Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

Monday 24 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **6669/01**

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. Find the exact values of x for which

$$\coth^2 x + 5 \operatorname{cosech} x + 5 = 0$$

giving your answers in terms of natural logarithms.

(6)

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Question 1 continued

Q1

(Total 6 marks)

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2. The curve C has equation

$$y = \mathrm{e}^{\operatorname{arcosh} x} \qquad x > 1$$

(a) Find, in terms of x, $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

(5)

(b) Hence show that

$$\left(x^2 - 1\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + x\frac{\mathrm{d}y}{\mathrm{d}x} - y = 0$$

(2)

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Question 2 continued

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Question 2 continued

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Question 2 continued

Q2

(Total 7 marks)

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3. The ellipse E has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad a > b > 0$$

The line l is the tangent to E at the point P $(a\cos\theta, b\sin\theta)$, where $0 < \theta < \frac{\pi}{2}$

(a) Use calculus to show that an equation of l is

$$xb\cos\theta + ya\sin\theta = ab$$

(4)

The line l meets the y-axis at the point Q and meets the x-axis at the point R.

The point M is the midpoint of QR.

(b) Find a Cartesian equation of the locus of M as θ varies, giving your answer in the form $y^2 = \mathrm{f}(x)$.

(4)

The normal to E at the point P meets the y-axis at the point T.

(c) Find the coordinates of the point T.

(2)

Given that $a = 4$, $b = 3$ and $\theta = \frac{\pi}{3}$

(d) find the exact area of triangle MTQ.

(3)

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Question 3 continued

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Question 3 continued

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Question 3 continued

Q3

(Total 13 marks)

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4. The plane Π has equation

$$2x - y + 2z = 7$$

The line l has equation

$$\frac{x+4}{3} = \frac{y-1}{2} = \frac{z-2}{1}$$

(a) Find the coordinates of the point of intersection of the line l with the plane Π.

(3)

(b) Calculate the acute angle between Π and l, giving your answer in degrees to one decimal place.

(5)

(c) Find the coordinates of the two points on the line l such that the shortest distance of each point from the plane is 3 units.

(5)

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

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5. Given that

$$I_n = \int_0^{\frac{\pi}{2}} x \sin^n x \, dx, \quad n \geqslant 0$$

(a) prove that, for $n \geqslant 2$

$$I_n = \frac{1}{n^2} + \frac{n-1}{n} I_{n-2}$$

(7)

(b) Hence, showing each step of your working, find the exact value of

$$\int_0^{\frac{\pi}{2}} x \sin^4 x \, dx$$

(3)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total 10 marks)

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6. The curve C has parametric equations

$$x = t\cos t, \qquad y = t\sin t, \qquad 0 \leqslant t \leqslant 2\pi$$

The length of the curve C is S.

(a) Show that

$$S = \int_0^{2\pi} \sqrt{1+t^2}\,\mathrm{d}t$$

(5)

(b) Use the substitution $t = \sinh\theta$ to find the exact value of S. Write your answer in the form $a + \frac{1}{2}\ln b$, where a and b are real constants given in terms of π.

(7)

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Question 6 continued

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Question 6 continued

Q6

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7.

$$\mathbf{A} = \begin{pmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{pmatrix}$$

(a) Show that 6 is an eigenvalue of the matrix **A** and find the other two eigenvalues of **A**. **(6)**

(b) Find an eigenvector corresponding to the eigenvalue 6 **(3)**

A transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is represented by the matrix **A**. The transformation T maps the line l_1 with equation

$$\left(\mathbf{r} - (2\mathbf{i} + \mathbf{j} - \mathbf{k})\right) \times (5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) = \mathbf{0}$$

onto the line l_2

(c) Find a Cartesian equation for the line l_2 **(5)**

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Question 7 continued

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Question 7 continued

Q7

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TOTAL FOR PAPER: 75 MARKS

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