Pearson Edexcel

Examiners' Report
Principal Examiner Feedback

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Pearson Edexcel International Advanced Level
In Further Pure Mathematics F1 (WFM01)
Paper 01

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Overview

In general, most candidates were able to have a good attempt at the paper with several very familiar topics on the paper, though some parts proved very challenging for even the very best candidates. Some notable points of difficulty were:

- question 4(a)(ii) explaining why the Newton-Raphson method fails for the given function and point,
- question 6(c) identifying a correct strategy proved to be challenging with frequently little headway made,
- question 8(c) again the coordinate geometry here proved challenging with errors in the area of BPA and often limited success in simplifying the expressions or knowing how to apply the ratio to the problem.
- question 9 with a summation induction involving logs and factorials being slightly unusual compared with previous summation induction questions.

It is also noteworthy that there were many cases of poor handwriting and/or cramped/chaotic layout that made it difficult for markers to find where credit was due.

Individual question reports

Question 1

Part (a) The vast majority of candidates were able to correctly calculate the matrix AB with two thirds attaining full marks for the question. Of those that didn't get the correct matrix, they were usually able to get at least two entries correct, thus achieving the M1 mark. A few obtained 3×3 matrices instead, but this was rare with only 6% scoring no marks for the question as a whole and only about 10% scoring fewer than 3 marks.

Part (b) Again this was usually done well, with both method marks usually following a slip in (a). The procedure to find the determinant of a 2×2 matrix was shown well, although in a few cases this was seen as part of an attempt at the inverse matrix scoring just the first M unless it was extracted. The determinant as "bc – ad" was also seen, giving a correct value for k, but not a correct solution.

Common mistakes included incorrect expansion to get an incorrect quadratic or in some case a linear expression, poor solving of the quadratic and confusion with signs. Despite having a correct equation, candidates frequently arrived at $\frac{9}{2}$ as an answer, rather than $-\frac{9}{2}$.

Question 2

In general, this question was answered very well, with most candidates able to apply the standard results to the given summation. It proved to be the most accessible question on the paper, just ahead of question 1, with nearly 75% of solutions fully correct and less than 5% zero scores. The use of summation formulae is a well rehearsed topic for many and a good source of marks for low grade candidates.

The common error of thinking $\sum_{r=1}^{n} 25 = 25$ instead of the correct *25n* was in fact rarely seen on this paper. Candidates making this error could only achieve the first two marks because they were unable to correctly factorise out at least the *"n"* from their three terms, but this did not affect many.

A small number made a slip in the expansion or one of the other summation formulae, but candidates who wrote down the correct three summation expression tended to correctly factorise out $\frac{n}{6}$, achieve the correct simplified, three term quadratic and then correctly factorise it into the required form. Those with the correct method but who failed to score the final accuracy marks had usually made a fairly basic algebraic error or simply misread their own writing.

Compared to previous years, fewer candidates took the approach of expanding out all the brackets to achieve a cubic in *n*, before then attempting to factorise.

Question 3

This question was generally well done and was accessible to many candidates. Zero mark scores were very rare (< 2%) though the mean was around 8/10, with just under 75% of candidates scoring at least 8 marks. It was positioned well in the paper.

Part (a) Most candidates reached $p = -8$ by using the factor theorem correctly (only a few substituted $z = 3$ in error), though some candidates obtained an incorrect value due to arithmetic errors. Long division was less frequently used and tended to be more error prone particularly as part of the solution to this part. Comparing coefficients was infrequently used.

Part (b) The correct quadratic factor was found by most and mainly by long division having found the correct p. It was possible to achieve a correct quadratic term even following an incorrect value of p. Those who had used long division in (a) tended to make more errors which caused problems finding the correct quadratic factor in (b). Since long division is problematic for some candidates, comparing coefficients or similar methods could be encouraged as these are less prone to algebraic error.

Many found the complex roots correctly showing appropriate method, though some simply stated them following a correct quadratic. It should be stressed that working should be shown, especially where questions insist on all steps of working in the preamble since calculator usage may not access all the marks in such cases.

The most common errors when finding roots were, not listing the real root $z = -3$, not showing any method which, with no/incorrect complex roots shown, resulted in no marks for solving the quadratic and careless in the placement of i in the solutions.

Part (c) Most applied the formula correctly with the exact values to achieve the correct value. The most common reason for loss of marks in this part was omission, whether by intent or oversight. Other typical mistakes included use of "i" in the Pythagoras formula resulting in a difference of squares, confusion between the argument of *z* and the modulus of *z*, or forgetting to take the square root.

Part (d) Many candidates achieved one or both of the marks, but the quality of the Argand diagrams was variable. The first mark was achieved by the majority, with candidates realising the complex conjugate pair are symmetric about the real line and plotting them correct for their roots. A small number had the complex roots symmetric about the imaginary axis instead, however.

The second mark was more problematic with many plotting the real root on the positive real axis instead of at -3 or not at all. Another common error was failing to plot their points (roughly) equidistant from the origin, not appreciating what they had found in part (c). The mark was not a follow through mark, so the correct relative lengths needed to be clear in the sketch.

Question 4

This was the first question of the paper where the modal mark was not full marks, but instead was 7/8, scored by about 33% of candidates. Full marks was achieved by about 28%, with the next most common score being 5/8 (by ~12%), so the question overall scored well and again was well placed in the paper. The mark in (a)(ii) was the key sticking point.

Part (a) Whilst almost all candidates showed that they understood the process of differentiating a polynomial expression, there were some who did not successfully convert the given expression into the correct index form. This resulted in answers for $f'(x)$ that contained incorrect coefficients, and with one of the two terms having an incorrect power, but would only lose two A marks. Such index manipulation should be a skill familiar to further mathematics candidates.

The explanation for the unsuitability of 0.25 as a starting point for the Newton-Raphson process was missed by many, and aside from this over 50% would have scored full marks for the question. It would be instructional for students to investigate conditions under which numerical methods can fail. A common misconception was that *"the upper bound cannot be used as the starting value because the next approximation must be higher" or that "the lower bound must always be the starting value".*

However, the majority of candidates were able to accurately apply the Newton-Raphson formula with the starting point of 0.15. Many candidates did not show any working for their Newton-Raphson equation, relying on the correct answer to imply the method mark.

Compared to previous years, fewer candidates failed to heed the instruction to give their answer to 3 decimal places and did their working to a greater degree of accuracy before correctly rounding.

Part (b) This was less well answered than part (a), though still most candidates were able to set up and solve a linear interpolation equation. Many used a sketch to help and these were the more successful candidates. A few candidates found the equation of the line joining the points (0.15,f(0.15)) and (02.5,f(0.25)) and went on to set y = 0, and then solve the resulting equation to find the approximate value of the root, α.

The most common error in this part was essentially a sign error, equating a positive ratio to a negative ratio by neglecting to account for the signs of f(0.15) and f(0.25), while a few attempted interval bisection instead of interpolation. Overall, though, the standard of work on this question was good.

Question 5
This question was also generally well done and was accessible to many candidates with a modal mark of 9/9 scored by a third of students and less than 15% scoring below 6 marks. The presence of a constant k in the equation did not seem to phase candidates in applying the methods, though it did prevent some from achieving a final answer in which the coefficients were necessarily integers.

Part (a) Was well done by most and very few errors were seen. A few put one or both results upside down or had incorrect signs, but this was not common. Sum and product were rarely confused

Part (b) The principle here appeared to be well understood and the main mistakes were with algebraic manipulation or misusing the powers or sign errors. The fractions were usually correctly combined and a correct identity for $\alpha^3+\beta^3$ quoted, and then used that to find an expression for the numerator of their sum of roots. There were some candidates who forgot the denominator $(\alpha\beta)^2$ and so did not achieve the final answer, but again these were in the minority. A correct simplified form was not always achieved or recognised by the candidate.

Part (c) Almost all the candidates who attempted this part of the question were successful in scoring the mark for the product of roots. The form $x^2-(\text{sum of roots})x+\text{product of roots}=0$ was well known but there were a few candidates who used "+(sum of roots)x" instead, but most were able to access the method mark. The final answer was not always given in the required form, with an equation having coefficients that are not necessarily integers being the most common reason for loss of mark here. A small number omitted the "=0" or missed an *x* but again these were rarities.

Question 6
This question proved more challenging than intended in part (c), with relatively few making progress. The modal mark was 5, being the first 5 marks of the question, which were found routine by many, was achieved by nearly 40% of candidates, with a little over 20% achieving full marks being the next most common score. The spread across over marks was narrow.

Part (a) Almost all candidates who attempted the question obtained $a = 5$, even if they made no further progress. It required quite a bit of working from some candidates, whereas others were able to just write it down, as was intended.

Part (b) This part was generally well answered, with most candidates being well versed in finding the tangents and normals of conics. It was good to see that working was shown via finding an expression for $\frac{dy}{dx}$ so that method marks were gained for this. A range of techniques were used, including implicit differentiation and parametric form, though most common was to write the equation with y as the subject first. From there the procedure to go on and find the equation of the normal was well demonstrated by the majority, though some did not write the derivative in terms of t and consequently struggled to proceed. There was some confusion, mainly with sign errors, when substituting into their expression or the equation of the line, and a few lost the final mark due to a lack of an intermediate step between their equation for the final M and writing down the final (given) answer. Evidence is needed in a "Show that…" question. In the main, students using $y = mx + c$ proceeded to find c correctly and successfully demonstrated the required equation.

Part (c) This part of the question caused a lot of confusion, although some very good, clear responses were seen. Of the valid approaches seen, the main method given in the scheme was the most common, but all the approaches were seen, while a significant proportion of candidates did not even attempt this part or gave up after very little work.

It was clear that many candidates did not appreciate the geometry with very few using sketches to help, and many simply tried to substitute the parameter for A and B into the normal equation to solve simultaneously. In such cases, it was not uncommon to see several attempts, with some candidates achieving, after some work, at least the first method by substituting for y in the equation of the normal at A. Much more successful, for those who realised what was needed, was the main method of substituting the coordinates of B into the equation of the normal at A, which usually led to correct solutions barring slips in solving at the end.

Question 7

Another question which was, in general, very well answered, with a modal mark of 10/11 scored by over 25% of candidates, a further 15% achieving full marks. However, the 8% scoring zero marks was an increase over the preceding questions, with the trend continuing hereafter. This indicates the last few questions were well placed late in the paper as ones which E grade candidates found more difficult to access mark.

Part (i) Most were able to correctly identify matrix **P** as representing a reflection in the line $y = -x$. The most common incorrect answers were *"A reflection in y = x", and "A rotation of XX degrees about the origin".* Some students appended *"about the origin"* to their otherwise correct answer but they were not penalised for the unnecessary/incorrect, additional description in this instance. Some candidates ignored the request for a single transformation, instead describing two separate transformations. These efforts could not achieve any marks.

Again in (b) most candidates were able to correctly determine matrix **Q**. Where errors were seen, they tended to be with the signs of the values within the matrix (despite the required matrix's form being given within the formula book) or simply forgetting to multiply by six in order to take into account the scale factor 6 enlargement. Another fairly common error was to only multiply the diagonal entries by 6.

Compared to previous years, fewer candidates made the basic mistake of multiplying the matrices in the incorrect order when working out matrix **R** in part (c), with sign slips again providing the main source of errors.

Part (ii) Although many candidates were able to obtain the correct value of λ for this part, the quality of answers varied more wildly. Some showed very little working, others almost seemed to stumble on it by accident while others took circuitous routes via finding the inverse matrix or even via use of eigenvalues and the identifying the transformation. However, most failed to either independently solve both equations, or to validate that their correct value from one equation worked in the other and it is this final mark that was most often lost in the 10/11 mark responses.

Setting up the matrix equation was shown by most, though some were unable to get even this far. Those that set up the equation often only extracted and solved one of the equations, and it was not uncommon that the value for λ was simply stated following the equation. This could potentially lose method marks, so candidates should be advised to show working - especially when a question is asking them to show something. Even in cases where both equations were extracted, many again then simply stated the value for λ with no demonstration of which equation or equations were used. The uniqueness element of the problem was lost on most candidates, with the focus on finding the value for λ dominating the candidates work.

Question 8

With a modal score of 0/11 scored by 17% of candidates this question saw a ramping up of difficulty at the tail end of the paper. At the other extreme 14% were able to score full marks as the second most common score, with an even spread of marks between. Many of the zero scores were attempts that did not make much progress, though 10% did score at least the first B mar for the x coordinate of P if nothing else.

Part (a) As noted above, of those who attempted the question the majority correctly found the coordinates of *P*, sometimes being the only mark attained. Disappointingly few candidates spotted the focus-directrix approach to finding *PS*, with the majority opting to attempt the distance formula, which was far more prone to error. Most of these were able to achieve the method for setting up the correct equation and proceeding to *PS* but errors or simply a lack of working shown meant fewer achieved the accuracy mark. A required intermediate step when using the Pythagoras approach was often omitted, losing the final mark. Evidence is needed in a ”Show that…” question.

Part (b) Although some candidates stopped after part (a), this part was actually more successfully completed than the former for those who attempted it. A correct expression for the gradient was usually found by those attempting this part, albeit sometimes embedded in an attempt at finding the equation of the line, but there was often confusion about the variables being used, with ill-defined or undefined letters. Most candidates formed the equation of the line using the point *S*. Those using instead the point *P* mostly rearranged their equation successfully. Candidates clearly understood the need to substitute $x = 0$ into their equation and that this corresponded to the *y*-intercept of their line. It was common for candidates to stop at the end of this part.

Part (c) This was another challenging coordinate geometry problem that many candidates left out, or made little progress with. A systematic approach to writing out key information or using a suitable sketch to work out the geometry of the situation, was not shown by many. However, most candidates attempting this part did successfully calculate the area of *OSP*, though a few did omit it. Finding the area of *BPA* was not quite so successful and where things usually ground to a halt, with the most common mistake being an incorrect value for *BP*. Several candidates struggled to apply the ratio given to form a clear equation, but those that did (and who had a correct value for the area of *BPA*) usually arrived at the correct value for a. Mistakes most commonly resulted from poor manipulation of the expression for the area for *BPA* or misapplication of the ratio to the areas formed.

Question 9

This proved to be one of the most challenging questions on the paper, correctly positioned as the final question. Many made attempts, indicating timing was not the issue, yet about 22% scored no marks at all, with just under 23% scoring only one mark and just over 23% the modal 3/6 marks. Only 20% score higher than the mode with a little under 10% able to achieve full marks.

Though induction questions about summations are a staple question, the use of the log term in the summation was a novelty in this question. However, the mark scheme gave good access to the first 3 marks for those who persisted. The final 3 marks were aimed at, and good discriminators for, the high end candidates.

An attempt at the $n = 1$ case was commonly attempted although careless approach cost many the first mark on this question; it was not always clear that n = 1 was being substituted in **both** sides of the given formula, giving LHS = RHS = log 1 = 0, and that this meant the proposition was true for $n = 1$. The assumption step and adding the $k + 1$-th term to the sum to *k* terms was often done correctly, by those who have learned the induction procedure, so that many candidates achieved the first 2 marks. However, there were 2 common errors which lost this mark and often hampered further progress; not adding the $k + 1$-th term to the assumed form and an incorrect statement of the $k + 1$-th term.

The next method mark was not dependent so all those who correctly applied a log law to combine two log terms could access this mark, rewarding those who persisted even with an incorrect $k+1$-th term so long as it was a log term.

A significant proportion of the candidates stalled at this point simply claiming the end result with no justification of the equivalence. Candidates struggled with introducing the cancelling factors and if they did then problems with algebra and bracketing often cost them the A marks. A number successfully worked backwards from the expected form and demonstrated equivalence by ‘meeting in the middle’

The four key elements of the final accuracy mark were sometimes muddled and not always included. Attempts at these were often included even when the induction step had not been carried out, though could gain no mark. The main failing in the conclusion was that the “**if** true for $n = k$, **then** true for $n = k + 1$” was not clear, with the assumption of both cases being true with no sense of implication of the latter from the former being common. Most did remember to include the $n = 1$ case in the conclusion and as long as an attempt at the check had been made could access the final mark even if the initial check was incorrect - commonly having assumed both sides equated to 1 rather than 0. Induction continues to be a challenging topic for candidates.

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