Examiners' Report
Principal Examiner Feedback

January 2023

Pearson Edexcel International Advanced Level
In Pure Mathematics P4 (WMA14) Paper 01

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January 2023
Publications Code WMA14_01_ER_2301

Overview

The paper overall provided good access for candidates across all ranges, with lower grade candidates able to score well in the first half of the paper, and a good ramping of difficulty over the last few questions to give higher grade candidates opportunity. The modal score was full marks for questions 2 through 5, with questions 1 and 7 being one mark away from full marks. However, questions 6, 8 and 9 performed less well in this respect, suggesting question 7 could have come before question 6, but otherwise the questions were well ordered.

Proof and vectors remain the most difficult topics for candidates, while partial fraction and implicit differentiation were very comfortable topics.

Report on Individual Questions

Question 1

Although the modal score was 8/9 for this question, achieved by nearly 35% of candidates, this was largely due to the lack of understanding of the validity conditions for the combination of series. The first 8 marks was accessed by most, and over 70% scored at least 6 marks.

The vast majority of candidates found part (a) accessible and were able to apply the method of partial fractions effectively. There were some who made arithmetic slips or sign errors which resulted in the loss of accuracy marks, but these were rarities. Almost all who obtained the correct values $A = 3$ and $B = 4$ wrote their answer in the correct partial fraction form as demanded, although sometimes this was only seen in part (b).

In part (b), scoring was generally lower, with a small number of candidates not tackling it at all. The expansion for $\left(1-x\right)^{-1}$ was usually correctly obtained, at least in an unsimplified form. For those with errors, it was usually the sign of the x term, which without an unsimplified expansion form being first written lost the B mark. For the expansion of $\left(2+3x\right)^{-1}$ the removal of the factor of 2 was sometimes a problem, with 2 instead of 2^{-1} being written outside the bracket being a common error, or simply ignoring the 2 entirely. There were also some issues with evaluating the coefficient of x^2, either due to incorrect binomial coefficient or, more often, incorrect simplification after a correct expression.

Those who achieved expansions in terms up to x^2 were generally able to use their values from part (a) to complete the expansion of $f(x)$ by multiplying and adding, but sign errors and other slips were not uncommon. A few did forget to use the coefficients from (a), but this was rare.

Only a very small number of candidates attempted alternative methods, such as expanding $\left(5x+10\right)\left(1-x\right)^{-1}\left(2+3x\right)^{-1}$, but rarely were such methods successfully completed.

Part (b)(ii) was an early discriminating mark on the paper with only a small proportion of candidates giving the correct answer. Though many did demonstrate that they knew the validity for each of the two series individually, many could not identify how these related to the combination of two series, with both intervals stated without choice. Other common answers included $x < \frac{2}{3}$, $\frac{2}{3} < x < 1$ and $x \neq \frac{2}{3}, x \neq 1$. Some even gave incompatible inequalities such as $|x| < -\frac{2}{3}$.

Question 2

The majority of candidates found this question accessible with 50% gaining at least 5 marks, and 33% achieving the mode of full marks. However, nearly 15% failed to score at all, with some not seeing how to get started and making little or no attempt. The most common mistake was the omission of a conclusion in part (a), saying that the equation they have arrived at is linear.

In part (a) most candidates opted for the substitution method and were able to rearrange to get *t* in terms of *x* and/or *y*, and then substitute into the other equation as appropriate. However, the rearranging was a challenge to some candidates, with some losing the constants from the original equations when substituting. There were often sign errors made, for instance instead of reaching $t = \frac{x+1}{1-2x}$ a lot of candidates arrived at $t = \frac{-x+1}{2x-1}$ or similar, and some got mixed up with negative signs after substituting into y=...

Despite these few instances of poor algebra, on the whole many candidates correctly manipulated the equation and reached $y = 2 - 4x$, or occasionally a correct multiple of this. The most common mistake was a failure to properly complete the proof and state that the points formed a straight line after they had reached the correct equation, losing the final mark in (a).

A smaller proportion of candidates attempted to complete (a) using parametric differentiation. Attempts at finding the derivatives often showed poor application of the rules or slips in algebraic manipulation but some did reach $\frac{\mathrm{d}y}{\mathrm{d}x} = -4$, though again a conclusion was often missing.

Other methods shown on the scheme were seen infrequently and with mixed success.

Part (b) was more successfully approached than part (a), in part due to the fact that it could be solved without part (a) at all. A few candidates who were not able to make any progress in (a) actually did much of the required work in (b) where they were unable to gain credit for it. However, many were able to successfully complete part (b) without a correct attempt at part (a).

Where an equation (whether linear or not) had been reached for part (a) candidates generally knew the correct approach to solve the equations simultaneously, usually achieving the correct answer if the equation in (a) was correct, though a few did make slips in the rearranging.

Some candidates, whether an equation had been obtained in (a) or not, instead used the given parametric equations along with $y = x + 12$ to find a value for *t*, usually correctly finding $t = -\frac{1}{5}$, before finding *x*. A small minority found *t* but did not go on to find *x*.

If parametric differentiation was used in (a) then candidates often established $y = 2 - 4x$ anyway in (b) (via methods that would have more easily accessed marks in (a)) and worked with Cartesian equations to find the required value of x.

Question 3

This question was generally done well with 40% able to score the modal full marks, and a further 25% able to access at least 3 marks. However, nearly 20% failed to score at all and about 12% just one mark, with many of these having made poor attempts rather than omitting the question.

Most candidates knew the formula for volume of revolution and could apply it well, though missing the π was not uncommon in those scoring 3/5, and the main reason for the zero scores was a failure to square y in the formula. However, condoning missing the dx and allowing for the π to be implied by later work allowed for most candidates to pick up the B mark.

Candidates who used the volume formula correctly were usually able to correctly integrate the function, or at least achieve a correct ln form with an incorrect constant. However there were numerous instances seen with integrals of the form $k\,x\ln(3x^2 + 5)$ or $k\,x^2\ln(3x^2 + 5)$, and also a few instances without a logarithm term at all. Some used a substitution, usually $u = 3x^2 + 5$, and were generally successful in achieving a correct integrated function in terms of u.

Almost all who attained a qualifying function were able to apply limits to score the M, either the original limits or correctly changed limits when substituting with u, though a few did use incompatible limits. Many were also able to write the answer in the required form $\pi\ln 2$, though many missed the instruction that the answer should be in the form $a\ln b$ and gave their final answer as $\frac{\pi}{2}\ln 4$.

Question 4

Whilst this question provided a high degree of discrimination between candidates, most were able to use an appropriate strategy. Once again, the modal mark was full marks, with over 33% achieving this. Performance among others was mixed with 0, 4 and 7 marks being the next most common score (~10-14%).

In part (a) most were able to change the limits successfully and were able to find a correct expression involving $\frac{\mathrm{d}u}{\mathrm{d}x}$ or equivalent, although there were occasional errors with the factor of 2. Most candidates recognised that they needed to substitute to obtain an integral in terms of u. A common error at this stage was to change $\sqrt{8x+4}$ to $4u$ instead of $2u$. There were however many well-presented, systematic, and fully correct solutions. A few candidates failed to give their answer in the form required, losing the final mark.

For part (b), most candidates recognised the need to use their answer to part (a) and attempted integration by parts. The first stage of integration by parts was generally tackled well. Those who began correctly, usually made a good attempt at the second stage, although sign errors and other slips were not uncommon. Mistakes in the use of limits were rare. After excellent integration some candidates lost the final mark due to a wrong value for k, usually 4, in part (a). A few failed to realise the need to use integration by parts and made no progress.

Question 5

Another accessible question with the modal full marks achieved by over 40%. Another common score was 4/7, achieved by over 35% usually correct part (a) with no progress in (b). Around 10% score no marks, but this was usually due to omission of the question.

Part (a) was answered very well, with many candidates (well over 80%) gaining full marks for this part. The majority of candidates were able to correctly differentiate implicitly and then rearrange to give the correct fractional form. Only a very small minority added an extra $\frac{dy}{dx}$, which is an improvement on some recent series. The most common error seen was making sign slips when rearranging the equation, though slips in differentiating one of the *y* terms were also relatively common among errors.

Part (b) proved more challenging for many candidates. Although there were some very good responses with fully correct answers, many were unable to see how their $\frac{dy}{dx}$ could approach infinity. It was clear that candidates are much more familiar with solving $\frac{dy}{dx} = 0$ as it was a common error to set the numerator equal to zero rather than the denominator. Those candidates who established $y = 5$ generally went on to substitute this into the equation of the curve to form a quadratic. Very few made errors in manipulating this equation.

Methods based on the symmetry of the hyperbola were occasionally seen, though in most of these it was doubtful that the candidates realised the geometry behind it but used that the diagram looks symmetric.

Question 6

The performance of candidates dipped on this question compared with the first 5 questions, with the modal score being 2 marks, scored by just over 25% of candidates. About 20% were able to score full marks, a further 14% losing just one mark (usually for not giving a suitable equation for the line in (a)(ii), and 12% scored 3/8. However, less than 10% scored no marks at all for the question with most attempting a difference of the vectors in (a) at the least.

In part (a)(i) the majority of students were able to work out $\overrightarrow{AB}$ correctly, often the only two marks scored. Where incorrect, students either did $\overrightarrow{OA} + \overrightarrow{OB}$ or subtracted with a slip in one of the components.

Part (a)(ii) was less well answered. A significant number of students did not achieve this mark as they stated '*l*:' or '*l* =', rather than '**r**=', and so did not have a correct vector equation. Careful heed should be paid to vector notation, however, tolerance was permitted with confusion between column vectors and **i**,**j**,**k** notations, allowing mixed notation throughout the question.

The majority of candidates who attempted the equation used $\overrightarrow{AB}$ rather than simplifying the direction vector by taking out the factor of 6 from each term. Most used $(\mathbf{r} =)\, \overrightarrow{OA} + \lambda\overrightarrow{AB}$ for the equation, though it was not uncommon for point *B* to be used as the point on the line instead.

Part (b) provided much more of a challenge, with many not determining how to get started and so scoring no marks. A majority did attempt an answer, though less half of the candidates managed a score for this part. Numerous attempts at applying the scalar product with inappropriate vectors were seen, which could not access marks, as candidates realised the perpendicularity condition is related to the scalar product. However, relatively few appreciated the condition for P to be on l in the first place.

Those that understood that they needed to use a general point on their line for $\overrightarrow{OP}$ generally found $\overrightarrow{CP}$ correctly for their equation. Although a few candidates then ground to a halt, most who reached this stage then attempted $\overrightarrow{CP}.d$ with their direction vector. However, some set this equal to ±1, rather than 0, and so lost the remaining marks. Others multiplied across each row and incorrectly found three values for λ instead of a single equation for λ.

Those who set the scalar product equal to 0 had a relatively good success rate in finding the correct λ for their vectors. Unfortunately, a number of these candidates lost the final two marks as they substituted back into $\overrightarrow{CP}$ instead of $\overrightarrow{OP}$. Likewise, those with an incorrect λ but otherwise correct solution to that point lost the method for this error too. Almost all gave their final answer as a vector rather than coordinates, and so did heed the questions demand in this instance.

Question 7

Question 7 saw much more variable performance but did perform better overall than question 6. The mean mark was 7/12 and modal mark was 11/12, showing most engaged well with the question, those losing just one often due to a lack of units on answers. The second most common mark by 11% of candidates was just one mark for the expression for $\frac{\mathrm{d}V}{\mathrm{d}r}$ in (a), sometime being all a candidate attempted. The distribution of marks beyond this was roughly even across all other scores.

Part (a) was answered correctly by almost all candidates who attempted it with only 4% scoring no marks at all for the question.

Part (b)(i) had a mix of responses, with some candidates separating variables, some integrating by recognition and a small number using substitution. The integration tended to be done well, although some candidates failed to use '+ c', but were still able to pick up three marks. Many did go on to substitute values to find the constant of integration, and there were some very good, clear solutions. Some, however, lost the final A mark for not showing sufficient working in getting from their + c to the final given answer, which perhaps indicated some lack of confidence in algebraic manipulation. Candidates sometimes struggled with part (b)(ii) and either left it out or failed to include units. This perhaps indicates a lack of familiarity with limits within the topic of modelling.

Even where candidates failed to complete part (b), many persisted with the rest of the question and often went on to score well in parts (c) and (d).

Part (c) was answered well, probably the most accessed part of the question after part (a), although finding a square root rather than a cube root was an occasional slip. Again, the omission of units was common and candidates should be advised always to appropriate units in their answers, even when the question does not specifically demand this.

Marks were available on part (d) even following lack of success in previous parts. Many candidates were able to demonstrate their knowledge of connected variables to find $\frac{\mathrm{d}r}{\mathrm{d}t}$, but some made no progress, not recognising this as a 'chain rule' application. Arithmetic slips sometimes accounted for the loss of the final mark, while a number of candidates failed to show the substitution or give an indication of what was being substituted and so forfeited the final two marks if the answer was incorrect.

Question 8

There was a marked tail off in standard of responses in the last two questions on the paper. With question 8 the modal mark was 5/11 (23%) with 0/11 (19%), 11/11 (14%) and 1/11 (11%) being the other common marks. Those who knew how to access part (b) usually did well with it, but it did prove a stumbling block for many.

Part (a) was generally well answered with over 50% scoring full marks. The majority of candidates found the coordinates of point *P* correctly, with this sometimes being as far as was progressed. However, many did attempt to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ as $\frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$, though the differentiation was sometimes poor, especially for $\frac{\mathrm{d}x}{\mathrm{d}t}$. Common expressions for this included $2\cos^2 t$ and $2\sin t$ amongst others. Some candidates differentiated correctly, but then made errors attempting to simplify their expression before substituting $t = \frac{\pi}{4}$ and consequently losing the A marks. Very few candidates who made an attempt at the part forgot to find the negative reciprocal of their gradient before attempting to find the equation of the normal, which was also done well. Candidates who struggled often tried to work backwards from the given answer to identify the gradient needed but failed to provide sufficient evidence for their gradient.

A small number of candidates attempt approaches via finding the Cartesian equation first, with mixed success in such answers. The differentiation again was the main issue, with the method for finding the normal shown well once a gradient had been established.

Part (b) was relatively poorly answered, with only about a third of candidates making significant progress into it. Again a few attempted Cartesian approaches but these were unlikely to make progress as no suitable stage for integrating was reached except in only a handful of rare cases.

Many candidates who did realise an integral was needed often did not set up the correct one. Of those that did attempt parametric integration, the majority proceeded at least as far as $k\sin^2 t$, followed often by sign slips when applying double angle formulas.

Some who did get a correct expression for $y\frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t$ struggled to simplify this to $4\sin^2 t$, instead often attempting integration by parts on their unsimplified expression. This was particularly true for those who used $\frac{\mathrm{d}x}{\mathrm{d}t} = \sin 2t$.

Even among those who reached the correct simplified integral, many did not know how to deal with the $4\sin^2 t$, with solutions such as $4\cos^2 t$ or $\frac{4}{3}\sin^3 t$ common. Those who realised that they needed to use the double angle formula were in general successful in reaching a suitable answer for the integral, though sign slips, or a slip with the coefficient (often 2 instead of 4 was found) led to the loss of the A mark. The substitution of limits often followed, but the final method required a full method for the area and this often was not present. Similarly, many correctly found the area of the triangle – the methods $\frac{1}{2}\times b\times h$ or using integration of the line were seen commonly – but this gained them no marks unless they also attempted to find the area under the curve and combine, which often did not happen when the attempts at the integral were not successful.

Question 9

In general, this proof question was better answered than in some previous exam series, yet just over 40% of candidates scored zero marks. Despite this, there were relatively few totally blank scripts, indicating a lack of time was not the issue. Aside from no marks, the most common score was 2/8, which would often be either the first two marks in (a) or the first two marks in (b), with only a few making a good attempt at both parts, and only about 11% of candidates were able to achieve full marks.

Part (a) was the more demanding part of the question for many, as it required a bit more thinking through the method required, opposed to the "bookwork" start to (b). The leading of the question aimed to set candidates on the right track, yet many did not appreciate or use the second possible case of a non-multiple of three integer. Repeating the cubing of $3k + 1$ was seen frequently with similar wording, or simply trying to write out a concluding statement from the given case, not realising at all the proof was only half complete. Of those candidates that cubed a second case appropriately $3k + 2$, was generally used, rather than $3k - 1$. The algebra, apart from the occasional slip, was usually good and most candidates were able to obtain the expansion in the form to show that it was not a multiple of three. Very few, however, scored the third mark as the first part of the proof was not incorporated into the final statements and an appreciation of the contradiction consequently was not shown.

Many candidates were able to make a reasonable attempt at part (b), recognising the similar pattern to the standard proof that $\sqrt{2}$ is irrational. The first two marks were accessible here and often achieved, even though a fully correct formal set up detailing the terms as integers and being in lowest form was seldom given. It was realised that it was necessary to assume that $\sqrt[3]{3}$ was rational, write it as a fraction, and proceed to $p^3 = 3q^3$, but at this point what was needed diverged from the known standard proof. Some jumped to state that p was a multiple of three without first saying that p^3 was a multiple of three, not using the result of (a), while others attempt to reason by oddness of p or q, not using multiples of 3 at all.

The better solutions did go on to state that $p = 3k$ and obtain the required form for q in terms of p. However, the final A mark was often not achieved, generally either as a result of not using clearly the result obtained in (a), or a failure to state that p/q had no common factors at the start. A common error for weaker candidates was to claim there was a contradiction too early, often after just establishing that p was a multiple of three, or even after just establishing that $p^3 = 3q^3$.

It was good to see that many knew the standard proof for irrationality of $\sqrt{2}$, but few were able to adapt the proof to $\sqrt[3]{3}$ successfully. However, the access was improved for this proof question compared to previous series and indicates candidates are beginning to pick up on some of the ideas required, at least when being faced with the familiarity of a proof they will have seen demonstrated in the classroom.

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