

Mark Scheme (Final)

October 2019

Pearson Edexcel IAL Mathematics

Mechanics 2 (WME02/01)



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October 2019 Mechanics 2 - WME02 Mark Scheme - Final

$3m\overline{x}$ or $3\overline{x}$ is a method error $(k+4)m\overline{x} = 2m + 12m + km$ A1Or equivalentMoments about x-axisM1Or a parallel axis. Require all terms and dimensionally correct. Condone sign error $3m\overline{y}$ or $3\overline{y}$ is a method error $(k+4)m\overline{y} = m + 9m + 5km$ A1Or equivalentUse of $\overline{y} = 2\overline{x}$ to form equation in kM1Allow the first 4 marks for a combined equation in vector formatUse of $\overline{y} = 2\overline{x}$ to form equation in kM1Allow the first 5 marks if they go straight an equation equating the right hand sides e.g. $m + 9m + 5km = 2(2m + 12m + km)$ wi incorrect statement about the total mass st $10 + 5k = 2(14 + k) \Rightarrow k = 6$ A1With no errors seen(6)Image: the parallel of the parallel	Q	Scheme	Marks	Notes
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Moments about x-axisM1dimensionally correct. Condone sign error $3m\overline{y}$ or $3\overline{y}$ is a method error $(k+4)m\overline{y} = m+9m+5km$ A1Or equivalentUse of $\overline{y} = 2\overline{x}$ to form equation in kM1Allow the first 4 marks for a combined equation in vector formatUse of $\overline{y} = 2\overline{x}$ to form equation in kM1Allow the first 5 marks if they go straight an equation equating the right hand sides e.g. $m+9m+5km = 2(2m+12m+km)$ wi incorrect statement about the total mass set $10+5k = 2(14+k) \Rightarrow k = 6$ A1With no errors seen(6)Image: Second		$(k+4)m\overline{x} = 2m + 12m + km$	A1	Or equivalent
Allow the first 4 marks for a combined equation in vector formatUse of $\overline{y} = 2\overline{x}$ to form equation in kM1Allow the first 5 marks if they go straight an equation equating the right hand sides e.g. $m + 9m + 5km = 2(2m + 12m + km)$ wi incorrect statement about the total mass sc $10 + 5k = 2(14 + k) \Rightarrow k = 6$ A1With no errors scen(6)[6][6][6][6][6][6][7]M1Equation of motion.M1F-30U - 750g sin $\theta = 0$ A1Correct unsimplified equation in F or theil $30U^2 + 490U - 16000 = 0$ U = 16.3 or U = 16 onlyA1Max 3 sf		Moments about <i>x</i> -axis	M1	dimensionally correct. Condone sign errors
Use of $\overline{y} = 2\overline{x}$ to form equation in kM1Use of $\overline{y} = 2\overline{x}$ to form equation in kM1Allow the first 5 marks if they go straight an equation equating the right hand sides e.g. $m + 9m + 5km = 2(2m + 12m + km)$ wi incorrect statement about the total mass so $10 + 5k = 2(14 + k) \Rightarrow k = 6$ A1With no errors seen(6)Image: the second secon		$(k+4)m\overline{y} = m+9m+5km$	A1	Or equivalent
Allow the first 5 marks if they go straight an equation equating the right hand sides e.g. $m + 9m + 5km = 2(2m + 12m + km)$ wi incorrect statement about the total mass sec $10 + 5k = 2(14 + k) \Rightarrow k = 6$ A1With no errors seen(6)[6][6][6][6][6][7]M1 $\frac{16}{U}$ scores M0Equation of motion.M1 $\frac{16}{U}$ scores M0F - 30U - 750g sin $\theta = 0$ A1Correct unsimplified equation in F or their $\Rightarrow \frac{16000}{U} - 30U - 490 = 0$ $30U^2 + 490U - 16000 = 0$ DM1Form and solve a quadratic equation in U Horizontal form seen or implied. Dependent on the two preceding M marks. Need to see working if solving an incorrect quadratic or obtain an incorrect answer.U = 16.3 or U = 16 onlyA1Max 3 sf				
an equation equating the right hand sides e.g. $m + 9m + 5km = 2(2m + 12m + km)$ wi incorrect statement about the total mass set $10+5k = 2(14+k) \Rightarrow k = 6$ A1With no errors seen(6)[6][6][6][6][7]M1 $10 + 5k = 2(14+k) \Rightarrow k = 6$ M1[6][6][7][6][8][6][9]M1 $10 + 5k = 2(14+k) \Rightarrow k = 6$ M1[10][6][9][10][10][10][10][11][11][12][12]Use of $F = \frac{16000}{U}$ [12][13][13][14][14][14][15][15][15][15]		Use of $\overline{y} = 2\overline{x}$ to form equation in k	M1	
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2Use of $F = \frac{16000}{U}$ M1 $\frac{16}{U}$ scores M02Use of $F = \frac{16000}{U}$ M1 $\frac{16}{U}$ scores M0Equation of motion.M1All terms required. Dimensionally correc Condone sign errors and sin/cos confusion $F - 30U - 750g \sin \theta = 0$ A1Correct unsimplified equation in F or their Form and solve a quadratic equation in U Horizontal form seen or implied. Dependent on the two preceding M marks Need to see working if solving an incorrect quadratic or obtain an incorrect answer. $U = 16.3$ or $U = 16$ onlyA1Max 3 sf				incorrect statement about the total mass seen
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Equation of motion.M1All terms required. Dimensionally correc Condone sign errors and sin/cos confusion $F - 30U - 750g \sin \theta = 0$ A1Correct unsimplified equation in F or their Form and solve a quadratic equation in U Horizontal form seen or implied. Dependent on the two preceding M marks Need to see working if solving an incorrect quadratic or obtain an incorrect answer. $U = 16.3$ or $U = 16$ onlyA1Max 3 sf				
Equation of motion.M1All terms required. Dimensionally correc Condone sign errors and sin/cos confusion $F - 30U - 750g \sin \theta = 0$ A1Correct unsimplified equation in F or their Form and solve a quadratic equation in U Horizontal form seen or implied. Dependent on the two preceding M marks Need to see working if solving an incorrect quadratic or obtain an incorrect answer. $U = 16.3$ or $U = 16$ onlyA1Max 3 sf	2	Use of $F = \frac{16000}{U}$	M1	$\frac{16}{U}$ scores M0
$\Rightarrow \frac{16000}{U} - 30U - 490 = 0$ $30U^{2} + 490U - 16000 = 0$ $U = 16.3 \text{ or } U = 16 \text{ only}$ $M = 16.3 \text{ on } U = 16$			M1	All terms required. Dimensionally correct. Condone sign errors and sin/cos confusion.
$\Rightarrow \frac{16000}{U} - 30U - 490 = 0$ $30U^{2} + 490U - 16000 = 0$ $U = 16.3 \text{ or } U = 16 \text{ only}$ $U = 16.3 \text{ or } U = 16 \text{ only}$ $U = 16.3 \text{ or } U = 16 \text{ only}$ $M = 16.3 \text{ on } U = 16 \text{ only}$ $M = 16.3 \text{ on } U = 16$		$F - 30U - 750g\sin\theta = 0$	A1	Correct unsimplified equation in F or their F
(5)		$30U^2 + 490U - 16000 = 0$		Horizontal form seen or implied. Dependent on the two preceding M marks Need to see working if solving an incorrect quadratic or obtain an incorrect answer.
		U = 16.3 or $U = 16$ only		Max 3 sf
			- 121	

Q	Scheme	Marks	Notes
3 a	Use of $\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} \left(\mathbf{a} = 2\mathbf{i} + \frac{9}{2}t^{-\frac{1}{2}}\mathbf{j}\right)$	M1	Powers going down by 1 Must be in vector form.
	Use of $\mathbf{F} = 2\mathbf{a}$ and $t = 4$	M1	For their a Must be working in vector form.
	$\mathbf{F} = 4\mathbf{i} + 4.5\mathbf{j}$	A1	Any equivalent simplified form ISW if go on to find magnitude
		(3)	
3b	Use of $\mathbf{r} = \int \mathbf{v} dt$	M1	Powers going up by 1 Must be in vector form.
	$\mathbf{r} = \left(t^2 + 10t\left(+a\right)\right)\mathbf{i} + \left(6t^{\frac{3}{2}}\left(+b\right)\right)\mathbf{j}$	A1	Condone missing constant of integration
	$\mathbf{r} = 56\mathbf{i} + 28\mathbf{j}$	A1	ISW if go on to find magnitude
		(3)	
3c	Equate components of v $\left(2t+10=9t^{\frac{1}{2}}\right)$	M1	
	Form quadratic in $t^{\frac{1}{2}}$ and solve for t $(2\alpha^2 - 9\alpha + 10 = 0)$	M1	Alternatively could square both sides to obtain $4t^2 - 41t + 100 = 0$ Need to see working if solving an incorrect quadratic equation or obtain incorrect answers.
	$((2\alpha - 5)(\alpha - 2) = 0)$ $t = 4$, $t = 6.25$ (6.3)	A1	
		(3)	
		[9]	

Q	Scheme	Marks	Notes
4a	$\begin{array}{c} & & & & \\ & & & & \\ & & & & \\ & & & & $		
	Take moments about A	M1	Or a complete alternative method to form an equation in <i>N</i> . Dimensionally correct. Condone sin/cos confusion.
	$3N = 2\cos\theta \times 160$	A1	Correct unsimplified equation
	$3N = 2 \times \frac{3}{4} \times 160 = 240$, $N = 80$ (newtons) (Given answer)	A1	Deduce given answer correctly. Must see evidence of fully substituted equation
		(3)	
4b			Two independent equations needed. Mark in the order seen.
	Resolve horizontally	M1	Form one equation in F and/or R
	$F = N\sin\theta \left(= 80 \times \frac{\sqrt{7}}{4} = 52.9 \right)$	A1	Correct unsimplified equation
	Resolve vertically	M1	Form a second equation in F and/or R
	$R + N\cos\theta = 160$ ($R = 160 - 60 = 100$)	A1	Correct unsimplified equation
	Use $F = \mu R$ to obtain μ	DM1	Dependent on two previous M marks
	$\mu = \frac{20\sqrt{7}}{100} = \frac{\sqrt{7}}{5}$	A1	0.53 or better (0.52915)
		(6)	
ļ	Alternative equations:		
	Resolving parallel to the rod: $R\sin\theta + F\cos\theta = 160\sin\theta$		
	Resolving perpendicular to the rod: $N + R\cos\theta = F\sin\theta + 160\cos\theta$		
	$M(C): 3R\cos\theta = 3F\sin\theta + 160\cos\theta$		
	M(midpt): $2R\cos\theta = 2F\sin\theta + N$		
		[9]	
<u> </u>			

Q	Scheme	Marks	Notes
5a	Mass ratioFrom BC Large Δ 60 (4)4aSmall Δ 15 (1)8arectangle15 (1)4.5aT30 (2) \overline{y}	B1 B1	Mass ratios correct Vertical distances correct
	Moments about BC	M1	Or a parallel axis. Signs correct for their split.
	$2\overline{y} = 16a - 8a - 4.5a = \frac{7}{2}a$	A1	Correct unsimplified equation
	$2\overline{y} = 16a - 8a - 4.5a = \frac{7}{2}a$ $\overline{y} = \frac{7}{4}a$	A1	Given Answer Need to see a linear equation in \overline{y}
		(5)	
5b			
	$\overline{x} = 5a$ from <i>B</i>	B1	Or equivalent. Seen or implied Might be seen in (a) but needs to be used here.
	$\alpha = \tan^{-1} \left(\frac{\frac{5}{2}a}{6a - \overline{y}} \right)$	M1	Trig ratio of a relevant angle in a triangle involving the c of m
	$\theta = \alpha + \tan^{-1} \frac{5}{12}$	M1	Correct method to find the required angle e.g $\theta = 180^{\circ} - \tan^{-1}\frac{6}{2.5} - \tan^{-1}\frac{6a - \overline{y}}{2.5a}$
	$=\tan^{-1}\frac{10}{17}+\tan^{-1}\frac{5}{12}$	A1	Correct unsimplified expression for θ seen
			$\tan^{-1}\frac{10}{17} = 30.46, \tan^{-1}\frac{17}{10} = 59.5,$ $\tan^{-1}\frac{5}{12} = 22.6, \tan^{-1}\frac{12}{5} = 67.38$
	$\theta = 53$	A1	Q asks for a whole number
		(5)	
			See over for alternatives

1 $x = 5a$ from B B1Might be seen in (a) but needs to be used h e.g.:Correct method to find all 3 sides of a triangle containing θ (or two sides of a right angled triangle containing θ)M1 $BO^2 = (5a)^2 + (\frac{7a}{4})^2 = \frac{449}{16}a^2, AB^2 = \frac{169}{4}$ $AO^2 = (\frac{5a}{2})^2 + (6a - \frac{7a}{4})^2 = \frac{389}{16}a^2$ $AO^2 = (\frac{5a}{2})^2 + (6a - \frac{7a}{4})^2 = \frac{389}{16}a^2$ Correct method to find the required angleM1e.g. $\cos \theta = \frac{AB^2 + AO^2 - OB^2}{2AB.AO}$ Correct unsimplified expression in θ A1($\cos \theta = 0.60062$) $\theta = 53$ A1Q asks for a whole number555If O is c of m, $\overline{AB} = \begin{pmatrix} -2.5a \\ -6a \end{pmatrix}$, $\overline{AO} = \begin{pmatrix} 2.5a \\ -4.25a \end{pmatrix}$ M1Use of scalar product to find $\cos \theta$ M1Use of scalar product to find $\cos \theta$ M1	to be used here.				5balt
Correct method to find all 3 sides of a triangle containing θ (or two sides of a right angled triangle containing θ)e.g.: $BO^2 = (5a)^2 + \left(\frac{7a}{4}\right)^2 = \frac{449}{16}a^2, AB^2 = \frac{169}{4}$ $AO^2 = \left(\frac{5a}{2}\right)^2 + \left(6a - \frac{7a}{4}\right)^2 = \frac{389}{16}a^2$ Correct method to find the required angleM1e.g. $\cos \theta = \frac{AB^2 + AO^2 - OB^2}{2ABAO}$ Correct unsimplified expression in θ A1($\cos \theta = 0.60062$) $\theta = 53$ A1Q asks for a whole number 5balt $\overline{x} = 5a$ from BB1If O is c of m, $\overline{AB} = \begin{pmatrix} -2.5a \\ -6a \end{pmatrix}, \overline{AO} = \begin{pmatrix} 2.5a \\ -4.25a \end{pmatrix}$ M1Lise of scalar product to find $\cos \theta$ M1Use of scalar product to find $\cos \theta$ M1			B1	$\overline{x} = 5a \text{ from } B$	
Correct method to find all 3 sides of a triangle containing θ (or two sides of a right angled triangle containing θ)M1 $BO^2 = (5a)^2 + \left(\frac{7a}{4}\right)^2 = \frac{449}{16}a^2, AB^2 = \frac{169}{4}$ $AO^2 = \left(\frac{5a}{2}\right)^2 + \left(6a - \frac{7a}{4}\right)^2 = \frac{389}{16}a^2$ $AO^2 = \left(\frac{5a}{2}\right)^2 + \left(6a - \frac{7a}{4}\right)^2 = \frac{389}{16}a^2$ Correct method to find the required angleM1e.g. $\cos \theta = \frac{AB^2 + AO^2 - OB^2}{2AB.AO}$ Correct unsimplified expression in θ A1($\cos \theta = 0.60062$) $\theta = 53$ A1Q asks for a whole number 5balt555balt $\overline{x} = 5a$ from BB1If O is c of m, $\overline{AB} = \begin{pmatrix} -2.5a \\ -6a \end{pmatrix}, \overline{AO} = \begin{pmatrix} 2.5a \\ -4.25a \end{pmatrix}$ M1Use of scalar product to find $\cos \theta$ M1Use of scalar product to find $\cos \theta$ M1	. 169 .				1
$AO^{2} = \left(\frac{3a}{2}\right) + \left(6a - \frac{7a}{4}\right) = \frac{367}{16}a^{2}$ Correct method to find the required angle $M1 = e.g. \cos\theta = \frac{AB^{2} + AO^{2} - OB^{2}}{2AB.AO}$ Correct unsimplified expression in θ $A1 = \left(\cos\theta = 0.60062\right)$ $\theta = 53$ $A1 = Q \text{ asks for a whole number}$ (5) $Sbalt = \frac{x}{x} = 5a \text{ from } B$ $B1 = Or \text{ equivalent. Seen or implied}$ $Mi \text{ be seen in (a) but needs to be used h}$ $If O \text{ is c of m,}$ $\overline{AB} = \begin{pmatrix} -2.5a \\ -6a \end{pmatrix}, \overline{AO} = \begin{pmatrix} 2.5a \\ -4.25a \end{pmatrix}$ $M1 = Expressions \text{ for } \overline{AB} \text{ and } \overline{AO}$	$,AB^{2}=\frac{109}{4}a^{2}$		M1	triangle containing θ (or two sides of a right	
Correct unsimplified expression in θ A1($\cos \theta = 0.60062$) $\theta = 53$ A1Q asks for a whole number $\theta = 53$ A1Q asks for a whole number $Sbalt$ $\overline{x} = 5a$ from BB1Or equivalent. Seen or implied Might be seen in (a) but needs to be used h $If O is c of m,$ $\overline{AB} = \begin{pmatrix} -2.5a \\ -6a \end{pmatrix}, \overline{AO} = \begin{pmatrix} 2.5a \\ -4.25a \end{pmatrix}$ M1Expressions for \overline{AB} and \overline{AO} Use of scalar product to find $\cos \theta$ M1	$\frac{389}{16}a^2$			angled triangle containing θ)	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		e.g. $\cos\theta = \frac{AB^2 + AO^2 - OB^2}{2AB.AO}$	M1	Correct method to find the required angle	
Sbalt 2 $\overline{x} = 5a \text{ from } B$ (5)Sbalt 2 $\overline{x} = 5a \text{ from } B$ B1Or equivalent. Seen or implied Might be seen in (a) but needs to be used hIf O is c of m, $\overline{AB} = \begin{pmatrix} -2.5a \\ -6a \end{pmatrix}, \overline{AO} = \begin{pmatrix} 2.5a \\ -4.25a \end{pmatrix}$ M1Expressions for \overline{AB} and \overline{AO} Use of scalar product to find $\cos \theta$ M1Correct unsimplified expression for $\cos \theta$		$(\cos\theta = 0.60062)$	A1	Correct unsimplified expression in θ	
5balt 2 $\overline{x} = 5a$ from BB1Or equivalent. Seen or implied Might be seen in (a) but needs to be used h1If O is c of m, $\overline{AB} = \begin{pmatrix} -2.5a \\ -6a \end{pmatrix}, \overline{AO} = \begin{pmatrix} 2.5a \\ -4.25a \end{pmatrix}$ M1Expressions for \overline{AB} and \overline{AO} Use of scalar product to find $\cos \theta$ M1Correct unsimplified expression for $\cos \theta$		Q asks for a whole number	A1	$\theta = 53$	
2 $X = 5a$ from B B1Might be seen in (a) but needs to be used hIf O is c of m, $\overrightarrow{AB} = \begin{pmatrix} -2.5a \\ -6a \end{pmatrix}$, $\overrightarrow{AO} = \begin{pmatrix} 2.5a \\ -4.25a \end{pmatrix}$ M1Expressions for \overrightarrow{AB} and \overrightarrow{AO} Use of scalar product to find $\cos \theta$ M1OutputCorrect unsimplified expression for $\cos \theta$			(5)		
$\overrightarrow{AB} = \begin{pmatrix} -2.5a \\ -6a \end{pmatrix}, \overrightarrow{AO} = \begin{pmatrix} 2.5a \\ -4.25a \end{pmatrix}$ $M1$ $Use of scalar product to find \cos\theta M1 Correct unsimplified expression for \cos\theta$			B1	$\overline{x} = 5a$ from B	
$\overrightarrow{AB} = \begin{pmatrix} -2.5a \\ -6a \end{pmatrix}, \overrightarrow{AO} = \begin{pmatrix} 2.5a \\ -4.25a \end{pmatrix} \qquad M1$ Use of scalar product to find $\cos\theta$ M1 $(AB) = \begin{pmatrix} -2.5a \\ -4.25a \end{pmatrix} \qquad Correct unsimplified expression for \cos\theta$		Expressions for \overrightarrow{AB} and \overrightarrow{AO}		If O is c of m,	
$\begin{array}{c} \hline \\ \hline $			M1	$\overrightarrow{AB} = \begin{pmatrix} -2.5a \\ -6a \end{pmatrix}, \overrightarrow{AO} = \begin{pmatrix} 2.5a \\ -4.25a \end{pmatrix}$	
			M1	Use of scalar product to find $\cos\theta$	
$\cos\theta = \frac{AB.AO}{\left \overline{AB}\right \cdot \left \overline{AO}\right } \left = \frac{-6.25 + 25.5}{\frac{13}{2} \times \frac{\sqrt{389}}{4}} \right $ A1	In for $\cos\theta$	Correct unsimplified expression f	A1	$\cos \theta = \frac{\overrightarrow{AB}.\overrightarrow{AO}}{\left \overrightarrow{AB}\right .\left \overrightarrow{AO}\right } \left(= \frac{-6.25 + 25.5}{\frac{13}{2} \times \frac{\sqrt{389}}{4}} \right)$	
$\theta = 53$ A1 Q asks for a whole number		Q asks for a whole number	A1	$\theta = 53$	
(5)			(5)		
5balt 3 $\overline{x} = 5a$ from BB1Or equivalent. Seen or implied Might be seen in (a) but needs to be used h		1 1	B1	$\overline{x} = 5a$ from B	
If O is c of m, find coordinates of point ofRelative to B:				If <i>O</i> is c of m, find coordinates of point of	
intersection between the line AO and the $A(2.5a, 6a), O(5a, 1.75a)$		A(2.5a, 6a), O(5a, 1.75a)			
perpendicular line through B M1 $\Rightarrow AO: y = -1.7x + 10.25a$		$\Rightarrow AO: y = -1.7x + 10.25a$	N/1	perpendicular line through B	
If perpendicular through <i>B</i> intersects <i>AO</i> at	ersects AO at L,	If perpendicular through B interse	IVI I		
$\Rightarrow BL: y = \frac{10}{17}x \qquad \Rightarrow L\left(\frac{3485}{778}, \frac{1025}{389}\right)$	$\left(\frac{35}{8}, \frac{1025}{389}\right)$	$\Rightarrow BL: y = \frac{10}{17}x \qquad \Rightarrow L\left(\frac{3485}{778}, \frac{10}{778}\right)$			
Correct method to find the required angle M1 e.g. $\sin \theta = \frac{BL}{BA}$		e.g. $\sin \theta = \frac{BL}{BA}$	M1	Correct method to find the required angle	
Correct unsimplified expression in θ A1 $= \frac{\sqrt{\left(\frac{3485}{778}\right)^2 + \left(\frac{1025}{389}\right)^2}}{\left(\frac{3485}{389}\right)^2}$	$\frac{1}{2}$		A1	Correct unsimplified expression in θ	
	<u>)</u>				
(5)	<u>)</u>	=6.5	Al	$\theta = 53$	
See over for alternative	<u>) </u>			$\theta = 53$	

5balt 4	$\overline{x} = 5a \text{ from } B$	B1	Or equivalent. Seen or implied Might be seen in (a) but needs to be used here.
	If <i>O</i> is c of m, find coordinates of point of intersection between the line <i>AB</i> and the perpendicular line through <i>O</i>	M1	$AB: y = \frac{12}{5}x$ If perpendicular through O intersects AB at H, $OH: y = -\frac{5}{12}x + \frac{23}{6}, H\left(\frac{230}{169}, \frac{552}{169}\right)$
	Correct method to find the required angle	M1	e.g. $\sin \theta = \frac{OH}{OA}$
	Correct unsimplified expression in θ	A1	$=\frac{\frac{205}{52}}{\sqrt{\frac{389}{16}}}$
	$\theta = 53$	A1	Q asks for a whole number
		[10]	

6(a)	$F = \begin{bmatrix} R & T \\ A & B \end{bmatrix} = \begin{bmatrix} R & T \\ 6 & B \end{bmatrix} = \begin{bmatrix} R & B \\ 6 & B \end{bmatrix}$		
	$R = 4g\cos 30^{\circ}$	B1	
	$F = \frac{1}{\sqrt{3}} R \left(= 2g\right)$	M1	Use of $F = \mu R$
	Work done $=1.5F$	M1	Independent. Correct method for work done seen or implied. Allow without a value for <i>F</i> found
	= 3g or 29.4 (J) or 29 (J)	A1	Not $\frac{147}{5}$
		(4)	
6(b)	PE lost by $B - PE$ gained by A	M1	Dimensionally correct. Condone sin/cos confusion and sign errors.
	$= 6g \times 1.5 - 4g \times 1.5 \sin 30^{\circ}$	A1	Correct unsimplified expression. Allow $\pm (6g \times 1.5 - 4g \times 1.5 \sin 30^\circ)$
	= 6g or 58.8 (J) or 59 (J)	A1	
		(3)	
6(c)	Work energy equation	M1	Q asks for work-energy. Need all terms. Dimensionally correct. Condone sign errors.
	$\frac{1}{2} \times 4v^2 + \frac{1}{2} \times 6v^2 + 3g = 6g$	A1ft	Correct unsimplified equation. Follow their (a) and (b). $\frac{1}{2} \times 10v^2 + \text{their}(a) = \text{their}(b) \text{ or starting}$ afresh
	$5v^2 = 3g \implies v = \sqrt{\frac{3g}{5}} \text{ or } 2.42 \text{ (m s}^{-1})$ or 2.4 (m s^{-1})	A1	not $\frac{7\sqrt{3}}{5}$
		(3) [10]	
-			

		r	
7a	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		
	Use of CLM	M1	Need all terms. Dimensionally correct. Condone sign errors
	$2mku - 3m \times 4u = -2m \times 2u + 3m \times u$ $(2k - 12 = -4 + 3)$	A1	Correct unsimplified
	$\Rightarrow k = \frac{11}{2}$	M1	Solve for <i>k</i>
	KE lost $=\frac{1}{2}2m(k^2-4)u^2+\frac{1}{2}3m(16-1)u^2$	M1	Complete expression. Dimensionally correct. Allow \pm . In k or their k
	$\left(=mu^2\left(k^2-4\right)+\frac{45mu^2}{2}\right)$		
	$=\frac{195mu^2}{4}$, $\lambda=\frac{195}{4}$	A1	Or equivalent (48.75). Accept 49, 48.8 ISW once correct answer seen.
		(5)	
7b	Use of CLM: $4mu = 8mw - 2mv$	M1	Dimensionally correct. Condone incorrect signs.
	Use of impact law:	M1	Used the right way round.
	v + w = 2eu	A1	Both equations correct unsimplified. Signs consistent.
	Solve for v or a multiple of v	M1	(v_B)
	$(w = 2eu - v \implies 10v = 16eu - 4u)$		
	Inequality in e: $10v > 10u \implies 16eu > 14u$	M1	Complete correct method with signs consistent with the direction of their v Upper limit of 1 not necessary No extra incorrect inequality
	$\Rightarrow \frac{7}{8} < e \le 1$	A1	Or equivalent. Want both limits. Must be ≤ 1 .
			NB: The last two M marks might appear in reverse order. The inequality $v > u$ can be used with an expression for <i>w</i> to form the inequality in <i>e</i> .
		(6)	
		[11]	

8a	$ \begin{array}{c} 12i+bj \\ \hline P \\ \hline X \\ \hline \\ 60 m \end{array} $		
	<i>P</i> and <i>Q</i> collide \Rightarrow at same vertical height $\Rightarrow bt - \frac{1}{2}gt^2 = dt - \frac{1}{2}gt^2$ $\Rightarrow bt = dt, b = d$	B1	Obtain given result correctly No incorrect statements seen
		(1)	
			Complete method arises succet to form on
8b	$10 = 3d - \frac{9}{2}g$	M1	Complete method using <i>suvat</i> to form an equation in <i>d</i> e.g. use of $s = ut + \frac{1}{2}at^2$ with
			correct values substituted but condone sign error
	$\Rightarrow d = 18 \ (18.0)$	A1	Follows use of 9.8. Not $\frac{541}{30}$
			Must be a scalar
		(2)	
8c	t = 3, vertical speed $v = d - 3g(=-11.4)$	M1	Condone sign errors
	-11.4 = 11.4 - gt	M1	Using symmetry with their 11.4
	t = 2.3 (2.32)(s)	Al	Not $\frac{341}{147}$
		(3)	
8calt1	Vertical distance $10 = dt - \frac{g}{2}t^2$	M1	Condone sign errors
	Solve for <i>t</i> and subtract $\left(3, \frac{200}{3 \times 98}\right)$	M1	
	$t = 2.3 \ (2.32)(s)$	A1	
		(3)	
8calt2	Time to top: $0 = d - gt$, $(t = 1.84)$	M1	Condone sign errors
	Solve for required time: $T = 2(3-1.84)$	M1	
	= 2.3 (s)	A1	
		(3)	
8d	Horizontal distance: $60 = 12 \times 3 + 3c$	M1	Allow if seen earlier and used here
	(c=8)	A1	Correct unsimplified equation for horizontal distance. Allow $c = -8$ if used correctly later
	v = -8i - 11.4j	M1	For their 8 and 11.4
	Direction $\tan^{-1}\frac{11.4}{8}$	DM1	Use of trig to find a relevant angle Dependent on the previous M1

8dalt	Downwards at 54.9° to the horizontal (55°) (35.1° to the vertical) Horizontal distance: $60 = 12 \times 3 + 3c$ ($c = 8$) $\mathbf{v} = -8\mathbf{i} + 18.0\mathbf{j}$ and conservation of energy	A1 (5) M1 A1	Or equivalent. Direction must be stated or clearly implied by a diagram. 0.959 rads (0.96 rads) Max 3 sf Allow if seen earlier and used here Correct unsimplified equation for horizontal distance. Allow $c = -8$ if used correctly later For their 8 and 18.0
	$\frac{1}{2}mv^{2} - \frac{1}{2}m(8^{2} + 18^{2}) = -mg \times 10$ (v = 13.9)	M1	
	Direction $\cos^{-1}\frac{8}{13.9}$	DM1	Use of trig to find a relevant angle Dependent on the previous M1
	Downwards at 54.9° to the horizontal (55°) (35.1° to the vertical)	A1	Or equivalent. Direction must be stated or clearly implied by a diagram. 0.959 rads (0.96 rads) Max 3 sf
		(5)	
8e			
	$\frac{-x}{12} \times \frac{18}{12} = -1$, $\left(\frac{-x}{12} \times \frac{d}{12} = -1\right)$	M1	Complete method (with their <i>d</i>) using velocity to find <i>x</i> Could be in scalar product format
	<i>x</i> = 8	A1	7.99 or better Allow +/-
	-8 = 18 - 9.8T, $(-x = d - 9.8T)$	DM1	Form equation in <i>T</i> . Signs correct Dependent on the previous M1
	T = 2.7 (2.65)	A1	Follows use of 9.8
	d	(4)	Complete method (with their <i>d</i>) to find the
8e alt	$\tan \alpha = \frac{d}{12}, \beta = 90 - \alpha$	M1	direction of motion
	= 33.64°	A1	
	$-\tan\beta = \frac{d-9.8T}{12}$	DM1	Form equation in <i>T</i> . Signs correct. Dependent on the previous M1
	T = 2.7 (2.65)	A1	CSO – need to be convinced by any changes of sign. Follows use of 9.8
		(4)	
		[15]	

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