Pearson
Edexcel

Mark Scheme (Results)

October 2021

Pearson Edexcel International A Level
In Pure Mathematics P2 (WMA12) Paper 01

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October 2021
Question Paper Log Number P69199A
Publications Code WMA12_01_2110_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Pearson Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ or ft will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given
   - $\llcorner$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by ‘MR’ in the body of the script.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of ‘0’ or ‘1’ for each mark, or “trait”, as shown:

	0	1
aM		●
aA	●	
bM1		●
bA1	●	
bB	●	
bM2		●
bA2		●

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the ‘0’ column when it was meant to be ‘1’ and all correct.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice

Question Number	Scheme	Marks
1.(a)	States/uses either $16k=-4$ or $\frac{16\times15}{2}k^2=p$	M1
	(i) $k=-\frac{1}{4}$	A1
	(ii) $p=\frac{15}{2}$	A1
		(3)
(b)	$g(x)=\left(2+\frac{16}{x}\right)(1+kx)^{16}$	
	Attempts either $2"p"$ or $16\times\frac{16\times15\times14}{3!}\times k^3$	M1
	Attempts sum of $2"p"$ and $16\times\frac{16\times15\times14}{3!}\times k^3$	dM1
	Term in x^2 $=(15-140)x^2=-125x^2$	A1
		(3)
		(6 marks)

(a)

M1: States or uses either $16k=-4$ or $\frac{16\times15}{2}k^2=p$. It may be implied by a correct value for *k* or *p* if no incorrect working is seen.

(i)

A1: $k=-\frac{1}{4}$ or -0.25

(ii)

A1: $p=\frac{15}{2}$ or 7.5 Allow if this follows from $k=\pm\frac{1}{4}$

.

(b)

M1: Attempts one of the two relevant terms or coefficients. Either $2p\left(x^2\right)$ or $16\times\frac{16\times15\times14}{3!}\times k^3\left(x^2\right)$ or exact equivalent. May be part of an attempt at a full expansion. May be in terms of *p* and *k*

dM1: Must have attempted at least one of the terms correctly. Attempts sum of their $2\times"p"\left(x^2\right)$ and their $16\times"\frac{16\times15\times14}{3!}\times k^3"\left(x^2\right)$ which must be arising from correct combination of powers from the expanded brackets though the coefficients need not be correct. Must be numerical terms. May be part of an attempt at a full expansion.

FYI If the k is not cubed in the second of these then you may see $2240x^2$ and can score the dM.

A1: Term in $-125x^2$. Must include the x^2. Do not allow as part of an expansion for this mark.

Question Number	Scheme	Marks
2.(a)	Attempts to substitute $u_2 = 6k+3$ in $u_3\left(=ku_2+3\right)$	M1
	$u_3 = k\left(6k+3\right)+3$	A1
		(2)
(b)	Uses $\sum_{n=1}^{3} u_n = 117 \Rightarrow 6+6k+3+k(6k+3)+3=117$	M1
	$6k^2+9k-105=0 \Rightarrow k=...$	dM1
	$k=\frac{7}{2}$	A1
		(3)
		(5 marks)

(a)

M1: Attempts a full method of finding u_3. Allow if the "+3" is missing once only. E.g. score for an attempt at substituting $u_2 = 6k+3$ into $u_3 = ku_2 \ (+3)$ or their u_2 into $u_3 = 6u_2 + 3$

May be implied by a correct answer, but an incorrect answer with no substitution is M0.

A1: $u_3 = k\left(6k+3\right)+3$ OR $u_3 = 6k^2+3k+3$ but isw after a correct answer is seen

(b)

M1: Sets their $u_1+u_2+u_3=117$ to produce an equation in just *k*.

dM1: Solves a 3TQ by any valid method to find at least one value for *k*.

A1: $k=\frac{7}{2}$ ONLY

Question Number	Scheme	Marks
3. (a)	States or uses $h=3$	B1
	Attempts $\frac{"3"}{2}\left\{\log_{10}2+\log_{10}14+2\times\left(.....\right)\right\}$	M1
	$=\frac{3}{2}\left\{\log_{10}2+\log_{10}14+2\times\left(\log_{10}5+\log_{10}8+\log_{10}11\right)\right\}$ = 10.10 (2dp)*	A1*
		(3)
(b)	Increase the number of strips	B1
		(1)
(c) (i)	$\int_2^{14}\log_{10}\sqrt{x}\,\mathrm{d}x=\frac{1}{2}\times 10.10=5.05$	B1
(ii)	$\log_{10}100x^3=2+3\log_{10}x$	B1
	$\int_2^{14}\log_{10}100x^3\,\mathrm{d}x=\left[2x\right]_2^{14}+3\times 10.10=54.30$	M1 A1
		(4)
		(8 marks)

(a)

B1: States or uses $h=3$. (If there is a conflict between what is stated and used award bod if one is correct.)

M1: An attempt at the trapezium rule with their value of h. Look for $\frac{"3"}{2}\left\{\log_{10}2+\log_{10}14+2\times\left(.....\right)\right\}$ where the ….. is an attempt at at least two intermediate terms and does not repeat the end points. The bracket structure must be correct or implied. (Use of h = 4 is likely to have two terms in this inner bracket.) Allow with values correct to 2 s.f. for the first and last terms.

A1*: Reaches 10.10 (or allow 10.1) following at least one correct intermediate line and no incorrect lines that would give a significantly different answer.

Example are $\frac{3}{2}\left\{\log_{10}2+\log_{10}14+2\times\left(\log_{10}5+\log_{10}8+\log_{10}11\right)\right\}$ or possibly $\frac{3}{2}\left\{\log_{10}28+2\log_{10}440\right\}$ or $\frac{3}{2}\left\{0.301+1.146+2\times\left(0.699+0.903+1.041\right)\right\}$ with values correct to at least 2d.p.

(Note that integration via the calculator gives 10.23)

(b)

B1: States "increase the number of strips", "decrease the width of the strips", “use more intervals” o.e.

(c)(i)

B1: (awrt) 5.05 or allow $\frac{1}{2}\times(a)$ if they make a slip miscopying the 10.10

(This is a B mark, so allow even if this answer comes from a repeat at using the trapezium rule.)

(c)(ii)

B1: States or implies that $\log_{10}100x^3 = 2+3\log_{10}x$

M1: $\int_2^{14} \log_{10}100x^3 \, dx = [ax]_2^{14} + 3\times 10.10$ or equivalent work for finding the area of the rectangle.

Use of the trapezium rule again here is M0. Must see use of (a).

A1: 54.30 Accept 54.3.

Question Number	Scheme	Marks
4.(a)	States -21	B1
		(1)
(b)	Attempts $\mathrm{f}(3)=\left(3^2-2\right)(2\times3-3)-21$	M1
	Achieves $\mathrm{f}(3)=0\Rightarrow$ $(x-3)$ is a factor of $\mathrm{f}(x)$ *	A1*
		(2)
(c) (i)	$\mathrm{f}(x)=2x^3-3x^2-4x-15=(x-3)\left(2x^2....\pm5\right)$	B1 M1
	$=(x-3)\left(2x^2+3x+5\right)$	A1
(ii)	Attempts b^2-4ac for their $2x^2+3x+5$	M1
	Achieves $b^2-4ac<0$ and states that only root is $x=3$ o.e. *	A1*
		(5)
		(8 marks)

(a)

B1: -21

(b)

M1: Attempts to substitute $x = 3$ into $\mathrm{f}(x)=\left(x^2-2\right)(2x-3)-21$ or its expanded form.

Condone slips but don't accept just $\mathrm{f}(3)=0$. Attempts via long division score M0.

A1*: Achieves $\mathrm{f}(3)=0$ and states that $(x-3)$ is a factor of $\mathrm{f}(x)$. If this latter is given in a preamble accept a minimal conclusion such as // or QED

(c)(i)

B1: Expands $\mathrm{f}(x)=\left(x^2-2\right)(2x-3)-21$ to reach correct 4 term cubic. Allow if the cubic is seen anywhere.

M1: Correct attempt to find quadratic factor using either division by $(x-3)$ or inspection.

To score the M mark look for e.g. correct first and last terms by inspection, or first two terms by division. (Two correct terms will imply the mark.) Allow for work seen in (b) a long as it is referred to in (c).

A1: $(\mathrm{f}(x)=)(x-3)\left(2x^2+3x+5\right)$ following a correct cubic. Must be seen together on one line.

(c) (ii)

M1: Attempts to show their quadratic factor has no real roots. Factorisation attempts are M0. Accept via

- an attempt at $b^2 - 4ac$ for their quadratic factor
- an attempt to solve their $2x^2 + 3x + 5$ using the quadratic formula or calculator
- an attempt to complete the square

A1*: Requires correct factorisation, correct calculation, reason and conclusion

For example after (c)(i) $\mathrm{f}(x) = (x-3)\left(2x^2 + 3x + 5\right)$ accept e.g.

- $2x^2 + 3x + 5$ has no roots as $3^2 - 4 \times 2 \times 5 < 0$ so $\mathrm{f}(x) = 0$ only has root at $x = 3$
- $2x^2 + 3x + 5 = 0 \Rightarrow x = -\frac{3}{4} \pm i\frac{\sqrt{31}}{4}$, $(x-3) = 0 \Rightarrow x = 3$. So only one real root.
- $2x^2 + 3x + 5 = 0 \Rightarrow 2\left(x + \frac{3}{4}\right)^2 - \frac{9}{8} + 5 \ldots \frac{31}{8} > 0$, so no roots hence 3 is the only root

There must be some reference to the root either by stating it or indicating the linear term has a root (e.g. writing “root” next to it), but do not accept incorrect statements such as only real root is $(x-3)$

Do not allow statements such as “Math error” without interpretation of what this means.

Question Number	Scheme	Marks
5.(a)	Attempts $480+13\times-15$ $= 285$ tonnes	M1 A1 **(2)**
(b)	Sets $\frac{N}{2}\{2\times480+(N-1)\times-15\}=7770$ $\frac{N}{2}\{960-15N+15\}=7770 \Rightarrow N^2-65N+1036=0$ *	M1A1 A1* **(3)**
(c)	States 28 only	B1 **(1)** **(6 marks)**

(a)

M1: Attempts to use $a+(n-1)d$ with at least two of *a*, *d* and *n* correct. (So allow with $a+nd$ if both *a* and *d* are correct, bod.) They may have $d>0$ for this mark.

A1: 285 tonnes. Condone the omission of the units

(b)

Note: Use of *n* instead of *N* is fine throughout part (b).

M1: Attempts to use a correct sum formula with $S=7770$ using their *a* and *d*, condoning slips.

A1: A correct equation for *N*. E.g. $\frac{N}{2}\{2\times480+(N-1)\times-15\}=7770$

A1*: Achieves $N^2-65N+1036=0$ (including the "=0") following a correct intermediate line (see scheme) and with no errors, although "=0" may be implied in intermediate steps.

(c)

B1: States 28 only. The 37 should be rejected if found. 29 is B0.

Question Number	Scheme	Marks
6 (i)	$x^2+y^2+10x-12y=k$	
(a)	Attempts $(x\pm5)^2+(y\pm6)^2.....=0$	M1
	Centre $(-5,6)$	A1
		(2)
(b)	Sets $k+(\pm5)^2+(\pm6)^2>0$	M1
	$k>-61$	A1
		(2)
(ii)	Centre $\left(\frac{-2+8}{2},\frac{10+(-14)}{2}\right)=...$; radius $\frac{1}{2}\sqrt{(-2-8)^2+(10-(-14))^2}=...$	M1
	Centre is $(3,-2)$; radius is 13	A1
	$\Rightarrow C_2:(x-3)^2+(y+2)^2=13^2\Rightarrow(p-3)^2+4=169\Rightarrow p=...$ Or $PX=r\Rightarrow(p-3)^2+(0-(-2))^2=169\Rightarrow p=...$	M1
	$p=3+\sqrt{165}$ only	A1
		(4)
		(8 marks)

(i) (a)

M1: Attempts to complete the square on both *x* and *y* or states the centre as $(\pm5,\pm6)$

For completing the square look for $(x\pm5)^2,(y\pm6)^2.....=...$

A1: Centre $(-5,6)$ This written down can score M1 A1

Allow written as separate coordinates ie. $x=-5, y=6$

(i)(b)

M1: $k+(\pm5)^2+(\pm6)^2>0$. Follow through on their $(-5,6)$. Allow $k+(\pm5)^2+(\pm6)^2 \ldots 0$

They must have an inequality for this mark.

A1: $k>-61$ but allow $k \ldots -61$

(ii)

M1: Attempts centre and radius (or radius squared) of C_2 using a correct method. (If only the diameter is found it is M0)

NB the attempt at the radius may arise from an attempt at *QX* or *RX* where *X* is the centre.

A1: Correct centre and radius. Centre at $(3,-2)$ and radius = 13

M1: Uses their centre and radius (allow if they think the diameter is the radius) in a correct method to find p (or x when $y = 0$). E.g.

- finds the equation of C_2, sets $y = 0$ and solves to find p (or x).
- uses the fact that $PX = r$ or $PX = QX$ or $PX = RX$ (where X is the centre) to form and solve an equation in p

For this mark allow if their centre and radius of the circle from (i) are used.

A1: $3+\sqrt{165}$ ONLY

Question Number	Scheme	Marks
Alt 6 (ii)	Uses either $PQ^2 + PR^2 = RQ^2$ or grad $PQ \times$ grad $PR = -1$	M1
	Correct equation. E.g. $(p+2)^2 + 10^2 + (p-8)^2 + 14^2 = (-2-8)^2 + (10+14)^2$ Alternatively $\dfrac{-10}{p+2} \times \dfrac{14}{p-8} = -1$	A1
	Correct method to set up and solve 3TQ. FYI $p^2 - 6p - 156 = 0$	dM1
	$p = 3+\sqrt{165}$ only	A1
		(4)

M1: Attempts

- either $PQ^2 + PR^2 = QR^2$ to set up an equation in p.
- or grad $PR \times$ grad $PQ = -1$ to set up an equation in p

Expect to see a correct attempt at the lengths or the gradients but condone slips.

A1: Correct equation which may be unsimplified. See scheme

dM1: Attempts to set up and solve a 3TQ resulting in a value for p.

A1: $3+\sqrt{165}$ ONLY

Question Number	Scheme	Marks
7. (i)	States or implies that $4\times 6^{n-1}>10^{100}$ Takes logs correctly to produce an equation without powers $4\times 6^{n-1}>10^{100}\Rightarrow \log 4+(n-1)\log 6>100\log 10$ $n=129$	B1 M1 A1 **(3)**
(ii) (a)	States $ar=-6$ and $\frac{a}{1-r}=25$ Combines to form an equation in $r \Rightarrow \frac{-6}{r(1-r)}=25$ $\Rightarrow -6=25r(1-r)\Rightarrow 25r^2-25r-6=0$ *	B1 M1 A1* **(3)**
(b)	$r=\frac{6}{5},-\frac{1}{5}$	B1 **(1)**
(c)	$r=-\frac{1}{5}$ as $\lvert r\rvert<1$ (for S_∞ to exist)	B1 **(1)**
(d)	Attempts $S_4=\frac{a(1-r^n)}{1-r}$ with $n=4, r=\text{their}(c)$ and $a=\frac{-6}{\text{their}(c)}$ $S_4=\frac{30\left(1-\left(-\frac{1}{5}\right)^4\right)}{1-\left(-\frac{1}{5}\right)}=24.96$ o.e.	M1 A1 **(2)**
		(10 marks)

(i)

B1: States or implies the solution can be found by solving $4\times 6^{n-1}...10^{100}$ where ... can be = or any inequality

M1: Shows a correct method of solving an equation of the form $4\times 6^{N}...10^{100}$ by correctly taking logs to produce an equation without powers. The log and index work must be correct, but allow slips in rearranging terms.

A1: $n=129$

(ii)(a)

B1: States or implies the correct two equations in a and r. $ar=-6$ and $\frac{a}{1-r}=25$

M1: Combines $ar=-6$ and $\frac{a}{1-r}=25$ to form a single equation in r

A1*: Proceeds to $25r^2-25r-6=0$ showing at least one correct simplified intermediate line and no errors (ii)(b)

B1: $r=\frac{6}{5},-\frac{1}{5}$ or exact equivalent – award when seen even if not in part (b).

(ii)(c)

B1: $r=-\frac{1}{5}$ as $|r|<1$ (for S_∞ to exist). Requires a minimal reason, but accept reasons that reject $\frac{6}{5}$ since it would mean all the terms are negative so cannot give a positive sum. Do not accept just "as the GS is convergent".

(ii)(d)

M1: Attempts $S_4=\frac{a\left(1-r^n\right)}{1-r}$ with $n=4, r=\text{their}(c)$ and $a=\frac{-6}{\text{their}(c)}$. If there was no attempt to answer (c) accept with either value from (b) for r.

If the correct formula is quoted then you can allow slips in substitution, but if the correct formula is not quoted then the equation should be correct for their r and a.

Alternatively, they may find and add the first 4 terms.

A1: 24.96 or equivalent such as $\frac{624}{25}$

Question Number	Scheme	Marks
8 (a)	$y=\frac{4}{3}x^3-11x^2+kx\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=4x^2-22x+k$	M1
	uses $x=2, \frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow 0=16-44+k\Rightarrow k=28$ *	dM1 A1*
		(3)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=4x^2-22x+28=0\Rightarrow(2x-4)(2x-7)=0\Rightarrow x=...$	M1
	$x<2, x>\frac{7}{2}$	A1
		(2)
(c)	$\int\left(\frac{4}{3}x^3-11x^2+28x\right)\mathrm{d}x\Rightarrow\frac{1}{3}x^4-\frac{11}{3}x^3+14x^2$	M1 A1
	Correct y coordinate of $M=\frac{68}{3}$	B1
	Complete method to find $R=2\times\text{"}\frac{68}{3}\text{"}-\int_0^2\left(\frac{4}{3}x^3-11x^2+28x\right)\mathrm{d}x$	M1
	$=2\times\text{"}\frac{68}{3}\text{"}-\left(\frac{1}{3}\times2^4-\frac{11}{3}\times2^3+14\times2^2\right)$	
	$=\frac{40}{3}$	A1
		(5) **(10 marks)**

(a)

M1: Attempts $\frac{\mathrm{d}y}{\mathrm{d}x}$ with one index correct. Must be seen in part (a).

dM1: Substitutes $x=2$ into $\frac{\mathrm{d}y}{\mathrm{d}x}$ and sets $=0$. The $\frac{\mathrm{d}y}{\mathrm{d}x}$ must be of the form ax^2+bx+k and the substitution must be clear. Going directly to $-28+k=0$ is M0. The "=0" may be implied by an attempt to solve the equation for this mark.

A1*: Achieves $k=28$ via a correct intermediate line and no missing "=0"

(b)

M1: Attempts to find the critical values.

A1: $x<2, x>\frac{7}{2}$ or $x\leqslant 2, x\geqslant\frac{7}{2}$ Do not accept $\frac{7}{2}<x<2$ Accept alternative set notations.

(c)

M1: Attempts to integrate with one index correct

A1: Correct integration $\int \left(\frac{4}{3}x^3 - 11x^2 + 28x\right)\mathrm{d}x \Rightarrow \frac{1}{3}x^4 - \frac{11}{3}x^3 + 14x^2$ need not be simplified

B1: Correct y coordinate of $M = \frac{68}{3}$. Accept awrt 22.7 for this mark. May be seen anywhere e.g. on the sketch

M1: Complete method to find $R =$ $2\times"\frac{68}{3}" - \int_0^2 \left(\frac{4}{3}x^3 - 11x^2 + 28x\right)\mathrm{d}x$ OR

$\int_0^2 \left(\frac{68}{3}\right) - \left(\frac{4}{3}x^3 - 11x^2 + 28x\right)\mathrm{d}x$ The lower limit may be implied. The integral must be a changed function.

A1: $\frac{40}{3}$ or exact equivalent

Question Number	Scheme	Marks
9.(a)	(If x and y are positive) $\left(\sqrt{x}-\sqrt{y}\right)^2 \ldots 0 \Rightarrow x - \ldots\sqrt{xy} + y \ldots 0$	M1
	$\Rightarrow x - 2\sqrt{xy} + y \ldots 0$	A1
	$\Rightarrow \dfrac{x+y}{2} \ldots \sqrt{xy}$	A1*
		(3)
(b)	States for example when $x=-8, y=-2,\ \dfrac{x+y}{2}=-5,\ \sqrt{xy}=4$ so $\dfrac{x+y}{2} < \sqrt{xy}$	B1
		(1)
		(4 marks)

(a)

M1: Sets up a correct inequality and attempts to expand $\left(\sqrt{x}-\sqrt{y}\right)^2$ leading to three terms.

A1: Correct expanded equation.

A1: Rearranges to the required equation with no errors seen.

If working in reverse allow the first M and A (if steps correct) but require also a minimal conclusion for the final A.

(b)

B1: Gives a suitable example with both sides evaluated correctly and a minimal conclusion.

There is no need to refer to x and y in the conclusion, so long as it has been shown the required inequality does not hold. E.g. $\dfrac{-8-2}{2}=-5 \not\geq 4=\sqrt{16}=\sqrt{-8\times-2}$ QED is fine.

A common response most likely to score 2 out of 3 marks

Question Number	Scheme	Marks
9.(a) **Alt 1**	$\frac{x+y}{2}\ldots\sqrt{xy} \Rightarrow \frac{(x+y)^2}{4}\ldots xy \Rightarrow \frac{x^2+\ldots xy+y^2}{4}\ldots xy$	M1
	$\Rightarrow x^2-2xy+y^2\ldots 0 \Rightarrow (x-y)^2\ldots 0$	A1
	States both of the following o.e • $(x-y)^2\ldots 0$ as it is a square number • so $\frac{x+y}{2}\ldots\sqrt{xy}$ is true	A1*
		(3)

M1: Assumes $\frac{x+y}{2}\ldots\sqrt{xy}$ true and attempts to square obtaining at least three terms.

A1: Correct expansion and rearranges the inequality correctly to factorise to a perfect square.

A1: A complete conclusion given.

Question Number	Scheme	Marks
9.(a) **Alt 2**	States $(x-y)^2\ldots 0 \Rightarrow x^2-2xy+y^2\ldots 0$	M1
	Rearranges $\Rightarrow x^2+2xy+y^2\ldots 4xy \Rightarrow (x+y)^2\ldots 4xy$	A1
	States that as x, y positive, so $x+y>0$ (and $xy>0$) $\Rightarrow (x+y)\ldots\sqrt{4xy} \Rightarrow \frac{x+y}{2}\ldots\sqrt{xy}$	A1*
		(3)

M1: Sets up an inequality using an appropriate perfect square and expands to at least three terms.

A1: Makes a correct rearrangement and factors the left hand side to produce the equation shown.

A1: Makes a full conclusion justifying why the square root gives $x+y$.

Question Number	Scheme	Marks
10 (i)	$\tan^2\left(2x+\frac{\pi}{4}\right)=3 \Rightarrow 2x+\frac{\pi}{4}=\pm\frac{\pi}{3},\pm\frac{2\pi}{3}$	M1
	$\Rightarrow x=\left(...-\frac{\pi}{4}\right)\div 2$	dM1
	$\Rightarrow x=\frac{\pi}{24},-\frac{11\pi}{24};\frac{5\pi}{24},-\frac{7\pi}{24}$	A1; A1, A1
		(5)
(ii)	$(2\sin\theta-\cos\theta)^2=1 \Rightarrow 4\sin^2\theta-4\sin\theta\cos\theta+\cos^2\theta=1$	
	Attempts to use $\sin^2\theta+\cos^2\theta=1 \Rightarrow 4\sin^2\theta-4\sin\theta\cos\theta=\sin^2\theta$	M1
	$\Rightarrow \sin\theta(3\sin\theta-4\cos\theta)=0$	dM1
	$\tan\theta=\frac{4}{3}$	A1
	$\theta=53.1°,233.1°,\underline{180°}$	A1, <u>B1</u>
		(5) **(10 marks)**

(i)

M1: Correct order of operations. Takes square root followed by arctan.

Implied by $2x+\frac{\pi}{4}=\frac{\pi}{3}$, $2x+\frac{\pi}{4}=1.047$. Condone for this mark only $2x+\frac{\pi}{4}=60$.

Allow if they use θ for $2x+\frac{\pi}{4}$

A longer method is to do $\dfrac{\sin^2\left(2x+\frac{\pi}{4}\right)}{\cos^2\left(2x+\frac{\pi}{4}\right)}=3 \Rightarrow \sin^2\left(2x+\frac{\pi}{4}\right)=3\cos^2\left(2x+\frac{\pi}{4}\right)$ and use $\sin^2\theta+\cos^2\theta=1$ to produce an equation in either sin or cos, before taking the inverse. They must have a correct method up to slips in rearranging, and reach the stage of taking arcsin or arccos in order to score the M.

dM1: Complete attempt to find one value for *x*.

This would involve an attempt to move the $\frac{\pi}{4}$ before dividing by 2. Condone

$x=\left(1.047\pm\frac{\pi}{4}\right)\div 2$

A1: One value of $\frac{\pi}{24},-\frac{11\pi}{24}$. Condone decimals here awrt 0.13, –1.44

A1: One value of $\frac{5\pi}{24},-\frac{7\pi}{24}$. Condone decimals here awrt 0.65, –0.92

A1: $\frac{\pi}{24},\frac{5\pi}{24},-\frac{7\pi}{24},-\frac{11\pi}{24}$ and no other values in the range

(ii)

M1: Attempts to multiply out to at least three terms, and use $\sin^2\theta+\cos^2\theta=1$ somewhere in the equation.

dM1: Cancels or factorises out the $\sin\theta$ term to produce a factor $a\sin\theta\pm b\cos\theta$ or an equation of the form $a\sin\theta\pm b\cos\theta=0$ oe (the "=0" may be implied)

A1: $\tan\theta=\frac{4}{3}$

A1: $\theta=53.1°, 233.1°$ and no others in the range

(Note: 0 and 360° are outside the range so ignore if given as solutions.)

B1: $\theta=180°$ Award when seen and allow however it arises.

Question Number	Scheme	Marks
10(ii) Alt	$(2\sin\theta-\cos\theta)^2=1\Rightarrow 2\sin\theta-\cos\theta=\pm1$	
	$\Rightarrow 2\sin\theta=\cos\theta\pm1$	
	$\Rightarrow 4\sin^2\theta=\cos^2\theta\pm2\cos\theta+1$	
	then attempts to use $\sin^2\theta+\cos^2\theta=1 \Rightarrow 4-4\cos^2\theta=\cos^2\pm2\cos\theta+1$	M1
	$\Rightarrow 5\cos^2\theta\pm2\cos\theta-3=0\Rightarrow\cos\theta=...$	dM1
	$\cos\theta=\frac{3}{5},-1$ or $\cos\theta=-\frac{3}{5},1$	A1
	$\theta=53.1°, 233.1°, \underline{180°}$	A1, <u>B1</u>
		(5) **(10 marks)**

M1: Attempts at least one of $2\sin\theta-\cos\theta=\pm1$, rearranges and squares then attempts to use $\sin^2\theta+\cos^2\theta=1$ to produce an equation in just $\cos\theta$ or just $\sin\theta$. FYI in $\sin\theta$ it is $4\sin^2\theta\pm4\sin\theta+1=1-\sin^2\theta$

dM1: Solves their quadratic in $\cos\theta$ or $\sin\theta$ to obtain at least one value for $\cos\theta$ or $\sin\theta$

A1: One correct pair of solutions. FYI in $\sin\theta$ it is $\sin\theta=0,\frac{4}{5}$ or $\sin\theta=0,-\frac{4}{5}$

A1: $\theta=53.1°, 233.1°$ and no others in the range. (Note via this method some extra solutions in the range are likely)

B1: $\theta=180°$ Award when seen and allow however it arises.

It is possible someone could have studied WMA13.

M1: Attempts to write $2\sin\theta-\cos\theta$ in the form $R\sin(\theta-\alpha)$ o.e. FYI $2\sin\theta-\cos\theta=\sqrt{5}\sin(\theta-26.6°)$

dM1: Proceeds from $R^2\sin^2(\theta-\alpha)=1$ to $\sin(\theta-\alpha)=\pm\frac{1}{R}$

A1: $\sin(\theta - 26.6°) = \pm\frac{1}{\sqrt{5}}$

A1: $\theta = 53.1°, 233.1°$ and no others in the range

(Note: 0 and 360° are outside the range so ignore if given as solutions.)

B1: $\theta = 180°$ Award when seen and allow however it arises.

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