calculator technology are not acceptable.]*

(5)

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Question 6 continued

Large ruled area for writing the answer to Question 6.





**Question 6 continued**

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Lined writing area with horizontal lines.

**(Total for Question 6 is 9 marks)**











8.

$$I_n = \int \cos^n x \, dx \quad n \geq 0$$

(a) Prove that for  $n \geq 2$ 

$$I_n = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2} \quad (4)$$

(b) Show that for positive even integers  $n$ 

$$\int_0^{\frac{\pi}{2}} \cos^n x \, dx = \frac{(n-1)(n-3)\dots 5 \times 3 \times 1}{n(n-2)(n-4)\dots 6 \times 4 \times 2} \times \frac{\pi}{2} \quad (4)$$

(c) Hence determine the exact value of

$$\int_0^{\frac{\pi}{2}} \cos^6 x \sin^2 x \, dx \quad (3)$$

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**Question 9 continued**

Lined writing area for the answer to Question 9.



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**Question 9 continued**

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**Question 9 continued**

Lined area for writing the answer to Question 9.

**(Total for Question 9 is 13 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

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