Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | | | | | |

Candidate Number | | | | |

Monday 24 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

P56480A

©2019 Pearson Education Ltd.

1/1/1/1/

P 5 6 4 8 0 A 0 1 3 2

Pearson

Leave blank

Answer ALL questions. Write your answers in the spaces provided.

1. A hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{9} = 1 \qquad \text{where } a \text{ is a positive constant}$$

The foci of H are at the points with coordinates (6, 0) and (–6, 0)

Find

(a) the exact value of the constant a,

(3)

(b) the equations of the directrices of H.

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 1 continued

Q1

(Total 6 marks)

Leave blank

2. (a) Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

(i) $\cosh 2x \equiv 2\cosh^2 x - 1$

(ii) $\sinh 2x \equiv 2\sinh x \cosh x$

(4)

(b) Solve the equation

$$\cosh 2x - 7\cosh x = -7$$

giving your answers as exact logarithms.

(5)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 2 continued

Leave blank

Question 2 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 2 continued

Q2

(Total 9 marks)

Leave blank

3. Find

(a) $\int \frac{1}{8 + 4x + x^2} \, dx$

(3)

(b) $\int \frac{1}{\sqrt{(8 - 4x - x^2)}} \, dx$

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 3 continued

Leave blank

Question 3 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 3 continued

Q3

(Total 7 marks)

Leave blank

4.

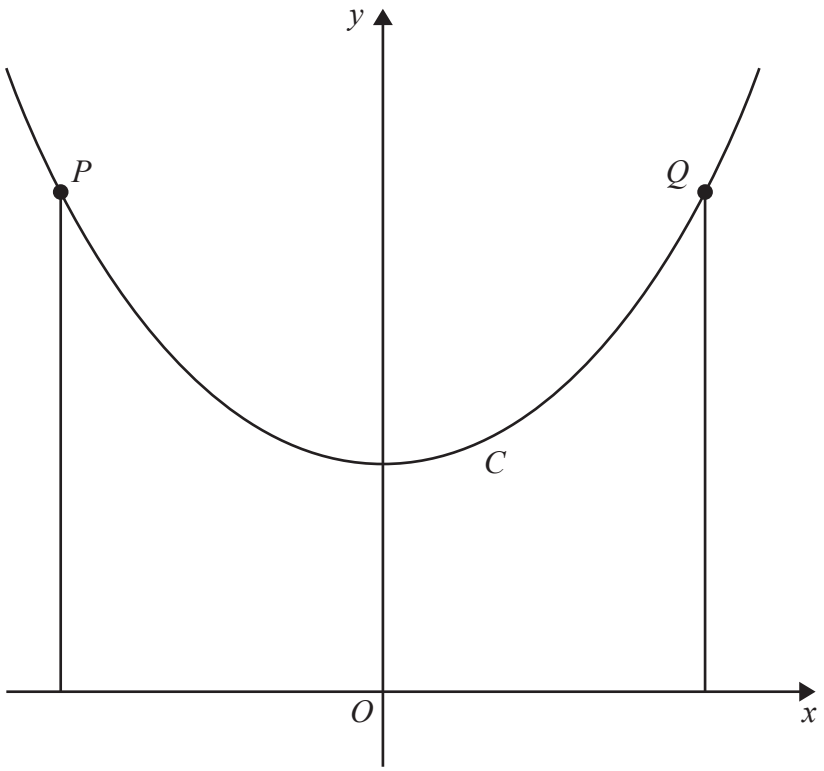

Figure 1

Figure 1 shows part of the curve C with equation

$$y = 3\cosh\left(\frac{x}{3}\right)$$

The point P and the point Q lie on the curve. The point P has x coordinate $-3a$ and the point Q has x coordinate $3a$.

(a) Find the length of the arc PQ, giving your answer as a multiple of $\sinh a$.

(5)

Given that the length of the arc PQ is 12

(b) show that the x coordinate of Q is $3\ln\left(p + \sqrt{q}\right)$, where p and q are integers to be found,

(2)

(c) show that the y coordinate of Q is $r\sqrt{s}$ where r and s are integers to be found.

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 4 continued

Leave blank

Question 4 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 4 continued

Q4

(Total 9 marks)

Leave blank

5. The plane Π has equation

$$\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Find a vector perpendicular to Π

(2)

The line l passes through the point A with coordinates $(2, -4, 0)$ and meets Π at the point with coordinates $(3, 2, -1)$.

The acute angle between the plane Π and the line l is α.

(b) Find α, giving your answer to the nearest degree.

(4)

(c) Find the perpendicular distance from A to Π

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 5 continued

Leave blank

Question 5 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 5 continued

(Total 10 marks)

Q5

Leave blank

6. The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 3 & 0 & 1 \\ 1 & 2 & 2 \\ 4 & 0 & 3 \end{pmatrix}$$

(a) (i) Show that 5 is an eigenvalue of **M**.

(ii) Find the other two eigenvalues of **M**.

(5)

(b) Find an eigenvector corresponding to the eigenvalue 5

(3)

The transformation represented by the matrix **M** maps the straight line l_1 onto the straight line l_2

The equation of l_1 is $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$ where $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$

(c) Find an equation for the line l_2, giving your answer in the form $(\mathbf{r} - \mathbf{c}) \times \mathbf{d} = \mathbf{0}$ where **c** and **d** are constant vectors.

(5)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 6 continued

Leave blank

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 6 continued

(Total 13 marks)

Q6

Leave blank

7. $$I_n = \int \cosh^n x \, dx \qquad n \geqslant 0$$

(a) Show that, for $n \geqslant 2$

$$nI_n = \sinh x \cosh^{n-1} x + (n-1) I_{n-2}$$

(6)

(b) Hence find

$$\int \cosh^4 x \, dx$$

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 7 continued

Leave blank

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 7 continued

(Total 10 marks)

Q7

Leave blank

8. An ellipse has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad a > 0,\ b > 0$$

The line l has equation $y = mx + c$ where $m < 0$ and $c > 0$

Given that l is a tangent to the ellipse,

(a) show that $c^2 = b^2 + a^2m^2$

(4)

The tangent l meets the positive x-axis at the point A and the positive y-axis at the point B. The origin is O.

Given that m varies,

(b) show that the minimum area of triangle OAB is ab.

(7)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 8 continued

Leave blank

Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA