Pearson Edexcel

Examiners' Report
Principal Examiner Feedback

January2020

Pearson Edexcel International GCE
In Further Pure Mathematics F1
(WFM01) Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Grade Boundaries

Grade boundaries for all papers can be found on the website at:
https://qualifications.pearson.com/en/support/support-topics/results-certification/grade-boundaries.html

January 2020
Publications Code WFM1_01_2001_ER*

General

This paper was accessible, and most students found plenty of opportunity to demonstrate their knowledge and understanding in this paper. There were some testing questions involving complex numbers, numerical methods, matrices, coordinate geometry and mathematical induction that allowed discrimination between the higher grades.

In summary, Q1, Q2, Q3, Q4(a), Q4(b), Q5(a), Q5(b), Q6(a), Q7(a), Q7(b) and Q8(a) were a good source of marks for the average student, mainly testing standard ideas and techniques, and Q4(c), Q5(c), Q6(b), Q6(c), Q7(c), Q7(d), Q8(b) and Q8(c) were discriminating at the higher grades. Q9 proved to be the most challenging question on the paper.

Report on Individual Questions

Question 1

Q1 proved accessible for most students with many fully correct solutions seen. A few students, who presumably did not understand the concept of a singular matrix, made no attempt at answering Q1(a).

In Q1(a), most students set their determinant of $\mathbf{A}$ equal to zero and successfully solved the resulting quadratic equation to find two values for p. Common mistakes included making sign errors when finding $\det(\mathbf{A})$; not setting their $\det(\mathbf{A})$ equal to zero; or making manipulation errors when solving the resulting quadratic equation.

In Q1(b), most students used $p = 3$ to find a correct $\mathbf{A}^{-1}$ with many giving their answer as $\frac{1}{8}\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$ and some writing their answer as $\begin{pmatrix} \frac{3}{4} & \frac{5}{8} \\ \frac{1}{4} & \frac{3}{8} \end{pmatrix}$. Some students applied an incorrect method of $\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})}\mathbf{A}$ or evaluated $\det(\mathbf{A})$ incorrectly as $18 + 10$. A few students did not use the given $p = 3$ and so expressed $\mathbf{A}^{-1}$ completely in terms of p, while others only applied $p = 3$ to their determinant and so wrote $\mathbf{A}^{-1} = \frac{1}{8}\begin{pmatrix} p+3 & 5 \\ 2 & p \end{pmatrix}$ as their final answer.

Question 2

Q2 proved generally accessible with most students scoring full marks. Those students who failed to obtain a correct $k = -17$ in Q2(a) tended to make little creditable progress in Q2(b).

In Q2(a), most students found a correct $k = -17$ by solving the linear equation that resulted from substituting $x = -\frac{1}{3}$ into the given cubic equation. Some students used a method of long division of the cubic expression by either $x + \frac{1}{3}$ or $3x + 1$. Those opting for this method were slightly more prone to error. It was rare to see incorrect divisors such as $x - \frac{1}{3}$ or $3x - 1$, but errors in the long division process were occasionally seen. Those students who used that fact that the constant term of the quadratic quotient had to be 13 when dividing the cubic expression by $3x + 1$ usually reached a correct equation $(k-1)x^2 - (-18x^2) = 0x^2$. Only a few students found $k = -17$ by using long division and setting an expression for the remainder (in terms of k) equal to zero. Most of these students found an incorrect remainder by making algebraic or manipulation errors on account of the complicated nature of the algebra that was involved. Other students used $x = -\frac{1}{3}$ to write a cubic expression such as $\left(x + \frac{1}{3}\right)(ax^2 + bx + c)$ or $(3x+1)(ax^2 + bx + c)$ which they equated to $3x^3 + kx^2 + 33x + 13$. Most of these students deduced the values of a and c by inspection and proceeded to consider both the x and x^2 terms in order to reach a value for k.

In Q2(b), most students used long division, while others compared coefficients or used direct factorisation by inspection to find the correct quadratic factor. Students either completed the square, used the quadratic formula or applied a direct calculator method to solve the quadratic factor equal to zero. Most students found the correct complex roots, although a few gave their answer in the wrong form as $\frac{6 \pm 4\mathrm{i}}{2}$, or processed this incorrectly to give either $3 \pm 4\mathrm{i}$ or $6 \pm 2\mathrm{i}$.

Question 3

Q3 proved accessible with most students scoring full marks.

In Q3(a), most students expanded $r^2(2r+3)$ and correctly substituted the standard formulae for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r^2$ into $\sum_{r=1}^{n}(2r^3+3r^2)$. Students who directly factorised out $\frac{1}{2}n(n+1)$ were generally more successful in obtaining the given answer. There were varying degrees of justification given from students obtaining $\frac{1}{2}n(n^3+4n^2+4n+1)$, with some going directly from $\frac{1}{2}n(n^3+4n^2+4n+1)$ to the given answer and others using algebraic long division to factorise out $(n+1)$.

In Q3(b), most students calculated $\sum_{r=10}^{25} r^2(2r+3)$ by applying $\mathrm{f}(25)-\mathrm{f}(9)$, where $\mathrm{f}(n)=\frac{1}{2}n(n+1)(n^2+3n+1)$, and many obtained the correct answer 222920. Some students used incorrect methods such as calculating $\mathrm{f}(25)-\mathrm{f}(10)$, $\mathrm{f}(25)+\mathrm{f}(9)$ or $\mathrm{f}(25)+\mathrm{f}(10)$. A few students answered Q2(b) by calculating only $\mathrm{f}(25)$ and some incorrectly applied $\mathrm{f}(9)$ as $\frac{1}{2}(9)(10)(100+30+1)$.

Question 4

Q4(a), Q4(b) and Q4(c)(i) proved accessible to students of all abilities. In Q4(c)(ii), a significant minority of students made either incomplete attempts or incorrect attempts at evaluating the exact value for the modulus of z_3.

In Q4(a), most students multiplied both the numerator and denominator of $z_3=\frac{p+5\mathrm{i}}{9+8\mathrm{i}}$ by $9-8\mathrm{i}$ and applied $\mathrm{i}^2=-1$ to give a rational fraction with a correct numerator and denominator. A few students incorrectly found the denominator as either '$81-64$' or '$81+8$'. Some students gave their final answer in the form $\frac{a+\mathrm{i}b}{c}$ rather than in the form $x+\mathrm{i}y$, as required by the question.

In Q4(b), nearly all students correctly found the modulus of z_2 as an exact $\sqrt{145}$ with some also writing both $\sqrt{145}$ and a decimal approximation such as 12.04.

In Q4(c)(i), many students used trigonometry to find a correct exact value for p. Some students, who usually did not sketch an Argand diagram to represent $\arg(p+5\mathrm{i})=\frac{\pi}{3}$, wrote down incorrect equations such as $\tan\left(\frac{\pi}{3}\right)=\frac{p}{5}$ or $\tan\left(\frac{5}{p}\right)=\frac{\pi}{3}$.

In Q4(c)(ii), those students who applied $|z_3|=\frac{|z_1|}{|z_2|}$ were more successful in finding a correct exact value for the modulus of z_3. Those students who substituted their exact value for p into their answer to Q4(a) struggled to achieve a correct $z_3=\left(\frac{8+3\sqrt{3}}{29}\right)+\left(\frac{27-8\sqrt{3}}{87}\right)\mathrm{i}$, although many applied a correct Pythagoras method to find $|z_3|$. A few students made no further progress after expressing z_3 in the form $a+\mathrm{i}b$. Some students found a final inexact answer of 0.479 without making any reference to an allowable exact value for $|z_3|$.

Question 5

Q5 proved accessible, with many students scoring full marks, although some struggled to make any creditable progress in Q5(c). In Q5(a) and Q5(c), the omission of ‘the curve is continuous’ as part of their reason was condoned on this occasion.

In Q5(a), almost all students correctly evaluated $\mathrm{f}(2)$ and $\mathrm{f}(3)$ with many indicating a change of sign and giving an acceptable conclusion. A few students did not give any conclusion while others did not give an acceptable reason such as e.g. ‘change of sign’, ‘< 0 and > 0’ or ‘$\mathrm{f}(2)\mathrm{f}(3)<0$’.

In Q5(b), most students differentiated $\mathrm{f}(x)$ correctly and applied the Newton-Raphson procedure correctly to give a second approximation for α as 2.54. A few students incorrectly differentiated $x^4-12x^{\frac{3}{2}}+7$ to give either $4x^3-18x^{\frac{5}{2}}$ or $4x^3-18x^{\frac{1}{2}}+7$, while others applied the incorrect Newton-Raphson formula of $\alpha_2\simeq 2.5+\frac{\mathrm{f}(2.5)}{\mathrm{f}'(2.5)}$. In some cases a lack of working meant that it was difficult for examiners to determine whether the Newton-Raphson procedure was applied correctly. For example, a few students stated the correct Newton-Raphson formula followed by their final answer while others just stated their final answer with no working.

In Q5(c), many students used a method of evaluating both $\mathrm{f}(2.535)$ and $\mathrm{f}(2.545)$ with many indicating a change of sign. Some students did not give a conclusion while others gave an incorrect conclusion of ‘root lies between 2.535 and 2.545’ without reference to ‘$\alpha=2.54$ is correct to 2 decimal places’. Incorrect methods included evaluating both $\mathrm{f}(2.53)$ and $\mathrm{f}(2.55)$; evaluating both $\mathrm{f}(2)$ and $\mathrm{f}(2.54)$; or just evaluating $\mathrm{f}(2.54)$. A few students correctly employed the alternative method of applying the Newton-Raphson procedure for a second time on 2.54.

Question 6

Q6 discriminated well between students of all abilities with some lower ability students struggling to make any creditable progress. Q6(a) and Q6(c) were more accessible to students than Q6(b).

In Q6(a), most students solved the matrix equation $\mathbf{AR} = \mathbf{R}'$ and some solved the matrix equation $\mathbf{R} = \mathbf{A}^{-1}\mathbf{R}'$. Many students who used these two methods achieved a correct answer $p = 5$. Some students multiplied out matrices in the wrong order and so attempted to solve incorrect equations such as $\begin{pmatrix} 3p-13 & p-4 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix} = \begin{pmatrix} 7 & -2 \end{pmatrix}$ or $\begin{pmatrix} 7 & -2 \end{pmatrix}\begin{pmatrix} \frac{4}{11} & \frac{3}{11} \\ \frac{1}{11} & -\frac{2}{11} \end{pmatrix} = \begin{pmatrix} 3p-13 & p-4 \end{pmatrix}$.

In Q6(b), most students used $p = 5$ to deduce a correct $R(2, 1)$. Many students attempted to plot the points O, R and $S(0, 7)$ on a set of coordinate axes. A correct $\text{area}(ORS) = 7$ was found by either applying $\frac{1}{2}(\text{base})(\text{height})$ to give $\frac{1}{2}(7)(2)$ or by using the shoelace method to give $\frac{1}{2}\begin{vmatrix} 0 & 2 & 0 & 0 \\ 0 & 1 & 7 & 0 \end{vmatrix} = \frac{1}{2}(14)$. Some students made the error of finding $\text{area}(ORS)$ as $\frac{1}{2}(7)(1)$. This error either arose from plotting $S(0, 7)$ incorrectly as $(7, 0)$ or by using $y_R = 1$ (instead of $x_R = 2$) as the height of triangle ORS.

In Q6(c), many students applied a correct '$\text{area}(OR'S') = |\det(\mathbf{A})| \times (\text{area}(ORS))$' to give an area for triangle $OR'S'$ which was 11 times their answer to part Q6(b). Some students either incorrectly evaluated $|\det(\mathbf{A})|$ as $|2(-4)+3(1)| = 5$ or applied an incorrect formula '$\text{area}(OR'S') = \dfrac{1}{|\det(\mathbf{A})|} \times (\text{area}(ORS))$'. A few students ignored the instruction "Hence, using your answer to part (b),". These students used the points O, $R'(7, -2)$ and $S'(21, -28)$ (and not their answer to part (b)) in a direct method to find the area of triangle $OR'S'$ and so received no credit, even when they found the correct answer of 77.

Question 7

Q7 proved very accessible with many students scoring at least 7 of the 9 marks available.

Most students wrote down a correct value for $\alpha\beta$ in Q7(a)(i) and then a correct expression, in terms of p, for $\alpha+\beta$ in Q7(b)(i).

Most students found the correct answers in Q7(a)(ii) and Q7(b)(ii) for the product and sum of $\left(\alpha+\frac{1}{\beta}\right)$ and $\left(\beta+\frac{1}{\alpha}\right)$. In Q7(b)(ii), only a few students did not simplify the sum expression of $\alpha+\beta+\frac{1}{\alpha}+\frac{1}{\beta}$ to become $\alpha+\beta+\frac{\alpha+\beta}{\alpha\beta}$. While many students gave a correct answer of $-\frac{2p}{15}$ in Q7(b)(ii), some gave their final answer as $-\frac{p}{3}+\frac{p}{5}$ which was condoned.

In Q7(c), most students who answered Q7(a) and Q7(b) correctly used the given equation to find the correct $p=4$, although a few obtained an incorrect $p=1$ as a consequence of manipulation or substitution errors.

In Q7(d), many students found a correct sum and product of $\left(\alpha+\frac{1}{\beta}\right)$ and $\left(\beta+\frac{1}{\alpha}\right)$ as $-\frac{8}{15}$ and $-\frac{4}{15}$ respectively. This was usually followed by a correct method to form the quadratic equation as described in the question. Some students, when forming their quadratic equation, used an incorrect sum of $-\frac{4}{3}$ or an incorrect product of $-\frac{5}{3}$. Other errors in attempting to establish the required quadratic equation included applying the incorrect method of x^2 + (sum)x + (product)$=0$; the omission of '$=0$'; and the failure to give integer coefficients.

Question 8

Q8 was accessible to most with Q8(c) providing good discrimination for the higher ability students. It was quite common to see fully correct solutions.

In Q8(a), almost all students used calculus, and most chose to make y the subject of $xy = 16$ before proceeding to differentiate explicitly. Implicit and parametric differentiation were also seen. A very small number who chose the implicit approach differentiated 16 incorrectly to give 16. The correct expression in terms t for the gradient of the tangent at point P was invariably achieved. Nearly all students deployed a correct perpendicular gradient rule to find the correct gradient of the normal. A correct straight method almost always followed which was mostly achieved via $y - y_1 = m_N(x - x_1)$ although the slightly more onerous $y = m_N x + c$ approach was occasionally seen. Only a very small number then failed to achieve the given answer without error.

In Q8(b), most students realised that the specific normal equation required was easily obtained by substituting $t = 2$ into the given answer to Q8(a), although a few slips were made. Some students needlessly repeated work from Q8(a) but errors were not common. The most popular approach to find the coordinates of B was to substitute the normal as $y = 4x - 30$ into $xy = 16$ to obtain a quadratic equation in x which was usually solved correctly. A small number obtained a quadratic equation in y but very few chose to produce a quadratic equation in t. A commonly occurring slip after achieving a correct $x_B = -\frac{1}{2}$ was to confuse this with t, leading to the incorrect $B(-2, -8)$. Most students applied Pythagoras correctly to $A(8, 2)$ and $B(-0.5, -32)$ to give an acceptable exact length for AB. There were a few cases of a decimal answer being given as a final answer with no sight of an acceptable exact length for AB.

Some students struggled to make any creditable progress in Q8(c). A common error was to use the equation of the normal again and set $x = 0$ in this equation to find C – perhaps as a result of the student not reading the question with enough care. Those who found the equation of the tangent at the correct $A(8, 2)$ almost always found the correct coordinates for C. Students who produced a reasonable sketch of the situation usually found that it helped them with devising a correct strategy to find the area of triangle ABC. Those who identified the right-angle between the tangent and the normal tended to find the length AC by using Pythagoras and then used their answer to Q8(b) to find the area of triangle ABC more efficiently. Other methods were also available. Finding the y-axis intercept of the normal to split triangle ABC into two triangles was common. The shoelace method was also seen although occasionally the multiplier of $\frac{1}{2}$ was absent. Some found the area of the rectangle that surrounds triangle ABC and subtracted off the areas of three right-angled triangles.

Question 9

Q9 discriminated well between students of all abilities with Q9(ii) more successfully answered than Q9(i).

In Q9(i), many students successfully showed that $\mathrm{f}(n) = 7^n(3n+1)-1$ is multiple of 9 for $n=1$. There were varying approaches to induction with many students gaining some credit for writing down a correct $\mathrm{f}(k+1) = 7^{k+1}(3(k+1)+1)-1$ as part of their working. Those students who worked directly from $\mathrm{f}(k+1)$ found it challenging to manipulate their $\mathrm{f}(k+1)$ into an expression where each term is an obvious multiple of 9. Many obtained the result $\mathrm{f}(k+1) = 7\mathrm{f}(k) + 6 + 21(7^k)$ but were unable to proceed further. There was a small number of students who, having obtained this result, went on to prove (by induction) that $\mathrm{g}(k) = 6+21(7^k)$ is in fact a multiple of 9. Some higher ability students used intricate algebra to manipulate $\mathrm{f}(k+1)$ to give $\mathrm{f}(k+1) = 18k(7^k) + 27(7^k) + 7^k(3k+1)-1$, where each term in $\mathrm{f}(k+1)$ is an obvious multiple of 9. A greater proportion of successful responses were seen from students who worked directly from $\mathrm{f}(k+1)-\mathrm{f}(k)$. Some of these students used correct algebra to achieve $\mathrm{f}(k+1) - \mathrm{f}(k) = 9(2k+3)(7^k)$, although not all went on to write an expression such as $\mathrm{f}(k+1) = 9(2k+3)(7^k) + \mathrm{f}(k)$, where each term in $\mathrm{f}(k+1)$ is an obvious multiple of 9. There were other valid methods that students employed with varying degrees of success, such as attempts to find $\mathrm{f}(k+1) - m\mathrm{f}(k)$ with a suitable value for m. The most sucessful of these attempts achieved a correct $\mathrm{f}(k+1) = 27 - 9k(7^{k+1}) + 28\mathrm{f}(k)$, where each term in $\mathrm{f}(k+1)$ is an obvious multiple of 9. Some students did not bring all strands of their proof together to give a fully correct proof. A minimal acceptable proof, following on from completely correct work, would incorporate the following: assuming the general result is true for $n=k$; then showing the general result is true for $n=k+1$; showing the general result is true for $n=1$; and finally concluding that the general result is true for all positive integers.

In Q9(ii), some students failed to demonstrate that the general result was true for both $n=1$ and $n=2$. Many students substituted $u_{k+1} = 2(2^{k+1}-1)$ and $u_k = 2(2^k-1)$ into $u_{k+2} = 3u_{k+1} - 2u_k$ and manipulated their expression to give the result $u_{k+2} = 2(2^{k+2}-1)$, although some students fudged this correct result from incorrect intermediate work. Some students did not bring all strands of their proof together to give a fully correct proof. In part (ii), a minimal acceptable proof, following on from completely correct work, would incorporate the following: assuming the general result is true for $n=k$ and $n=k+1$; then showing the general result is true for $n=k+2$; showing the general result is true for $n=1$ and $n=2$; and finally concluding that the general result is true for all positive integers.

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom