Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
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Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

**Time** 1 hour 30 minutes

**Paper reference** **WFM03/01**

# Mathematics

## International Advanced Subsidiary/Advanced Level
## Further Pure Mathematics F3

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over*

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**1** (a) Use the definitions of hyperbolic functions in terms of exponentials to prove that

$$8\cosh^4 x = \cosh 4x + p\cosh 2x + q$$

where $p$ and $q$ are constants to be determined.

**(3)**

(b) Hence, or otherwise, solve the equation

$$\cosh 4x - 17\cosh 2x + 9 = 0$$

giving your answers in exact simplified form in terms of natural logarithms.

**(5)**

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**Question 1 continued**

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**Question 1 continued**

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**Question 1 continued**

Q1

**(Total 8 marks)**

**2.**

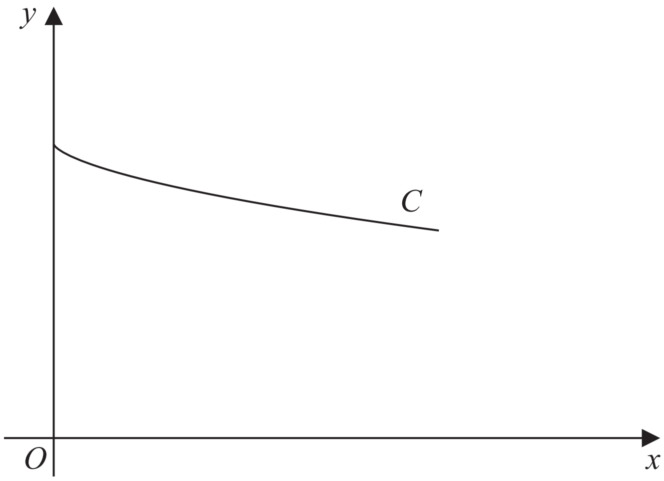


**Figure 1**

Figure 1 shows a sketch of the curve $C$ with parametric equations

$$x = \ln(\sec\theta + \tan\theta) - \sin\theta \qquad y = \cos\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{4}$$

The curve $C$ is rotated through $2\pi$ radians about the $x$-axis and is used to form a solid of revolution $S$.

Using calculus, show that the **total** surface area of $S$ is given by

$$\frac{\pi}{2}\left(p + q\sqrt{2}\right)$$

where $p$ and $q$ are integers to be determined.

**(8)**

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**Question 2 continued**



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**Question 2 continued**

Q2

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**3.** (a) Given that $y = \text{arsech}\left(\frac{x}{2}\right)$, where $0 < x \leqslant 2$, show that

$$\frac{dy}{dx} = \frac{p}{x\sqrt{q - x^2}}$$

where $p$ and $q$ are constants to be determined.

**(4)**

**In part (b) solutions based entirely on calculator technology are not acceptable.**

$$f(x) = \text{artanh}(x) + \text{arsech}\left(\frac{x}{2}\right) \qquad 0 < x \leqslant 1$$

(b) Determine, in simplest form, the exact value of $x$ for which $f'(x) = 0$

**(5)**

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**Question 3 continued**

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**Question 3 continued**

Q3

**(Total 9 marks)**

**4.** $$\mathbf{M} = \begin{pmatrix} 6 & k & 2 \\ k & 5 & 0 \\ 2 & 0 & 7 \end{pmatrix}$$

where $k$ is a constant.

Given that 3 is an eigenvalue of $\mathbf{M}$,

(a) determine the possible values of $k$.

**(3)**

Given that $k < 0$

(b) determine the other eigenvalues of $\mathbf{M}$.

**(3)**

(c) Determine a normalised eigenvector corresponding to the eigenvalue 3

**(3)**

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**Question 4 continued**

Q4

**(Total 9 marks)**



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**5.** Determine

(i) $\displaystyle\int \frac{1}{\sqrt{x^2 - 3x + 5}}\,dx$

**(3)**

(ii) $\displaystyle\int \frac{1}{\sqrt{63 + 4x - 4x^2}}\,dx$

**(4)**

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**Question 5 continued**

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**Question 5 continued**

Q5

**(Total 7 marks)**

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**6.**

$$I_n = \int e^x \sin^n x \, dx \qquad n \in \mathbb{Z} \qquad n \geqslant 0$$

(a) Show that

$$I_n = \frac{e^x \sin^{n-1} x}{n^2+1}(\sin x - n\cos x) + \frac{n(n-1)}{n^2+1} I_{n-2} \qquad n \geqslant 2$$

**(6)**

(b) Hence find the exact value of

$$\int_0^{\frac{\pi}{2}} e^x \sin^4 x \, dx$$

giving your answer in the form $Ae^{\frac{\pi}{2}} + B$ where $A$ and $B$ are rational numbers to be determined.

**(4)**

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**Question 6 continued**

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**Question 6 continued**

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**Question 6 continued**

Q6

**(Total 10 marks)**



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**7.** The line $l_1$ has equation

$$\frac{x-3}{4} = \frac{y-5}{-2} = \frac{z-4}{7}$$

The plane $\Pi$ has equation

$$2x + 4y - z = 1$$

The line $l_1$ intersects the plane $\Pi$ at the point $P$

(a) Determine the coordinates of $P$

**(3)**

The acute angle between $l_1$ and $\Pi$ is $\theta$ degrees.

(b) Determine, to one decimal place, the value of $\theta$

**(3)**

The line $l_2$ lies in $\Pi$ and passes through $P$

Given that the acute angle between $l_1$ and $l_2$ is also $\theta$ degrees,

(c) determine a vector equation for $l_2$

**(5)**

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**Question 7 continued**

Q7

**(Total 11 marks)**

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**8.** The ellipse $E$ has equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

(a) Determine the eccentricity of $E$

**(2)**

(b) Hence, for this ellipse, determine

(i) the coordinates of the foci,

(ii) the equations of the directrices.

**(2)**

The point $P$ lies on $E$ and has coordinates $(3\cos\theta, 2\sin\theta)$.

The line $l_1$ is the tangent to $E$ at the point $P$

(c) Using calculus, show that an equation for $l_1$ is

$$2x\cos\theta + 3y\sin\theta = 6$$

**(3)**

The line $l_2$ passes through the origin and is perpendicular to $l_1$

The line $l_1$ intersects the line $l_2$ at the point $Q$

(d) Determine the coordinates of $Q$

**(3)**

(e) Show that, as $\theta$ varies, the point $Q$ lies on the curve with equation

$$(x^2 + y^2)^2 = \alpha x^2 + \beta y^2$$

where $\alpha$ and $\beta$ are constants to be determined.

**(3)**

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**Question 8 continued**

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**Question 8 continued**

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**Question 8 continued**

Q8

**(Total 13 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**