Candidate surname   Other names

Pearson Edexcel
International
Advanced Level

Centre Number   Candidate Number

Tuesday 14 January 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WFM01/01**

Mathematics
**International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1**

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations.
Calculators must not have the facility for symbolic algebra manipulation,
differentiation and integration, or have retrievable mathematical
formulae stored in them.**

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear.
  Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question*.

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1.
$$A = \begin{pmatrix} p & -5 \\ -2 & p+3 \end{pmatrix}$$

(a) Determine the values of the constant p for which $\mathbf{A}$ is singular.

(3)

Given that $p = 3$

(b) determine $\mathbf{A}^{-1}$

(3)

Question 1 continued

Q1

(Total 6 marks)

2. Given that $x = -\dfrac{1}{3}$ is a root of the equation

$$3x^3 + kx^2 + 33x + 13 = 0 \qquad k \in \mathbb{R}$$

determine

(a) the value of k,

(2)

(b) the other 2 roots of the equation in the form $a + ib$, where a and b are real numbers.

(4)

Question 2 continued

(Total 6 marks)

Leave blank

3. (a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that for all positive integers n

$$\sum_{r=1}^{n} r^2 (2r + 3) = \frac{n}{2}(n + 1)(n^2 + 3n + 1)$$

(4)

(b) Hence calculate the value of $\sum_{r=10}^{25} r^2 (2r + 3)$

(2)

P 6 2 9 1 0 A 0 6 2 8

Question 3 continued

Q3

(Total 6 marks)

4. $$z_1 = p + 5i, \quad z_2 = 9 + 8i \quad \text{and} \quad z_3 = \frac{z_1}{z_2}$$

where p is a real constant.

(a) Determine z_3 in the form $x + iy$, where x and y are in terms of p

(3)

(b) Determine the exact value of the modulus of z_2

(1)

Given that the argument of z_1 is $\dfrac{\pi}{3}$

(c) (i) determine the exact value of p

(ii) determine the exact value of the modulus of z_3

(3)

Question 4 continued

(Total 7 marks)

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5.
$$f(x) = x^4 - 12x^{\frac{3}{2}} + 7 \qquad x \geqslant 0$$

(a) Show that the equation $f(x) = 0$ has a root, α, in the interval $[2, 3]$.

(2)

(b) Taking 2.5 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 2 decimal places.

(4)

(c) Show that your answer to (b) gives α correct to 2 decimal places.

(2)

Question 5 continued

Q5

(Total 8 marks)

6.
$$A = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$$

The transformation represented by **A** maps the point $R(3p - 13, p - 4)$, where p is a constant, onto the point $R'(7, -2)$

(a) Determine the value of p

(3)

The point S has coordinates $(0, 7)$

Given that O is the origin,

(b) determine the area of triangle ORS

(2)

The transformation represented by **A** maps the triangle ORS onto the triangle $OR'S'$

(c) Hence, using your answer to part (b), determine the area of triangle $OR'S'$

(2)

Question 6 continued

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Question 6 continued

Question 6 continued

(Total 7 marks)

Q6

7. The equation $3x^2 + px - 5 = 0$, where p is a constant, has roots α and β.

(a) Determine the value of

(i) $\alpha\beta$

(ii) $\left(\alpha + \dfrac{1}{\beta} \right)\left(\beta + \dfrac{1}{\alpha} \right)$

(3)

(b) Obtain an expression, in terms of p, for

(i) $\alpha + \beta$

(ii) $\left(\alpha + \dfrac{1}{\beta} \right) + \left(\beta + \dfrac{1}{\alpha} \right)$

(3)

Given that

$$\left(\alpha + \frac{1}{\beta} \right) + \left(\beta + \frac{1}{\alpha} \right) = 2\left(\alpha + \frac{1}{\beta} \right)\left(\beta + \frac{1}{\alpha} \right)$$

(c) determine the value of p.

(1)

(d) Using the value of p found in part (c), obtain a quadratic equation, with integer coefficients, that has roots $\left(\alpha + \dfrac{1}{\beta} \right)$ and $\left(\beta + \dfrac{1}{\alpha} \right)$

(2)

Question 7 continued

P 6 2 9 1 0 A 0 1 7 2 8

Question 7 continued

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Question 7 continued

P 6 2 9 1 0 A 0 1 9 2 8

8. A rectangular hyperbola, H, has Cartesian equation $xy = 16$

   The point $P\left(4t, \dfrac{4}{t}\right)$, $t \neq 0$, lies on H.

   (a) Use calculus to show that an equation of the normal to H at P is

   $$ty - t^3x = 4 - 4t^4$$

   (5)

   The point A on H has parameter $t = 2$

   The normal to H at A meets H again at the point B.

   (b) Determine the exact value of the length of AB.

   (6)

   The tangent to H at A meets the y-axis at the point C.

   (c) Determine the exact area of triangle ABC.
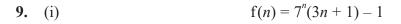
   (3)

Question 8 continued

P 6 2 9 1 0 A 0 2 1 2 8

Question 8 continued

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Question 8 continued

Q8

(Total 14 marks)

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9. (i)
$$f(n) = 7^n(3n + 1) - 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$, $f(n)$ is a multiple of 9

(6)

(ii) A sequence of numbers is defined by

$$u_1 = 2 \qquad u_2 = 6$$

$$u_{n+2} = 3u_{n+1} - 2u_n \qquad n \in \mathbb{Z}^+$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 2(2^n - 1)$$

(6)

Question 9 continued

P 6 2 9 1 0 A 0 2 5 2 8

Question 9 continued

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Question 9 continued

P 6 2 9 1 0 A 0 2 7 2 8

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Question 9 continued

Q9

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END