Pearson Edexcel

Mark Scheme (Results)

October 2020

Pearson Edexcel International Advanced Level
in Core Mathematics C34 (WMA02) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = …

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks
1(a)	$\frac{dy}{dx}=-\frac{80}{(5x-2)^4}$	M1 A1
		[2]
(b)	$x=\frac{4}{5}\Rightarrow y=\frac{2}{3}$	B1
	$x=\frac{4}{5}\Rightarrow \frac{dy}{dx}=-5$	M1
	Hence equation of tangent is $\left(y-\frac{2}{3}\right)=-5\left(x-\frac{4}{5}\right)$	M1
	$y=-5x+\frac{14}{3}$	A1 cso
		[4]
		(6 marks)

(a)

M1 Differentiates using the chain rule (or quotient rule) to a form $\frac{\pm C}{(5x-2)^4}$ or equivalent

If the quotient rule is used they must proceed from $\frac{P(5x-2)^3\times 0\pm Q(5x-2)^2}{R(5x-2)^6}\rightarrow\frac{\pm Q(5x-2)^2}{R(5x-2)^6}$

A1 Correct and in simplest form. Accept $-80(5x-2)^{-4}$ or $\frac{-80}{(5x-2)^4}$

(b)

B1 y coordinate of P is $y=\frac{2}{3}$

M1 Attempts to find the gradient of the curve at P. Substitututes $x=\frac{4}{5}$ into their $\frac{dy}{dx}$

Condone $x=\frac{4}{5}\rightarrow\frac{dy}{dx}=\ldots$ as long as you don't have evidence to the contrary, for example using y.
Can be awarded even when they go on to find the equation of the normal.

M1 Attempts to find the equation of the tangent using their $\left(\frac{4}{5},\frac{2}{3}\right)$ and their numerical $\frac{dy}{dx}$

Look for $\left(y-their\,\frac{2}{3}\right)=-5\left(x-\frac{4}{5}\right)$ where -5 is the value of their $\frac{dy}{dx}$ at $x=\frac{4}{5}$
If the form $y=mx+c$ is used they must proceed as far as $c=\ldots$

A1 CSO $y=-5x+\frac{14}{3}$ ISW after a correct answer.

Question Number	Scheme	Marks
2 (a)	240 (m^2)	B1
		[1]
(b)	$260=\dfrac{1200e^{0.04t}}{4e^{0.04t}+1}\Rightarrow 160e^{0.04t}=260$	M1 A1
	$\Rightarrow 0.04t=\ln\left(\dfrac{260}{160}\right)\Rightarrow t=12.14$	dM1 A1
		[4]
(c)	300(m^2)	B1
		[1]
		(6 marks)

(a)

B1 240(m^2) Units are not required

(b)

M1 Proceeds from $260=\dfrac{1200e^{0.04t}}{4e^{0.04t}+1}$ to a form $Pe^{\pm 0.04t}=Q$

Condone P or Q negative. Condone slips

A1 $160e^{0.04t}=260$ or exact equivalent for example $e^{0.04t}=\dfrac{13}{8}$.

dM1 Dependent upon the first M. It is for correctly taking lns and proceeding to a value of t.

Eg proceeding from $e^{0.04t}=...$ to $0.04t=\ln(...)$ to $t=...$

$160e^{0.04t}=260\Rightarrow \ln160+0.04t=\ln260\Rightarrow t=...$ is also fine.

It can only be scored from a solvable equation E.g $Pe^{\pm 0.04t}=Q,\quad P,Q>0$

A1 Awrt 12.14. Allow for the exact answer of $25\ln\left(\dfrac{13}{8}\right)$ or $25\ln\left(\dfrac{a}{b}\right)$ where $\dfrac{a}{b}\equiv\dfrac{13}{8}$

(c)

B1 300(m^2) Units are not required. Condone 299.99

Part (b) No working

Going from $260=\dfrac{1200e^{0.04t}}{4e^{0.04t}+1}$ to 12.14 scores SC 1000

Question Number	Scheme	Marks
3.	$f(x)=\dfrac{2x^2+21}{(1-4x)(3+x)^2} \equiv \dfrac{A}{(1-4x)}+\dfrac{B}{(3+x)^2}+\dfrac{C}{(3+x)};\ \lvert x\rvert<\dfrac{1}{4}$	
(a) (i)	$A=2,\ B=3$	B1 B1
(a) (ii)	$2x^2+21\equiv A(3+x)^2+B(1-4x)+C(1-4x)(3+x)$ $x=-3\Rightarrow 39=13B\Rightarrow B=3$ $x=\dfrac{1}{4}\Rightarrow 21\dfrac{1}{8}=\left(3\dfrac{1}{4}\right)^2 A\ \Rightarrow \dfrac{169}{8}=\dfrac{169}{16}A\Rightarrow A=2$ Eg Compare $x^2: 2=A-4C\Rightarrow 2=2-4C\Rightarrow C=0$	M1 A1*
		[4]
(b)	$f(x)=\dfrac{2x^2+21}{(1-4x)(3+x)^2}\equiv\dfrac{2}{(1-4x)}+\dfrac{3}{(3+x)^2}$	
	$=2(1-4x)^{-1}+\dfrac{3}{9}\left(1+\dfrac{1}{3}x\right)^{-2}$	B1 ft
	$=\{2\}\left(1+(-1)(-4x)+\dfrac{(-1)(-2)}{2!}(-4x)^2+\ldots\right)+\left\{\dfrac{1}{3}\right\}\left(1+(-2)\left(\dfrac{1}{3}x\right)+\dfrac{(-2)(-3)}{2!}\left(\dfrac{1}{3}x\right)^2+\ldots\right)$	M1, A1 A1
	$=2\left(1+4x+16x^2+\ldots\right)+\dfrac{1}{3}\left(1-\dfrac{2}{3}x+\dfrac{1}{3}x^2+\ldots\right)$	
	$=\dfrac{7}{3}+\dfrac{70}{9}x+\dfrac{289}{9}x^2+\ldots$	A1, A1
		[6]
		10

(a)(i)

B1 At least one of $A=2$ or $B=3$

B1 Both $A=2$ and $B=3$

(a)(ii)

M1 Writes down a correct identity and attempts to find the value of either one of A or B or C

This can be scored from $2x^2+21\equiv A(3+x)^2+B(1-4x)+C(1-4x)(3+x)\Rightarrow A=2$

A1* Using a correct identity with a correct numerical equation to show that $C=0$

(b)

B1ft $\frac{\text{their } B}{(3+x)^2}\rightarrow\frac{\text{their } B}{9}\left(1+\frac{1}{3}x\right)^{-2}$ with the 3^{-2} processed. Condone sign slip $\frac{\text{their } B}{(3-x)^2}\rightarrow\frac{\text{their } B}{9}\left(1-\frac{1}{3}x\right)^{-2}$

M1 Evidence of correct binomial structure for either a power of -1 or -2.

Evidence is the correct binomial coefficient with the correct power of x

Look for $\left(1\pm{*x}\right)^{-1}=1\pm\left(-1\right)*x\pm\frac{(-1)(-2)}{2}*x^2$ or $\left(1\pm{*x}\right)^{-2}=1\pm\left(-2\right)*x\pm\frac{(-2)(-3)}{2}*x^2$

A1 Correct (un-simplified or simplified) expansion for at least one of either $(1-4x)^{-1}$ or $\left(1+\frac{1}{3}x\right)^{-2}$

A1 Correct (un-simplified or simplified) expansion for both $(1-4x)^{-1}$ and $\left(1+\frac{1}{3}x\right)^{-2}$

A1 Two of $\frac{7}{3}+\frac{70}{9}x+\frac{289}{9}x^2$ Must be simplified.

ISW after sight of a correct answer.

A1 Correct simplified expansion up term in x^2 $\frac{7}{3}+\frac{70}{9}x+\frac{289}{9}x^2$

If the candidate then multiplies by 9 this mark is withheld.

Question Number	Scheme	Marks
4.(a)	$f(8) = -0.3, \quad f(8.5) = (+)0.7$	M1
	Change of sign (and continuous), hence root	A1
		[2]
(b)	$\frac{dy}{dx} = \ln x + x \times \frac{1}{x} - 3x^{-\frac{1}{2}}$	M1 A1
	At Q, $\frac{dy}{dx} = 0 \Rightarrow \ln x = 3x^{-\frac{1}{2}} - 1$	M1
	$\Rightarrow x = e^{\frac{3}{\sqrt{x}} - 1}$ *	A1*
		[4]
(c)	Substitutes $x_1 = 2.5$ in $\Rightarrow x = e^{\frac{3}{\sqrt{x}} - 1}$	M1
	$\Rightarrow x_2 =$ awrt 2.453, $x_3 =$ awrt 2.498	A1
		[2]
		(8 marks)

(a)

M1 Attempts both f(8) and f(8.5) getting at least one correct to one decimal place rounded or truncated
Note $f(8) = -0.3350...$, $f(8.5) = (+)0.6977...$

A1 Both correct with a reason (change of sign) and a minimal conclusion.
Examples of a minimal conclusion are " root in between", "proven", " ✓" but don't award incorrect statements.
Allow as a minimum "As $f(8) \times f(8.5) < 0$, P must lie between these values."

(b)

M1 Attempts the product rule on $x \ln x$ achieving $\ln x + 1$ or unsimplified equivalent
Any efforts to work backwards will score 0 marks until there is some attempt to differentiate $x \ln x$

A1 $\frac{dy}{dx} = \ln x + x \times \frac{1}{x} - 3x^{-\frac{1}{2}}$ which may be left unsimplified

M1 Sets their $\frac{dy}{dx} = 0$ and makes $\ln x$ the subject of the formula. This may be implied by an equation.
Setting $y = 0$ is M0

A1* Proceeds to $x = e^{\frac{3}{\sqrt{x}} - 1}$ with no errors. This is a given answer

(c)

M1 Substitutes $x_1 = 2.5$ in $\Rightarrow x = e^{\frac{3}{\sqrt{x}} - 1}$. Implied by sight of $e^{\frac{3}{\sqrt{2.5}} - 1}$ or awrt 2.45

A1 $x_2 =$ awrt 2.453, $x_3 =$ awrt 2.498

Question Number	Scheme	Marks
5(a)	$\mathrm{f}(x+2)-2\mathrm{f}(x)=\frac{3}{x+2}-1-\frac{6}{x}+2$	M1
	$=\frac{3x-6(x+2)+x(x+2)}{x(x+2)}=\frac{x^2-x-12}{x(x+2)}$	M1, A1
		[3]
(b)	Either attempts to solve $\frac{3}{x}-1=7$ or substitute 7 in $\frac{3}{x+1}$	M1
	$\mathrm{f}^{-1}(7)=\frac{3}{8}$	A1
		[2]
(c)	Sets $\mathrm{fg}(x)=8\Rightarrow 3\mathrm{e}^{-4x}-1=8$	M1
	$\Rightarrow \mathrm{e}^{-4x}=3$	dM1
	$\Rightarrow x=-\frac{1}{4}\ln(3)$ oe	A1
		[3]
		(8 marks)

(a)

M1 Attempts $\mathrm{f}(x+2)-2\mathrm{f}(x)$ achieving $\frac{3}{x\pm2}\pm\frac{6}{x}\pm...$ or $\frac{3}{x\pm2}\pm2\times\frac{3}{x}\pm...$

M1 Attempts to add algebraic fractions (with 3 or 4 terms) of the form $\frac{A}{x\pm2}\pm\frac{B}{Cx}\pm D\pm...$

The denominator must be correct for their fractions and there must be an attempt to "correctly" adapt a**t least two of the numerators. (Condone bracketing slips)**

E.g. $\frac{A}{x+2}\pm\frac{B}{x}\pm C\rightarrow\frac{Ax\pm B(x+2)\pm C}{(x+2)x}$ "third term incorrectly adapted"

A1 Final answer $\frac{x^2-x-12}{x(x+2)}$ or equivalent such as $\frac{(x-4)(x+3)}{x^2+2x}$ ISW after a correct answer

(b)

M1 Either attempts to solve $\frac{3}{x}-1=7$ condoning errors on signs leading to $x=$..

Or attempts to find $\mathrm{f}^{-1}(x)=\frac{3}{x\pm1}$ oe and substitutes in $x=7$

A1 $\mathrm{f}^{-1}(7)=\frac{3}{8}$ oe

(c)

M1 Sets $\mathrm{fg}(x)=8\Rightarrow 3\mathrm{e}^{-4x}-1=8$ or equivalent such as $\frac{3}{\mathrm{e}^{4x}}-1=8$

dM1 Reaches $\mathrm{e}^{\pm4x}=C,\quad C>0$ It is dependent upon the previous M mark

A1 $x=-\frac{1}{4}\ln(3)$ or exact equivalent such as $x=\frac{1}{4}\ln\left(\frac{1}{3}\right),\quad -\frac{1}{2}\ln\left(\sqrt{3}\right),\quad \frac{1}{2}\ln\left(\frac{1}{\sqrt{3}}\right)$

ISW after a sight of a correct answer.

Question Number	Scheme	Marks
6.	$2y^3 + 3x^2y - 3x + 2 = 0$	
(a)	$6y^2\frac{dy}{dx} + 6xy + 3x^2\frac{dy}{dx} - 3 = 0$	M1 dM1 A1
	$6xy - 3 + (6y^2 + 3x^2)\frac{dy}{dx} = 0$	M1
	$\frac{dy}{dx} = \frac{1-2xy}{x^2+2y^2}$ or $\frac{dy}{dx} = \frac{2xy-1}{-x^2-2y^2}$ **o.e.**	A1 **cso**
		[5]
(b)	{Curve cuts x-axis at $P \Rightarrow y = 0$} $x_P = \frac{2}{3}$	B1
	At $P\left(\frac{2}{3}, 0\right)$, $m_T = \frac{1-0}{\left(\frac{2}{3}\right)^2 + 0} = \frac{9}{4} \Rightarrow m_N = -\frac{4}{9}$ $y - 0 = -\frac{4}{9}\left(x - \frac{2}{3}\right)$ and $x = 0 \Rightarrow y_Q = -\frac{4}{9}\left(-\frac{2}{3}\right)$	M1
	$\text{Area}(OPQ) = \frac{1}{2}\left(\frac{2}{3}\right)\left(\frac{8}{27}\right) = \frac{8}{81}$	dM1 A1 **cao**
		[4]
		9

(a) Part (a) now starts M, M A and not M, A, B

M1 Differentiates either $2y^3 \to ay^2\frac{dy}{dx}$ or $3x^2y \to px^2\frac{dy}{dx} + qxy$

dM1 Differentiates both $2y^3 \to ay^2\frac{dy}{dx}$ and $3x^2y \to px^2\frac{dy}{dx} + qxy$

A1 Correct differentiation

M1 An attempt to make $\frac{dy}{dx}$ the subject. For this to be awarded there must be **at least two terms in** $\frac{dy}{dx}$ coming from differentiating $2y^3$ and $3x^2y$

Look for an attempt to collect terms and factorise

A1 CSO $\frac{dy}{dx} = \frac{1-2xy}{x^2+2y^2}$ or $\frac{dy}{dx} = \frac{2xy-1}{-x^2-2y^2}$

This must be in simplest form so the 3's must be cancelled in $\frac{dy}{dx} = \frac{3-6xy}{3x^2+6y^2}$

(b)

B1 Deduces a correct x_P Can be simplified or un-simplified

M1 Uses (their x_P, 0) and the gradient of the **normal** in a complete method to find $y_Q = ...$

This mark can be implied if they choose the "c" from their $y = mx + c$

dM1 dependent on previous M mar**k.** Applies $\frac{1}{2}(x_P)(y_Q)$ $\frac{1}{2}(x_P)("c")$ or ignoring signs

A1 $\frac{8}{81}$ NOT $-\frac{8}{81}$ BUT can be awarded if $x_P = -\frac{2}{3}$ and they proceed to area $= \frac{8}{81}$

Question Number	Scheme	Marks
7 (a)	$\tan(2A+A) \equiv \dfrac{\tan 2A + \tan A}{1-\tan 2A \tan A}$	M1
	$\equiv \dfrac{\dfrac{2\tan A}{1-\tan^2 A}+\tan A}{1-\dfrac{2\tan A}{1-\tan^2 A}\times \tan A}$	dM1
	$\equiv \dfrac{2\tan A + \tan A\left(1-\tan^2 A\right)}{1-\tan^2 A - 2\tan A \times \tan A}$	ddM1
	$\equiv \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$ *	A1*
		[4]
(b)	$\dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x} = 4\tan x$	
	$11\tan^3 x - \tan x = 0$	M1 A1
	$\tan x = \pm\dfrac{1}{\sqrt{11}} \Rightarrow x = ...$ (one correct x for their equation)	M1
	$x = \pm 0.293, 0$	A1
		[4]
		(8 marks)

(a)

M1 Attempts $\tan(2A+A) = \dfrac{\tan 2A \pm \tan A}{1 \mp \tan 2A \tan A}$

dM1 Follows with an attempt at substituting $\tan(2A) = \dfrac{2\tan A}{1-\tan^2 A}$ in the above

ddM1 Multiplies **all terms** on both the numerator and denominator by $1-\tan^2 A$ or equivalent

Alternatively creates a "single fractions " of the form $\dfrac{....}{1-\tan^2 A}$ on both numerator and denominator

This must be done correctly for their fraction condoning bracketing slips.

Do not allow the candidate to write out either of the simplified lines (numerator or denominator)

A1* Proceeds to given answer with no errors showing **ALL** required steps. Allow a full solution with $A \to \theta$

The penultimate line in almost all cases should be $\dfrac{2\tan A + \tan A\left(1-\tan^2 A\right)}{1-\tan^2 A - 2\tan A \times \tan A}$ or $\dfrac{2\tan A + \tan A - \tan^3 A}{1-\tan^2 A - 2\tan^2 A}$

There should be no notational errors such as $\tan A^2 \leftrightarrow \tan^2 A$ or mixed variables (unless they state e.g. $A = \theta$.)

There should be no obvious bracketing errors within the body of the script.

(b)

M1 Uses the given identity and proceeds to an equation of the form $A\tan^3 x \pm B\tan x = 0$ oe
Accept $A\tan^2 x \pm B = 0$ oe

A1 $11\tan^3 x - \tan x = 0$ or $11\tan^2 x = 1$ or exact equivalent. It is implied by $\tan x = (\pm)\frac{1}{\sqrt{11}}$

M1 Uses a correct method of solving their $\tan^2 x = D$ using correct order of operations
This may be implied by one correct answer from their $\tan x = (\pm)\sqrt{D}$, $D > 0$ in either degrees or radians. Degree answer is 16.8. Condone 0.29 as sufficient accuracy for the M1

A1 $x = \pm 0.293, 0$ ONLY in the range . Condone $x = \pm 0.0932\pi, 0$. **This is not awrt**

....

Other methods are available in part (a). This is an example of how you can apply the mark scheme

M1 Attempts $\tan(2A + A) = \frac{\tan 2A \pm \tan A}{1 \mp \tan 2A \tan A}$ or $\frac{\sin(2A + A)}{\cos(2A + A)} = \frac{\sin 2A\cos A \pm \cos 2A\sin A}{\cos 2A\cos A \mp \sin 2A\sin A}$

dM1 Follows with an attempt at substituting $\tan(2A) = \frac{\sin 2A}{\cos 2A} = \frac{2\sin A\cos A}{\cos^2 A - \sin^2 A}$ AND $\tan A = \frac{\sin A}{\cos A}$

in the above. Allow any **correct** substitution for $\cos 2A$

ddM1 Creates a single fraction in single angles. E.g.

$$\tan(3A) = \frac{\tan 2A + \tan A}{1 - \tan 2A\tan A} = \frac{\frac{2\sin A\cos A}{\cos^2 A - \sin^2 A} + \frac{\sin A}{\cos A}}{1 - \frac{2\sin A\cos A}{\cos^2 A - \sin^2 A} \times \frac{\sin A}{\cos A}} = \frac{2\sin A\cos^2 A + \sin A\left(\cos^2 A - \sin^2 A\right)}{\cos A\left(\cos^2 A - \sin^2 A\right) - 2\sin^2 A\cos A} = \frac{3\sin A\cos^2 A - \sin^3 A}{\cos^3 A - 3\sin^2 A\cos A}$$

It must be in a form that requires just one more process (e.g. $\div\cos^3 A$) before reaching the given answer

A1* Divides both numerator and denominator by $\cos^3 A$ to reach the given answer

Question Number	Scheme	Marks
8 (a)	$h=\frac{3}{2}$	B1
	$\text{Area}(R)=\frac{1}{2}\times\frac{3}{2}\left\{\underline{4+2(2.5829+2.6612+3.0514)+3.4026}\right\}$	M1
	$\{=0.75(23.9936)=17.9952\}=18.0\ (3\text{ sf})$	A1 **cao**
		[3]
(b)	$\int 4-2xe^{-\frac{1}{2}x}\,dx=\underline{4x}-\left[-4xe^{-\frac{1}{2}x}+\int 4e^{-\frac{1}{2}x}\,dx\right]$	<u>B1</u> M1
	$=4x-\left[-4xe^{-\frac{1}{2}x}-8e^{-\frac{1}{2}x}\right]$	dM1, A1
	$\text{Area}(R)=\int_0^6 4-2xe^{-\frac{1}{2}x}\,dx=\left[4x+4xe^{-\frac{1}{2}x}+8e^{-\frac{1}{2}x}\right]_0^6$	
	$=(24+24e^{-3}+8e^{-3})-(0+0+8)$	M1
	$=16+32e^{-3}$	A1
		[6]
		9 marks

(a)

B1: Correct h seen or implied.

M1: Full application of the trapezium rule with their h.

Look for an attempt at $\frac{"h"}{2}\left\{y_0+y_6+2\times\left(y_{1.5}+y_3+y_{4.5}\right)\right\}$ condoning slips.

Bracketing errors can be recovered but an answer of 26.39...OR 29.79.. is M0

A1: awrt 18.0

Note that the calculator answer is 17.6

(b)

B1: Integrates $4\to 4x$

M1: Attempts to integrate by parts the $xe^{-\frac{1}{2}x}$ term

Look for $-2xe^{-\frac{1}{2}x}\to \pm Axe^{-\frac{1}{2}x}\pm B\int e^{-\frac{1}{2}x}dx;\ A,\ B\neq 0$.

Condone incorrect signs as long as you don't see an incorrect rule stated.

Occasionally you may see the second integration being done at the same time. The same constant appearing twice is usually evidence of this. In such a case M1 dM1 is scored at the same time.

dM1: Fully integrates the $xe^{-\frac{1}{2}x}$ term to a form $\lambda xe^{-\frac{1}{2}x}\pm\mu e^{-\frac{1}{2}x};\ \lambda,\mu\neq 0$

A1: Correct integration $4x-\left[-4xe^{-\frac{1}{2}x}-8e^{-\frac{1}{2}x}\right]$ oe $4x+4xe^{-\frac{1}{2}x}+8e^{-\frac{1}{2}x}$

M1: Applies limits of 6 and 0 to an integrated function of the form $\pm\alpha x\pm\beta xe^{-\frac{1}{2}x}\pm\mu e^{-\frac{1}{2}x};\ \alpha,\ \beta,\ \mu\neq 0$ and subtracts. There must be explicit evidence that the 0 has been used. It cannot just be set = 0

A1: Correct answer $16+32e^{-3}$ (or exact simplified equivalent)

Question Number	Scheme	Marks
9.	$x = 1 - 8\cos 2t,\ y = 9\sin t;\ 0 \leqslant t \leqslant \frac{\pi}{2}$	
(a)	$\frac{dx}{dt} = 16\sin 2t,\quad \frac{dy}{dt} = 9\cos t$	B1, B1
	$\frac{dy}{dx} = \frac{9\cos t}{16\sin 2t}\ \left\{= \frac{9}{32\sin t}\right\}$ $x = 5 \Rightarrow t = \frac{\pi}{3},\ \frac{dy}{dx} = \frac{9\cos\left(\frac{\pi}{3}\right)}{16\sin\left(2\left(\frac{\pi}{3}\right)\right)}\ \left\{= \frac{\frac{9}{2}}{16\left(\frac{\sqrt{3}}{2}\right)}\right\}$	M1
	$\left\{= \frac{9}{16\sqrt{3}}\right\} = \frac{3}{16}\sqrt{3}$	A1
		[4]
(b)	Uses $\cos 2t = 1 - 2\sin^2 t$ with $\cos 2t = \frac{1-x}{8},\ \sin t = \frac{y}{9} \Rightarrow \frac{1-x}{8} = 1 - 2\left(\frac{y}{9}\right)^2$	M1, A1
	Proceeds correctly to $y = \frac{9}{4}\sqrt{7+x}$ *	A1 *
		[3]
(c)	$\{-7 \le x \le 9 \Rightarrow\}\ a = -7,\ b = 9$	B1, B1
		[2]
(d)	E.g. $0 \le f(x) \le 9$ or $0 \le y \le 9$ or $[0, 9]$	B1
		[1]
		10 marks

(a) **Note that the demand is via parametric differentiation, so cartesian equivalents score 0 marks**

B1: At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct (Can be implied or recovered by sight of a correct $\frac{dy}{dx}$)

If $x = 1 - 8\cos 2t$ is adapted , this must be done correctly $x = 1 - 8\cos 2t = 16\sin^2 t - 7 \Rightarrow \frac{dx}{dt} = 32\sin t\cos t$

B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct (Can be implied or recovered by sight of a correct $\frac{dy}{dx}$)

M1: Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **and** substitutes $t = \frac{\pi}{3}$ or $t = 60°$ into their $\frac{dy}{dx}$

A1: Achieves a correct gradient in the correct form $\frac{3}{16}\sqrt{3}$ Alternatively gives $k = \frac{3}{16}$ or exact equivalent

(b) Part (b) is M M A on epen. We are marking it M A A

M1: Uses $\cos 2t = \pm 1 \pm 2\sin^2 t$ in an attempt to form an equation in y and x

E.g. $x = 1 - 8\left(1 - 2\sin^2 t\right) = 1 - 8 + 16\sin^2 t = -7 + 16\left(\frac{y}{9}\right)^2$ condoning bracketing slips and sign slips on $\cos 2t$

A1: A correct unsimplified equation linking x and y

A1*: Proceeds correctly to the given answer.

E.g. $\frac{1-x}{8} = 1 - 2\left(\frac{y}{9}\right)^2 \Rightarrow y = \frac{9}{4}\sqrt{7+x}$ would require an intermediate line of $y^2 = \frac{81}{16}(7+x)$

Alt (b) using given equation

(b)	$y = \frac{9}{4}\sqrt{7+x} = \frac{9}{4}\sqrt{7+1-8\cos 2t}$ $= \frac{9}{4}\sqrt{7+1-8(1-2\sin^2 t)}$		M1, A1
	$= \frac{9}{4}\sqrt{16\sin^2 t} = 9\sin t = y$ with conclusion *		A1 *
			[3]

M1: Substitutes $x = 1 - 8\cos 2t$ into $y = \frac{9}{4}\sqrt{7+x}$ and attempts $\cos 2t = \pm 1 \pm 2\sin^2 t$

A1: A fully correct expression for y in terms of just $\sin t$. E.g. $\frac{9}{4}\sqrt{7+1-8(1-2\sin^2 t)}$

A1*: Progresses from $\frac{9}{4}\sqrt{7+x}$ to $9\sin t = y$ with no algebraic errors seen in their working and a minimal conclusion. E.g. Hence shown

(c)

B1: At least one of $a = -7$ or $b = 9$. Implied by one correct of $-7 \le x \le 9$

B1: Both $a = -7$ and $b = 9$ Implied by both correct of $-7 \le x \le 9$

(d)

B1: Correct range stated using allowable notation. See scheme. Allow $\mathrm{f} \leftrightarrow \mathrm{f}(x)$

Do not allow for example $0 \le x \le 9$ or $(0, 9)$

Question Number	Scheme		Marks
10 (a)	$\frac{\mathrm{d}h}{\mathrm{d}t} \propto \sqrt{h}$ or $\frac{\mathrm{d}h}{\mathrm{d}t} = k\sqrt{h}$ o.e.	$\frac{\mathrm{d}h}{\mathrm{d}t} \propto -\sqrt{h}$ or $\frac{\mathrm{d}h}{\mathrm{d}t} = -k\sqrt{h}$	M1
	$\left\{\frac{\mathrm{d}h}{\mathrm{d}t} = kh^{\frac{1}{2}} \Rightarrow\right\} \int \frac{1}{h^{\frac{1}{2}}}\,\mathrm{d}h = \int k\,\mathrm{d}t \Rightarrow \ldots$	$\left\{\frac{\mathrm{d}h}{\mathrm{d}t} = -kh^{\frac{1}{2}} \Rightarrow\right\} \int \frac{1}{h^{\frac{1}{2}}}\,\mathrm{d}h = \int -k\,\mathrm{d}t \Rightarrow \ldots$	M1
	$\int h^{-\frac{1}{2}}\,\mathrm{d}h = \int k\,\mathrm{d}t \Rightarrow \frac{h^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt\ \{+c\}$ or $2h^{\frac{1}{2}} = kt\ \{+c\}$	$\int h^{-\frac{1}{2}}\,\mathrm{d}h = \int -k\,\mathrm{d}t \Rightarrow \frac{h^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = -kt\ \{+c\}$ or $2h^{\frac{1}{2}} = -kt\ \{+c\}$	A1
	$\{t = 0,\ h = 225 \Rightarrow\}$ $2\sqrt{225} = k(0) + c\ \{\Rightarrow c = 30\}$	$\{t = 0,\ h = 225 \Rightarrow\}$ $2\sqrt{225} = -k(0) + c\ \{\Rightarrow c = 30\}$	M1
	$t = 125,\ h = 100$ $\Rightarrow 2\sqrt{100} = k(125) + 30$ $\Rightarrow 20 = k(125) + 30 \Rightarrow k = -0.08$	$t = 125,\ h = 100$ $\Rightarrow 2\sqrt{100} = -k(125) + 30$ $\Rightarrow 20 = -k(125) + 30 \Rightarrow k = 0.08$	dM1
	$\Rightarrow 2h^{\frac{1}{2}} = -0.08t + 30$ $\Rightarrow h^{\frac{1}{2}} = -0.04t + 15$ CSO	$\Rightarrow 2h^{\frac{1}{2}} = -0.08t + 30$ $\Rightarrow h^{\frac{1}{2}} = -0.04t + 15$ CSO	A1*
	$a = 375$ or $0 \le t \le 375$		B1
			[7]
(b)	$\{h = 50 \Rightarrow\}\ 50 = (15 - 0.04t)^2 \Rightarrow t = (198.2233047\ldots)$		M1
	Time = $198.2233047\ldots - 125$		dM1
	$= 73.2233047\ldots = 73$ (minutes)		A1
			[3]
			10 marks

(a) **Note that k can be set as positive (above left) or negative (above right) at the start of (a)**

M1: Converts the given information into maths. Do not award if k has been given a pre defined value.

Condone $\mathrm{d}h = \pm k\sqrt{h}\ \mathrm{d}t$ and allow versions such as $\frac{\mathrm{d}t}{\mathrm{d}h} = \pm\frac{k}{\sqrt{h}}$

M1: Separates the variables for their differential equation (which in the form $\frac{\mathrm{d}h}{\mathrm{d}t} = \mathrm{f}(h)$) and attempts to integrate at least one side. Condone lack of integral signs. One side must be of the correct form

A1: Correct integration. with or without a constant of integration.

Watch for $\frac{h^{\frac{1}{2}}}{2} = \pm kt\ \{+c\}$ which leads to the correct answer. This scores A0.

Follow through on their k if this has been assigned a value. This would occur when $k = 1$ for instance.

M1: Substitutes $t = 0,\ h = 225$ into a changed equation and finds c

Can be scored when k has been set to 1, for example, so these solutions generally score 011100-

dM1: Substitutes $t = 125,\ h = 100$ and their value of 'c' into their changed equation and find a value for k

A1*: CSO Proceeds without errors to the given answer. A penultimate line of $h^{\frac{1}{2}} = -0.04t + 15$ or equivalent should be seen.

B1: States $a = 375$ or $0 \le t \le 375$ Condone if any units are attached to this value

(b)

M1: Substitutes $h = 50$ into the given equation and rearranges to find $t = ...$

May be scored from a 3TQ

dM1: ...and then subtracts 125 from their value for t. This can be implied by $t_{h=50} - 125$

A1: awrt 73 (minutes) following correct work .

Do not need units but withhold if they state a different unit. E.g 73 seconds

Question Number	Scheme	Marks
11 (a)	$x-2a=\pm 3b \Rightarrow x=..$	M1
	$x=2a\pm 3b$	A1
		[2]
(b)	$P=(2a,-3b)$	B1
		[1]
(c) (i)	$P'=(2a,6b)$	B1ft
(ii)	$P''=(a,-9b)$	B1ft
		[2]
(d)	Correct equation $-x+2a-3b=2x+a$	M1
	Collects terms and proceeds to value for x $3x=a-3b \Rightarrow x=...$	dM1
	$x=\frac{1}{3}a-b$ oe **only**	A1
		[3]
		(8 marks)

(a)
M1: Considers either solution of $\lvert x-2a\rvert=3b \Rightarrow x=..$

May be awarded from a solution to the equation $(x-2a)^2=(3b)^2 \Rightarrow x=..$

A1: Both $x=2a+3b$ and $x=2a-3b$

(b)
B1: $P=(2a,-3b)$ may be given separately $x=2a,\ y=-3b$

(c)(i)
B1ft $P'=(\textit{their}\ 2a, 2\times\lvert \textit{their} 3b\rvert)$ It is a follow through but only from a coordinate in quadrant 4

You may ft on non algebraic part (b) 's so $(2,-3)\to(2,3)$ would be OK here

(c)(ii)
B1ft $P''=\left(\frac{1}{2}\times \textit{their} 2a, 3\times \textit{their} -3b\right)$ Follow through on a coordinate in any quadrant

You may ft on non algebraic part (b) 's so $(2,-3)\to(1,-9)$ is OK

(d)
M1 Correct equation. This may be one of a pair of equations (or more)
Note that $x-2a+3b=-2x-a$ is also correct.
Award for this equation even if the solution is later rejected.

dM1 Attempts to solve a correct equation $-x+2a-3b=2x+a$ o.e.
Collects terms and proceeds to $x=...$
Award for a solution to a correct equation even if it is later rejected.

A1 $x=\frac{1}{3}a-b$ oe such as $x=\frac{1}{3}(a-3b)$ **only.** If other values have been found they must be rejected

Part (d) via squaring is very difficult but possible

M1 If $(x-2a)^2=(2x+a+3b)^2$ is attempted the candidate must proceed as far as the "correct" intermediate equation $x=\dfrac{-8a-12b\pm\sqrt{100a^2+120ab+36b^2}}{6}$

dM1 Chooses $x=\dfrac{-8a-12b+(10a+6b)}{6}$

Question Number	Scheme	Notes	Marks
12 (i)	$\int 4(5y-7)^{-4}\,dy = -\frac{4}{15}(5y-7)^{-3}\ \{+c\}$		M1, A1
			[2]
(ii)	Attempts to multiple out $(1-4\tan 3x)^2 = 1-8\tan 3x+16\tan^2 3x$		M1
	Integrates with two of the three (different) terms in the correct form $\int .....dx = \pm\alpha x \pm \beta\ln\lvert\sec 3x\rvert \pm \lambda\tan 3x$ o.e.		M1
	Integrates to a correct form $\int 1-8\tan 3x+16\sec^2 3x-16\,dx = \pm\alpha x\pm\beta\ln\lvert\sec 3x\rvert\pm\lambda\tan 3x$		dM1
	$= -15x-\frac{8}{3}\ln\lvert\sec 3x\rvert+\frac{16}{3}\tan 3x\ \{+c\}$ or $-15x+\frac{8}{3}\ln\lvert\cos 3x\rvert+\frac{16}{3}\tan 3x$		A1
			[4]
(iii)	$\{u=1+2\cos\theta \Rightarrow\}\ \frac{du}{d\theta}=-2\sin\theta$ o.e.		B1
	Applies $\sin 2\theta = 2\sin\theta\cos\theta$ to the integral		B1
	Full attempt at substitution $\int \frac{2\sin 2\theta}{1+2\cos\theta}d\theta = \pm k\int \frac{u-1}{u}.du$		M1
	Correct integral in u $= \int \frac{1-u}{u}\,du$ o.e.		A1
	Correct attempt at integration with an attempt to apply "u" limits of 3 and 1 to an integral of the required form $\int_3^1 \frac{1}{u}-1\,du = [\ln u - u]_3^1 = ...$		dM1
	$= 2-\ln 3 = \ln e^2 - \ln 3 = \ln\left(\frac{1}{3}e^2\right)$; $\left\{A=\frac{1}{3}\right\}$		A1
			[6]
			12

(i)

M1: Integrates to a form $\pm\lambda(5y-7)^{-3}$; $\lambda \neq 0$

If they use a substitution $u=5y-7$ look for $\pm\delta u^{-3}$; $\delta\neq 0$

Also "correct" are versions such as $\pm\kappa\left(y-\frac{7}{5}\right)^{-3}$; $\kappa\neq 0$ and $\pm\alpha(20y-28)^{-3}$; $\alpha\neq 0$

A1: Correct integration and in simplest form. Look for $-\frac{4}{15}(5y-7)^{-3}$ o.e. and condone the omission of $+c$

If the question is done in another variable $y \leftrightarrow x$ then score as above but M1 A0

(ii)

M1: Multiplies out to give $A + B\tan 3x + C\tan^2 3x;\ A,\ B,\ C \neq 0$

M1: Integrates giving any two terms in the correct form. Award for any two of

either $A \to Ax$,

$B\tan 3x \to ...\ln|\sec 3x|$ or $-...\ln|\cos 3x|$

or $C\tan^2 3x \to ...\tan 3x \pm ...x$

dM1: Awarded for integrating all three terms into the required form

- writing $(1-4\tan 3x)^2$ in the form $A + B\tan 3x + C\tan^2 3x$
- attempting to use $\pm 1 \pm \tan^2 3x = \pm\sec^2 3x$
- and then integrating to a form $Ax \pm \beta\ln|\sec 3x| \pm \lambda\tan 3x$ or $Ax \pm \beta\ln|\cos 3x| \pm \lambda\tan 3x$

A1: Correct answer

either $-15x - \frac{8}{3}\ln|\sec 3x| + \frac{16}{3}\tan 3x\ \{+c\}$

or $-15x + \frac{8}{3}\ln|\cos 3x| + \frac{16}{3}\tan 3x\{+c\}$ with or without $+c$

Condone a lack of modulus sign throughout this part.

(iii) We are now marking this B1 B1 M1 A1 M1 A1

B1: $\frac{\mathrm{d}u}{\mathrm{d}\theta} = -2\sin\theta$ or exact equivalent, for example $\mathrm{d}u = -2\sin\theta\,\mathrm{d}\theta$

B1: Applies $\sin 2\theta = 2\sin\theta\cos\theta$ to the integral, not just written in isolation.

M1: Full method in forming an integral in just u.

Expect to see $\sin 2\theta = P\sin\theta\cos\theta$ proceeding to an integral in u of the form $\pm k\int \frac{u-1}{u}.(\mathrm{d}u)$

Condone a missing du as long as the integrand is of the correct form

A1: Correct integrand $\int \frac{1-u}{u}(\mathrm{d}u)$ or equivalent such as $-\int \frac{u-1}{u}(\mathrm{d}u)$ ignoring limits but watch for **

Watch for cases where the limits are "flipped".

** So accept $\int \frac{u-1}{u}(\mathrm{d}u)$ ONLY IN CASES where you see the limits as well $\int_1^3 \frac{u-1}{u}(\mathrm{d}u)$

dM1: Correct method of integrating an expression of the form $\pm k\int \frac{u-1}{u}.\mathrm{d}u$ between $u = 1$ and $u = 3$

The limits can be applied either way around.

Expect to see division by u followed by $\int \frac{1}{u}.\mathrm{d}u \to \ln u$ or $\ln ku$

An alternative to is to change the form correctly back to θ's and use 0 and $\frac{\pi}{2}$. If this is done expect the $\cos\left(\frac{\pi}{2}\right)'s$ etc to be processed.

A1: $\ln\left(\frac{1}{3}\mathrm{e}^2\right)$ or $A = \frac{1}{3}$

Question Number	Scheme	Marks
13(a) (i)	$\frac{\mathrm{d}H}{\mathrm{d}t} = (0) + \frac{\mathrm{e}^{0.2t} \times -25\sin(0.5t) - 10\mathrm{e}^{0.2t}\cos(0.5t)}{(\mathrm{e}^{0.2t})^2}$	M1 A1
	$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{-25\sin(0.5t) - 10\cos(0.5t)}{\mathrm{e}^{0.2t}}$ oe	A1
		[3]
(ii)	Sets $\frac{\mathrm{d}H}{\mathrm{d}t} = 0 \Rightarrow a\sin(0.5t) + b\cos(0.5t) = 0$	M1
	$-25\sin(0.5t) - 10\cos(0.5t) = 0 \Rightarrow \tan(0.5t) = -\frac{10}{25} = -0.4$ *	A1*
		[2]
(b)	$\tan(0.5t) = -0.4 \Rightarrow 0.5t = 2\pi - 0.381$	M1
	$\Rightarrow t = 11.8$	A1
	$H = 64.4$	A1
		[3]
		(8 marks)

(a)(i)

M1 Attempts quotient rule $\frac{vu'-uv'}{v^2}$ to reach a form $\frac{\pm\alpha e^{0.2t}\sin(0.5t)\pm\beta e^{0.2t}\cos(0.5t)}{\left(e^{0.2t}\right)^2}$

Ignore reference to the 60 for this mark and condone slips on the v^2 (e.g. $e^{0.04t^2}$)

If a candidate states or implies $\frac{vu'+uv'}{v^2}$ then it is M0

Alt via product rule $50e^{-0.2t}\cos(0.5t)\to\pm\alpha e^{-0.2t}\cos(0.5t)\pm\beta e^{-0.2t}\sin(0.5t)$

A1 Correct and unsimplified use of either the product or quotient rule. Ignore the 60 for this mark.

$\frac{dH}{dt}=(0)+\frac{e^{0.2t}\times-25\sin(0.5t)-10e^{0.2t}\cos(0.5t)}{\left(e^{0.2t}\right)^2}$ or $\frac{dH}{dt}=(0)-25e^{-0.2t}\sin(0.5t)-10e^{-0.2t}\cos(0.5t)$

A1 $\frac{dH}{dt}=-25e^{-0.2t}\sin(0.5t)-10\cos(0.5t)e^{-0.2t}$ oe BUT it must be in simplest form.

Allow common factors to be taken out so allow forms such as $\frac{dH}{dt}=-5\times\frac{5\sin(0.5t)+2\cos(0.5t)}{e^{0.2t}}$

(a)(ii)

M1 Sets their $\frac{dH}{dt}=0$ and factorises out/ divides by $e^{-0.2t}$ and proceeds to an equation of the form $a\sin(0.5t)+b\cos(0.5t)=0$

This can implied by sight by $\frac{dH}{dt}=\pm\alpha e^{-0.2t}\cos(0.5t)\pm\beta e^{-0.2t}\sin(0.5t)\Rightarrow\pm\alpha\cos(0.5t)=\pm\beta\sin(0.5t)$

A1* $\tan(0.5t)=-0.4$ via a correct intermediate line. correct intermediate line is one of the form $-25\sin(0.5t)-10\cos(0.5t)=0$ or $-25\sin(0.5t)=10\cos(0.5t)$

This can be awarded from an incorrect denominator in part (i) e.g. of $e^{0.2t^2}$ but not from an incorrect numerator

(b)

M1 Takes arctan and selects an angle from the 2nd or 4th quadrant.

Award for $0.5t=2\pi-0.381$ or $0.5t=\pi-0.381$ implied by 11.8 or 5.52

Degree answers will score 0 marks here unless the 0.5 is converted to radians

A1 $t=$ awrt 11.8

A1 $H=$ awrt 64.4

....

Note that errors on $\left(e^{0.2t}\right)^2$ will be common with $e^{0.2t^2}$ and $e^{0.04t^2}$ being common.

They can recover in part (a) to score M1 A0 A1 and go on to score all marks in (ii).

....

Question Number	Scheme	Marks
14 (a)	$\overrightarrow{OQ}=\overrightarrow{OP}+\overrightarrow{PQ}==\begin{pmatrix}3\\2\\9\end{pmatrix}+\begin{pmatrix}1\\-2\\4\end{pmatrix}=\begin{pmatrix}4\\0\\13\end{pmatrix}\Rightarrow Q(4,0,13)$	B1
		[1]
(b)	$\overrightarrow{PA}=\overrightarrow{OA}-\overrightarrow{OP}=\begin{pmatrix}8\\2\\6\end{pmatrix}-\begin{pmatrix}3\\2\\9\end{pmatrix}=\begin{pmatrix}5\\0\\-3\end{pmatrix}$ or $\overrightarrow{AP}=\begin{pmatrix}-5\\0\\3\end{pmatrix}$	M1
	$\left\{\cos\theta=\dfrac{\overrightarrow{PA}\bullet\overrightarrow{PQ}}{\lvert\overrightarrow{PA}\rvert\lvert\overrightarrow{PQ}\rvert}\right\}=\dfrac{5\times1+0\times-2+-3\times4}{\sqrt{(5)^2+(0)^2+(-3)^2}.\sqrt{(1)^2+(-2)^2+(4)^2}}$	dM1
	$\theta=$ awrt $105.19°$	A1
		[3]
(c)	$\{l_2:\}\ \mathbf{r}=\begin{pmatrix}8\\2\\6\end{pmatrix}+\mu\begin{pmatrix}1\\-2\\4\end{pmatrix}$	M1, A1
		[2]
(d)	**Attempts** $\begin{pmatrix}3\\2\\9\end{pmatrix}-\begin{pmatrix}1\\-2\\4\end{pmatrix}$	M1
	$\overrightarrow{OX}=\begin{pmatrix}3\\2\\9\end{pmatrix}-\begin{pmatrix}1\\-2\\4\end{pmatrix}=\begin{pmatrix}2\\4\\5\end{pmatrix}\Rightarrow X(2,4,5)$	A1
		[2]
(e)	$\overrightarrow{PY}="\begin{pmatrix}8+\mu\\2-2\mu\\6+4\mu\end{pmatrix}"-\begin{pmatrix}3\\2\\9\end{pmatrix}\left\{=\begin{pmatrix}5+\mu\\-2\mu\\-3+4\mu\end{pmatrix}\right\}$	M1
	$\overrightarrow{PY}\bullet\mathbf{d}_2=0\Rightarrow\begin{pmatrix}5+\mu\\-2\mu\\-3+4\mu\end{pmatrix}\bullet\begin{pmatrix}1\\-2\\4\end{pmatrix}=0\Rightarrow\mu=...$	dM1
	$\Rightarrow 5+\mu+4\mu-12+16\mu=0\Rightarrow -7+21\mu=0\Rightarrow\mu=\dfrac{1}{3}$	A1 o.e.
	$\overrightarrow{OY}=\begin{pmatrix}8+\frac{1}{3}\\2-2\left(\frac{1}{3}\right)\\6+4\left(\frac{1}{3}\right)\end{pmatrix}=\begin{pmatrix}\frac{25}{3}\\\frac{4}{3}\\\frac{22}{3}\end{pmatrix}\Rightarrow Y\left(\frac{25}{3},\frac{4}{3},\frac{22}{3}\right)$	ddM1 A1
		[5]
		13

(a)

B1: $(4, 0, 13)$ but condone vectors here. Allow for $\begin{pmatrix}4\\0\\13\end{pmatrix}$ or $4\mathbf{i}+13\mathbf{k}$ but not $\begin{pmatrix}4\mathbf{i}\\0\mathbf{j}\\13\mathbf{k}\end{pmatrix}$

(b)

M1: ‘Subtraction’ attempt either way around) to find $\overrightarrow{PA}$ or $\overrightarrow{AP}$. If no method is shown imply for 2 correct

dM1: Applies dot product formula between their $\left(\overrightarrow{PA} \text{ or } \overrightarrow{AP}\right)$ and $\left(\overrightarrow{PQ} \text{ or } \overrightarrow{QP}\right)$ or a multiple of these vectors leading to value for $\cos\theta$. Condone slips

An answer of 74.81° implies this dM mark but will lose the A. Radian values are 1.84(105°) and 1.31 (75°)

An alternative is via the cosine rule with $\cos APQ = \dfrac{34+21-69}{2\times\sqrt{34}\times\sqrt{21}}$

A1: $\theta = \text{awrt } 105.19°$ Do not accept both the obtuse and acute angles being given.

(c)

M1: Attempts $\begin{pmatrix}8\\2\\6\end{pmatrix} + \mu\begin{pmatrix}1\\-2\\4\end{pmatrix}$ condoning slips.

Allow exact equivalents such as $\begin{pmatrix}8\\2\\6\end{pmatrix} + \mu\begin{pmatrix}2\\-4\\8\end{pmatrix}$ and any scalar constant.

A1: Correct **equation** $\mathbf{r} = \begin{pmatrix}8\\2\\6\end{pmatrix} + \mu\begin{pmatrix}1\\-2\\4\end{pmatrix}$ or equivalent $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}8\\2\\6\end{pmatrix} + \mu\begin{pmatrix}1\\-2\\4\end{pmatrix}$ Note $l_2 = \begin{pmatrix}8\\2\\6\end{pmatrix} + \mu\begin{pmatrix}1\\-2\\4\end{pmatrix}$ is A0

(d)

M1: Attempts $\begin{pmatrix}3\\2\\9\end{pmatrix} - \begin{pmatrix}1\\-2\\4\end{pmatrix}$ or equivalent such as $\begin{pmatrix}4\\0\\13\end{pmatrix} - 2\times\begin{pmatrix}1\\-2\\4\end{pmatrix}$.

May be implied by two "correct" coordinates. Solutions from area formulae need to reach a similar point before the M mark is scored. Eg. finding $\lambda = -1$ and attempting to find the coordinates

A1: Correct coordinate $(2, 4, 5)$ but condone vector $\begin{pmatrix}2\\4\\5\end{pmatrix}$ or $2\mathbf{i}+4\mathbf{j}+5\mathbf{k}$ Multiple answers will lose this mark

(e)

M1: Attempts $\overrightarrow{PY}$ or $\overrightarrow{YP}$ where Y is a general point on "their" l_2 condoning slips.

Follow through on their answer for part (c) here

dM1: Dependent upon the previous M. It is a full method to find μ

For example via scalar product $\pm\begin{pmatrix}5+\mu\\-2\mu\\-3+4\mu\end{pmatrix}\bullet\begin{pmatrix}1\\-2\\4\end{pmatrix} = 0 \Rightarrow \mu = ...$ Condone $\pm\begin{pmatrix}5+\mu\\-2\mu\\-3+4\mu\end{pmatrix}\bullet\begin{pmatrix}1\mu\\-2\mu\\4\mu\end{pmatrix} = 0 \Rightarrow \mu = ...$

Again, if they have an incorrect equation for l_2 follow through on their answer to (c)

Alternatively minimises $\left|\overrightarrow{PY}\right|$ via differentiation

$$d^2 = (5+\mu)^2 + (-2\mu)^2 + (-3+4\mu)^2 \Rightarrow 2(5+\mu)+8\mu+8(-3+4\mu)=0 \Rightarrow \mu = ...$$

A1: Correct value of $\mu = \dfrac{1}{3}$ which may be left unsimplified

ddM1: Substitutes their value of μ into their equation for l_2.

If this is not explicitly shown imply by two correct values for their μ

A1: $\left(\frac{25}{3}, \frac{4}{3}, \frac{22}{3}\right)$ but condone $\begin{pmatrix} \frac{25}{3} \\ \frac{4}{3} \\ \frac{22}{3} \end{pmatrix}$ or $\frac{25}{3}\mathbf{i} + \frac{4}{3}\mathbf{j} + \frac{22}{3}\mathbf{k}$

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