Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE Mathematics

Core Mathematics C4 (6666/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- o.e. – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark
- dM1 denotes a method mark which is dependent upon the award of the previous method mark.
- aef "any equivalent form"

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = …

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners’ reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners’ reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

June 2019
6666/01 Core Mathematics 4 Mark Scheme

Question Number	Scheme	Notes	Marks
1.	$(1+ax)^{\frac{2}{3}} \approx 1+\frac{1}{2}x+kx^2$; $\mathrm{f}(x)=(4-9x)(1+ax)^{\frac{2}{3}}$, $\lvert ax\rvert<1$		
	$\left\{(1+ax)^{\frac{2}{3}} \approx 1+\left(\frac{2}{3}\right)(ax)+\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(ax)^2+\ldots=1+\frac{2}{3}ax-\frac{1}{9}a^2x^2+\ldots\right\}$		
(a)	$\frac{2}{3}a=\frac{1}{2}$	**see notes**	M1
	$a=\frac{3}{4}$	$a=\frac{3}{4}$ or 0.75	A1 o.e.
			(2)
(b)	$\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(a)^2$	**Either** $\frac{(\frac{2}{3})(\frac{2}{3}-1)}{2!}(a)^2$ **or** $\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(a)^2$ **or** $\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(ax)^2$ **or** $\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(\text{their } a)^2$ **or** $-\frac{1}{9}a^2$	M1
	$\left\{k=\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}\left(\frac{3}{4}\right)^2\right\} \Rightarrow k=-\frac{1}{16}$	$k=-\frac{1}{16}$ or -0.0625	A1
			(2)
(c)	$\left\{(4-9x)\left(1+\frac{1}{2}x-\frac{1}{16}x^2\right)\right\}$		
	$\{x^2:\}$ $-\frac{1}{4}-\frac{9}{2}$; $=-\frac{19}{4}$ or -4.75	Either $4(\text{their } k)-\frac{9}{2}$ or $4(\text{their } k)x^2-\frac{9}{2}x^2$	M1
		$-\frac{19}{4}$ or -4.75	A1
			(2)
			6

	Question 1 Notes	
1.	**Note**	Writing down $\left\{(1+ax)^{\frac{2}{3}}\right\}=1+\left(\frac{2}{3}\right)(ax)+\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(ax)^2+\ldots$ gets (a) M1 and (b) M1
(a)	**Note**	Give M1 for any of • writing down $\frac{2}{3}a=\frac{1}{2}$ • expanding $(1+ax)^{\frac{2}{3}}$ to give $1+\left(\frac{2}{3}\right)(ax)$ • writing down $\frac{2}{3}ax=\frac{1}{2}$ or $\frac{2}{3}a=\frac{1}{2}x$ or $\frac{2}{3}ax=\frac{1}{2}x$
	Note	Give M1 A1 $a=\frac{3}{4}$ from no working
(b)	**Note**	Give A0 for $k=-\frac{1}{16}x^2$ or $-0.0625x^2$ without reference to $k=-\frac{1}{16}$ or -0.0625
	Note	Allow A1 for $k=-\frac{1}{16}x^2$ or $-0.0625x^2$ followed by $k=-\frac{1}{16}$ or -0.0625
(c)	**Note**	Give A0 for $-\frac{19}{4}x^2$ or $-4.75x^2$ without reference to $-\frac{19}{4}$ or -4.75
	Note	Allow A1 for $-\frac{19}{4}x^2$ or $-4.75x^2$ followed by $-\frac{19}{4}$ or -4.75

Question Number	Scheme	Notes	Marks

Question Number	Scheme	Notes	Marks
2.	x: $\frac{\pi}{3}$, $\frac{7\pi}{18}$, $\frac{4\pi}{9}$, $\frac{\pi}{2}$ y: 0.75, 0.60402, 0.33682, 0		
(a)	$\frac{1}{2}\times\frac{\pi}{18}\times\left[\underline{0.75+0+2(0.60402+0.33682)}\right]$ {**Note:** The "0" does not have to be included in [......] }	Outside brackets $\frac{1}{2}\times\frac{\pi}{18}$ or $\frac{\pi}{36}$ or awrt 0.087	B1 o.e.
		For structure of [……] Condone one copying slip	M1
	$\left\{=\frac{\pi}{36}(2.63168)\right\}=0.22965740...=0.2297$ (4 sf)	anything that rounds to 0.2297	A1 isw
			(3)
(b) **Way 1**	$\left\{\int\sin 2x\sin x\,dx=\int 2\sin x\cos x\sin x\,dx=\int 2\sin^2 x\cos x\,dx\right\}$		
	$=\frac{2}{3}\sin^3 x\ \{+c\}$	Integrates to give $\pm\lambda\sin^3 x;\ \lambda\neq 0$ or λu^3 where $u=\sin x$	M1
		$\frac{2}{3}\sin^3 x$ or $\frac{2}{3}u^3$ where $u=\sin x$	A1
	$\left\{\text{Area}(R)=\left[\frac{2}{3}\sin^3 x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}\right\}$		
	$=\left(\frac{2}{3}\sin^3\left(\frac{\pi}{2}\right)\right)-\left(\frac{2}{3}\sin^3\left(\frac{\pi}{3}\right)\right)$	**dependent on the previous M mark** Some evidence of applying limits of $\frac{\pi}{2}$ and $\frac{\pi}{3}$ and subtracts the correct way round	dM1
	$=\left(\frac{2}{3}\right)-\left(\frac{2}{3}\left(\frac{3\sqrt{3}}{8}\right)\right)$		
	$=\frac{2}{3}-\frac{1}{4}\sqrt{3}$	$\frac{2}{3}-\frac{1}{4}\sqrt{3}$ or $\frac{2}{3}\left(1-\frac{3}{8}\sqrt{3}\right)$ or $\frac{1}{12}\left(8-3\sqrt{3}\right)$ isw	A1 o.e. cso
			(4)
(b) **Way 2**	$\left\{\int\sin 2x\sin x\,dx=-\frac{1}{2}\int(\cos 3x-\cos x)\,dx\right\}$		
	$=-\frac{1}{2}\left(\frac{1}{3}\sin 3x-\sin x\right)\{+c\}$	Integrates to give $\pm\alpha\sin 3x\pm\beta\sin x;\ \alpha,\beta\neq 0$	M1
		$-\frac{1}{2}\left(\frac{1}{3}\sin 3x-\sin x\right)$, simplified or un-simplified	A1
	$\left\{\text{Area}(R)=\left[-\frac{1}{6}\sin 3x+\frac{1}{2}\sin x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}\right\}$		
	$=\left(-\frac{1}{6}\sin\left(\frac{3\pi}{2}\right)+\frac{1}{2}\sin\left(\frac{\pi}{2}\right)\right)-\left(-\frac{1}{6}\sin(\pi)+\frac{1}{2}\sin\left(\frac{\pi}{3}\right)\right)$	**dependent on the previous M mark** Some evidence of applying limits of $\frac{\pi}{2}$ and $\frac{\pi}{3}$ and subtracts the correct way round	dM1
	$\left\{=\left(\frac{1}{6}+\frac{1}{2}\right)-\left(0+\frac{1}{4}\sqrt{3}\right)\right\}=\frac{2}{3}-\frac{1}{4}\sqrt{3}$	$\frac{2}{3}-\frac{1}{4}\sqrt{3}$ or $\frac{2}{3}\left(1-\frac{3}{8}\sqrt{3}\right)$ or $\frac{1}{12}\left(8-3\sqrt{3}\right)$	A1 o.e. cso
			(4)
			7

Question Number	Scheme	Notes	Marks

2. (b) **Way 3**	$\left\{ \text{A} = \int \sin 2x \sin x \, dx \right\}$ $\left\{ \begin{array}{l} u = \sin 2x \Rightarrow \frac{du}{dx} = 2\cos 2x \\ \frac{dv}{dx} = \sin x \Rightarrow v = -\cos x \end{array} \right\}$ $\left\{ \begin{array}{l} u = 2\cos 2x \Rightarrow \frac{du}{dx} = -4\sin 2x \\ \frac{dv}{dx} = \cos x \Rightarrow v = \sin x \end{array} \right\}$		
	$\text{A} = -\sin 2x \cos x + \int 2\cos 2x \cos x \, dx$		
	$\text{A} = -\sin 2x \cos x + 2\cos 2x \sin x + \int 4\sin 2x \sin x \, dx$		
	$\text{A} = -\sin 2x \cos x + 2\cos 2x \sin x + 4\text{A}$		
	$-3\text{A} = -\sin 2x \cos x + 2\cos 2x \sin x$		
	$\text{A} = -\frac{1}{3}(-\sin 2x \cos x + 2\cos 2x \sin x)$	Uses integration by parts twice to give $\pm\alpha \sin 2x \cos x \pm \beta \cos 2x \sin x;\ \alpha, \beta \neq 0$	M1
		$-\frac{1}{3}(-\sin 2x \cos x + 2\cos 2x \sin x)$ simplified or un-simplified	A1
	$\left\{ \text{Area}(R) = \left[\frac{1}{3}(\sin 2x \cos x - 2\cos 2x \sin x) \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \right\}$		
	$= \frac{1}{3}\left(\sin(\pi)\cos\left(\frac{\pi}{2}\right) - 2\cos(\pi)\sin\left(\frac{\pi}{2}\right) \right) - \frac{1}{3}\left(\sin\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{3}\right) - 2\cos\left(\frac{2\pi}{3}\right)\sin\left(\frac{\pi}{3}\right) \right)$	**dependent on the previous M mark** Some evidence of applying limits of $\frac{\pi}{2}$ and $\frac{\pi}{3}$ and subtracts the correct way round	dM1
	$= \frac{1}{3}(0+2) - \frac{1}{3}\left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} \right)$		
	$= \frac{2}{3} - \frac{1}{4}\sqrt{3}$	$\frac{2}{3} - \frac{1}{4}\sqrt{3}$ or $\frac{2}{3}\left(1 - \frac{3}{8}\sqrt{3}\right)$ or $\frac{1}{12}\left(8 - 3\sqrt{3}\right)$	A1 o.e. cso
			(4)

		Question 2 Notes
2. (a)	**Note**	For M1, no errors are allowed (e.g. an omission of a y-ordinate or an extra y-ordinate or a repeated y-ordinate).
	Note	Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 0.2336539648…)
	Note	Give B1 M1 A1 for $\frac{\pi}{36}(0.75) + \frac{\pi}{18}(0.60402 + 0.33682)$ = awrt 0.2297
	Note	Using exact values of y at $x = \frac{\pi}{3}, \frac{7\pi}{18}, \frac{4\pi}{9}$ and $\frac{\pi}{2}$ gives $0.229658... = 0.2297$ (4 sf)

	Question 2 Notes Continued	
2. (a)	**Note**	**Bracketing mistake:**
		Unless the final calculated answer implies that the method has been applied correctly
		Give B1 M0 A0 for $\frac{\pi}{36}(0.75)+2(0.60402+0.33682)+0$ (answer of 1.947129847…)
		Give B1 M0 A0 for $\frac{\pi}{36}+(0.75)+2(0.60402+0.33682)+0$ (answer of 2.718946463…)
	Alternative method: Adding individual trapezia	
	Area $\approx \frac{\pi}{18}\times\left[\frac{0.75+0.60402}{2}+\frac{0.60402+0.33682}{2}+\frac{0.33682+0}{2}\right] = 0.22965740...$	
	B1	$\frac{\pi}{18}$ and a divisor of 2 on all terms inside brackets
	M1	First and last ordinates once and two of the middle ordinates inside brackets ignoring the 2, with no omissions of y ordinates or no extra y ordinates.
	A1	Anything that rounds to 0.2297
(b)	**Note**	$\left[\frac{2}{3}\sin^3 x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ followed by awrt 0.234 with no correct answer seen is dM1 A0

Question Number	Scheme	Notes	Marks
3.	$x^2 - y^3 - x - x\sin(\pi y) = -2$		
(a) **Way 1**	$\left\{\frac{\not{d}y}{\not{d}x} \times\right\}$ $\underline{2x - 3y^2\frac{dy}{dx}} - 1 - \underline{\underline{\sin(\pi y) - \pi x\cos(\pi y)\frac{dy}{dx}}} = \underline{0}$	**(2nd M1 is B1 on epen)**	M1 A1 M1
	$\frac{dy}{dx}(-3y^2 - \pi x\cos(\pi y)) + 2x - 1 - \sin(\pi y) = 0$	**dependent on the first M mark**	dM1
	$\left\{\frac{dy}{dx} =\right\}$ $\frac{2x - 1 - \sin(\pi y)}{3y^2 + \pi x\cos(\pi y)}$ or $\frac{-2x + 1 + \sin(\pi y)}{-3y^2 - \pi x\cos(\pi y)}$	Correct answer or equivalent	A1 isw
	Note: Condone omission of brackets or inconsistent use of brackets		**(5)**
(b)	At $(3, 2)$, $m_T = \frac{dy}{dx} = \frac{2(3) - 1 - \sin(2\pi)}{3(2)^2 + \pi(3)\cos(2\pi)}$ $\left\{ = \frac{5}{12 + 3\pi}\right\}$	Substitutes $x = 3$ & $y = 2$ into an equation involving $\frac{dy}{dx}$	M1
	• $y - 2 = \left(\frac{5}{12 + 3\pi}\right)(x - 3)$ Cuts y-axis $\Rightarrow x = 0 \Rightarrow y - 2 = \left(\frac{5}{12 + 3\pi}\right)(-3)$ • $2 = \left(\frac{5}{12 + 3\pi}\right)(3) + c$ $\left\{\Rightarrow c = 2 - \frac{15}{12 + 3\pi}\right\}$	**dependent on the previous M mark** Using a numerical m_T $(\neq m_N)$, where m_T is in terms of π, **either** $y - 2 = m_T(x - 3)$ and sets $x = 0$ in their tangent equation **or** $2 = (\text{their } m_T)(3) + c$	dM1
	So, $\left\{y_Q = \frac{-5}{4 + \pi} + 2 \Rightarrow\right\}$ $y_Q = \frac{2\pi + 3}{\pi + 4}$	**dependent on the final A mark in (a)** $\frac{2\pi + 3}{\pi + 4}$ $\{a = 2, b = 3, c = 4\}$	A1
			(3)
			8
(a) **Way 2**	$\left\{\frac{\not{d}x}{\not{d}y} \times\right\}$ $\underline{2x\frac{dx}{dy} - 3y^2} - \frac{dx}{dy} - \underline{\underline{\sin(\pi y)\frac{dx}{dy} - \pi x\cos(\pi y)}} = \underline{0}$		M1 A1 M1
	$\frac{dx}{dy}(2x - 1 - \sin(\pi y)) - 3y^2 - \pi x\cos(\pi y) = 0$	**dependent on the first M mark**	dM1
	$\frac{dy}{dx} = \frac{2x - 1 - \sin(\pi y)}{3y^2 + \pi x\cos(\pi y)}$ or $\frac{-2x + 1 + \sin(\pi y)}{-3y^2 - \pi x\cos(\pi y)}$	Correct answer or equivalent	A1
			(5)

		Question 3 Notes
3. (a)	**Note**	Writing down ***from no working*** • $\frac{dy}{dx} = \frac{2x - 1 - \sin(\pi y)}{3y^2 + \pi x\cos(\pi y)}$ or $\frac{-2x + 1 + \sin(\pi y)}{-3y^2 - \pi x\cos(\pi y)}$ scores M1A1B1M1A1 • $\frac{dy}{dx} = \frac{-2x + 1 + \sin(\pi y)}{3y^2 + \pi x\cos(\pi y)}$ scores M1A0B1M1A0
	Note	Some will write $2x\,dx - 3y^2 dy - 1dx - \sin(\pi y)dx - \pi x\cos(\pi y)dy = 0$ leading to $\frac{dy}{dx} = \frac{2x - 1 - \sin(\pi y)}{3y^2 + \pi x\cos(\pi y)}$ or equivalent. This should score full marks.

		Question 3 Notes Continued
3. (a)	**M1**	Differentiates implicitly to include either $\pm\pi x\cos(\pi y)\frac{dy}{dx}$ or $y^3 \to \pm 3y^2\frac{dy}{dx}$. $\left(\text{Ignore}\left(\frac{dy}{dx}=\right)\right)$
	1st A1	$x^2 - y^3 - x \to 2x - 3y^2\frac{dy}{dx} - 1$ and $-2 \to 0$
	Note	$2x - 3y^2\frac{dy}{dx} - 1 - \sin(\pi y) - \pi x\cos(\pi y)\frac{dy}{dx} \to 2x - 1 - \sin(\pi y) = 3y^2\frac{dy}{dx} + \pi x\cos(\pi y)\frac{dy}{dx}$ will get 1st A1 (implied) as the " = 0 " can be implied by the rearrangement of their equation.
	M1	$-x\sin(\pi y) \to -\sin(\pi y) - \pi x\cos(\pi y)\frac{dy}{dx}$ or $+\sin(\pi y) - \pi x\cos(\pi y)\frac{dy}{dx}$ or $-\sin(\pi y) + \pi x\cos(\pi y)\frac{dy}{dx}$ or $+\sin(\pi y) + \pi x\cos(\pi y)\frac{dy}{dx}$
	Note	If an extra term appears then give 1st A0
	dM1	**dependent on the first M mark** An attempt to factorise out **all the terms in** $\frac{dy}{dx}$ as long as there are ***at least two terms*** in $\frac{dy}{dx}$. i.e. $\frac{dy}{dx}(-3y^2 - \pi x\cos(\pi y)) + ... = ...$
	Note	Writing down an extra $\frac{dy}{dx} = ...$ and including it in their factorisation is fine for the dM1 mark.
	Note	**Final A1 cso:** If their solution is not completely correct, then do not give this mark.
	Note	**Final A1 isw:** You can, however, ignore subsequent working following on from a correct solution.
(a)	**Way 2**	Apply the mark scheme for Way 2 in the same way as Way 1
(b)	**1st M1**	1st M1 can be gained by seeing at least one example of substituting $x = 3$ and at least one example of substituting $y = 2$; unless it is clear that they are instead applying $x = 2$ and $y = 3$. Otherwise, you will need to check (with your calculator) that $x = 3$, $y = 2$ has been substituted into their expression for $\frac{dy}{dx}$.
	Note	$\sin(2\pi)$ or $\cos(2\pi)$ are not considered as being in terms of π for the 2nd M1 mark, but $\pi(3)\cos(2\pi)$ is considered to be in terms of π.

Question Number	Scheme	Notes	Marks
4.	$y = \dfrac{8}{5(2x+3)^2}$; $x > -\dfrac{3}{2}$		
	$\pi \displaystyle\int \left(\dfrac{8}{5(2x+3)^2}\right)^2 \mathrm{d}x$	For $\pi \displaystyle\int \left(\dfrac{8}{5(2x+3)^2}\right)^2 \mathrm{d}x$ or $\dfrac{64\pi}{25}\displaystyle\int \left(\dfrac{1}{(2x+3)^2}\right)^2$ or equivalent. Ignore the limits and $\mathrm{d}x$. Can be implied.	B1
	$\left\{= \dfrac{64\pi}{25}\displaystyle\int (2x+3)^{-4}\mathrm{d}x\right\}$		
	$= \left\{\dfrac{64\pi}{25}\right\}\left(\dfrac{(2x+3)^{-3}}{(-3)(2)}\right)$	$(2x\pm3)^{-4} \to \pm\lambda(2x\pm3)^{-3}$; $\lambda \neq 0$ **or** $(2x\pm3)^{-4} \to \pm\lambda u^{-3}$; $\lambda \neq 0$, where $u = 2x\pm3$ **Note:** Allow $(K(2x\pm3))^{-4} \to \pm\lambda(K(2x\pm3))^{-3}$; $\lambda, K \neq 0$	M1
		$(2x+3)^{-4} \to \dfrac{(2x+3)^{-3}}{(-3)(2)}$ **or** $(2x+3)^{-4} \to \dfrac{u^{-3}}{(2)(-3)}$ where $u = 2x+3$ which can be simplified or un-simplified **Note:** Allow $(K(2x+3))^{-4} \to \dfrac{(K(2x+3))^{-3}}{(K)(-3)(2)}$; $K \neq 0$	A1
	$\left\{V = \left[-\dfrac{64\pi}{150}\left(\dfrac{1}{(2x+3)^3}\right)\right]_{-1}^{\frac{1}{2}}\right\}$		
	$= -\dfrac{64\pi}{150}\left(\dfrac{1}{(2(\frac{1}{2})+3)^3} - \dfrac{1}{(2(-1)+3)^3}\right)$	**dependent on the previous M mark** Applies limits of 0.5 and -1 to an expression of the form $\pm C(2x\pm3)^{-3}$ or $\pm C(K(2x\pm3))^{-3}$; $C, K \neq 0$ **or** applies limits of 4 and 1 to an expression of the form $\pm Cu^{-3}$ where $u = 2x+3$ **or** applies limits of -2 and -5 to an expression of the form $\pm Cu^{-3}$ where $u = 2x-3$ **Note:** Subtraction of limits is not required	dM1
	$\left\{= -\dfrac{64\pi}{150}\left(\dfrac{1}{64} - 1\right) = -\dfrac{64\pi}{150}\left(-\dfrac{63}{64}\right)\right\}$		
	$= \dfrac{21}{50}\pi$	$\dfrac{21}{50}\pi$ or $\dfrac{63}{150}\pi$ or $\dfrac{42}{100}\pi$ or 0.42π	A1 **cso**
			(5)

Question 4 Notes

	Note:	B1 can be implied by seeing a correct formula $\pi\displaystyle\int y^2\,\mathrm{d}x$ (with or without $\mathrm{d}x$) and a correct expression for y^2 seen in their working. E.g. $y^2 = \left(\dfrac{8}{5(2x+3)^2}\right)^2$ or even ... $= \left(\dfrac{8}{5(2x+3)^2}\right)^2$

Question Number	Scheme	Notes	Marks
5.	$\dfrac{dh}{dt}=\dfrac{k}{h+4}$, $0\le h\le 35$		
(a)	$\left\{h=16,\ \dfrac{dh}{dt}=0.6 \Rightarrow 0.6=\dfrac{k}{16+4} \Rightarrow\right\}$ $k=12$	$k=12$	B1
			(1)
(b) **Way 1**	$\left\{\dfrac{dh}{dt}=\dfrac{12}{h+4} \Rightarrow \int (h+4)dh=\int 12\ dt\right\}$		
	$\dfrac{h^2}{2}+4h=12t\ \{+c\}$	$h\pm 4\to \alpha h^2+\beta h;\ \alpha,\beta\ne 0$	M1
		Correct integration with a follow through on their k found in (a). Ignore limits or $+c$	A1ft
	$\left\{\left[\dfrac{h^2}{2}+4h\right]_0^{30}=\left[12t\right]_0^T\right\}$		
	$\dfrac{900}{2}+120=12T \Rightarrow T=...$	**dependent on the previous M mark** Applies $h=30$ to an integrated equation of the form $\alpha h^2+\beta h=\gamma t+c;\ \alpha,\beta,\gamma\ne 0$ (c can be 0) and solves to find a value for the time taken	dM1
	$\left\{T=\dfrac{570}{12}\Rightarrow\right\}$ $T=47.5$ (seconds)	47.5	A1
			(4)
(b) **Way 2**	$\left\{\dfrac{dh}{dt}=\dfrac{12}{h+4} \Rightarrow \int (h+4)dh=\int 12\ dt\right\}$		
	$\dfrac{(h+4)^2}{2}=12t\ \{+c\}$	$h\pm 4\to \alpha(h+4)^2;\ \alpha\ne 0$	M1
		Correct integration. Ignore limits or $+c$	A1
	$\left\{\left[\dfrac{(h+4)^2}{2}\right]_0^{30}=\left[12t\right]_0^T\right\}$		
	$\dfrac{(34)^2}{2}-\dfrac{(4)^2}{2}=12T \Rightarrow T=...$	**dependent on the previous M mark** Applies $h=30$ and $h=0$ correctly to an integrated equation of the form $\alpha(h\pm 4)^2=\gamma t+c;\ \alpha,\gamma\ne 0$ (c can be 0) and solves to find a value for the time taken	dM1
	$\left\{T=\dfrac{570}{12}\Rightarrow\right\}$ $T=47.5$ (seconds)	47.5	A1
			(4)
(c) **Way 1**	$\left\{\dfrac{dV}{dt}=96\pi \Rightarrow\right\}$ Volume $=96\pi(47.5)$	96π(their "47.5"), where their "47.5" is positive	M1
	$\{=4560\pi=14325.6625...\}=14300\ (cm^3)$ (3 sf)	14300	A1 **cao**
			(2)
(c) **Way 2**	$\left\{\dfrac{dV}{dh}=\dfrac{dV}{dt}\div\dfrac{dh}{dt}\Rightarrow\right\}$ $\dfrac{dV}{dh}=96\pi\times\left(\dfrac{h+4}{12}\right)$	Applies $96\pi\div\left(\text{their } \dfrac{dh}{dt}\right)$, integrates to find V (with or without $+c$), and substitutes $h=30$ into their expression for V	M1
	$\dfrac{dV}{dh}=8\pi h+32\pi \Rightarrow V=4\pi h^2+32\pi h\ \{+c\}$		
	So, volume $=4\pi(30)^2+32\pi(30)$		
	$\{=4560\pi=14325.6625...\}=14300\ (cm^3)$ (3 sf)	14300	A1 **cao**
			(2)
			7

Question Number	Scheme	Notes	Marks
5. (b) **Way 3**	$\left\{\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{12}{h+4} \Rightarrow \frac{\mathrm{d}t}{\mathrm{d}h}=\frac{1}{12}(h+4) \Rightarrow \frac{\mathrm{d}t}{\mathrm{d}h}=\frac{1}{12}h+\frac{1}{3}\right\}$		
	$t=\frac{1}{24}h^2+\frac{1}{3}h\ \{+c\}$	$\lambda(h\pm 4)\rightarrow \alpha h^2+\beta h;\ \alpha,\beta,\lambda\neq 0$	M1
		Correct integration. Ignore limits or $+c$. You can imply $t=...$	A1
	$\left\{[t]_0^T=\left[\frac{1}{24}h^2+\frac{1}{3}h\right]_0^{30}\right\}$		
	$T=\frac{1}{24}(30)^2+\frac{1}{3}(30)$	**dependent on the previous M mark** Applies $h=30$ correctly to an integrated equation of the form $t=\alpha h^2+\beta h+c;\ \alpha,\beta\neq 0$ (c can be 0)	dM1
	$T=47.5$ (seconds)	47.5	A1
			(4)
5. (b) **Way 4**	$\left\{\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{12}{h+4} \Rightarrow \frac{\mathrm{d}t}{\mathrm{d}h}=\frac{1}{12}(h+4)\right\}$		
	$t=\frac{1}{24}(h+4)^2\ \{+c\}$	$\lambda(h\pm 4)\rightarrow \alpha(h+4)^2;\ \alpha,\lambda\neq 0$	M1
		Correct integration. Ignore limits or $+c$. You can imply $t=...$	A1
	$\left\{[t]_0^T=\left[\frac{1}{24}(h+4)^2\right]_0^{30}\right\}$		
	$T=\frac{1}{24}(34)^2-\frac{1}{24}(4)^2$	**dependent on the previous M mark** Applies $h=30$ and $h=0$ correctly to an integrated equation of the form $t=\alpha(h\pm 4)^2+c;\ \alpha\neq 0$ (c can be 0)	dM1
	$T=47.5$ (seconds)	47.5	A1
			(4)

	Question 5 Notes	
5. (b)	**Note**	Give M1 A1 dM1 for the solution $\left\{\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{k}{h+4} \Rightarrow \int (h+4)\mathrm{d}h=\int k\ \mathrm{d}t \Rightarrow\right\}\ \frac{h^2}{2}+4h=kt\ \{+c\}$ $\left\{\left[\frac{(h+4)^2}{2}\right]_0^{30}=[kt]_0^T\right\} \Rightarrow \frac{900}{2}+120=kT \Rightarrow T=...$ (where they do not use the value of k) followed by final A1 for $\left\{T=\frac{570}{k}\Rightarrow\right\}\ T=47.5$ (seconds)

Question Number	Scheme	Notes	Marks
6. (i)	$\int \frac{5}{6e^{3x}}\,dx = \int \frac{5}{6}e^{-3x}dx = -\frac{5}{18}e^{-3x}\ \{+c\}$	Integrates to give $\pm\alpha e^{-3x},\ \alpha \neq 0, \frac{5}{6}, 30$	M1
		$-\frac{5}{18}e^{-3x}$ or $-\frac{5}{18e^{3x}}$ with or without $+\ c$	A1
			(2)
(ii)(a)	$\frac{4y^2+3y-4}{y(2y-1)} \equiv A + \frac{B}{y} + \frac{C}{(2y-1)}$		
	$\{y^2: 4 = 2A \Rightarrow\}\ A = 2$	Their constant term = 2	B1
	$4y^2+3y-4 \equiv Ay(2y-1) + B(2y-1) + Cy$	Forming a correct identity	B1
	Either • constant: $-4 = -B \Rightarrow B = 4$ • $y: -A + 2B + C \Rightarrow 3 = -2 + 8 + C \Rightarrow C = -3$ • $y = 0 \Rightarrow -4 = -B \Rightarrow B = 4$ • $y = \frac{1}{2} \Rightarrow 1 + \frac{3}{2} - 4 = \frac{1}{2}C \Rightarrow C = -3$	Uses their identity in an attempt to find the value of at least one of either their B or their C	M1
	$\left\{\frac{4y^2+3y-4}{y(2y-1)} \equiv\right\}\ 2 + \frac{4}{y} - \frac{3}{(2y-1)}$	Correct partial fractions. Can be seen anywhere in part (ii)	A1
			(4)
(b) **Way 1**	$\left\{\int \frac{4y^2+3y-4}{y(2y-1)}\,dy\right\}$ $= \int\left(2 + \frac{4}{y} - \frac{3}{(2y-1)}\right)dy$ $= 2y + 4\ln y - \frac{3}{2}\ln(2y-1)\ \{+c\}$	Integrates to give at least one of either $\frac{B}{y} \to \pm\lambda \ln y$ or $\frac{C}{2y-1} \to \pm\mu \ln(2y-1)$ or $\gamma \ln(y-\frac{1}{2}),\ B \neq 0,\ C \neq 0$	M1
		Correct follow through integration for at least two terms from their $A \neq 0$ or from their B or from their C	A1 ft
		$2y + 4\ln y - \frac{3}{2}\ln(2y-1)$ **or** $2y + 4\ln y - \frac{3}{2}\ln(y-\frac{1}{2})$ can apply isw	A1
	Final A1: Correct bracketing required. Can be simplified or un-simplified, with/without $+c$		**(3)**
(iii)	$\left\{\int \frac{1}{\sqrt{x}}\ln(2x)\,dx\right\}$, $\left\{\begin{array}{l} u = \ln(2x) \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^{-\frac{1}{2}} \Rightarrow v = 2x^{\frac{1}{2}} \end{array}\right\}$		
	$= 2x^{\frac{1}{2}}\ln(2x) - \int 2x^{\frac{1}{2}}\left(\frac{1}{x}\right)\{dx\}$	Either $\frac{1}{\sqrt{x}}\ln(2x) \to \pm\lambda x^{\frac{1}{2}}\ln(kx) \pm \int \mu x^{\frac{1}{2}}\left(\frac{\alpha}{\beta x}\right)\{dx\}$ or $\pm\lambda x^{\frac{1}{2}}\ln(kx) \pm \int \mu x^{-\frac{1}{2}}\{dx\};\ \lambda, \mu, k \neq 0$	M1
	$= 2x^{\frac{1}{2}}\ln(2x) - 4x^{\frac{1}{2}}$	**dependent on the previous M mark** Integrates the second term to give $Ax^{\frac{1}{2}};\ A \neq 0$	dM1 A1 on epen
		$2x^{\frac{1}{2}}\ln(2x) - 4x^{\frac{1}{2}}$, simplified or un-simplified	A1
	$\left\{\left[2\sqrt{x}\ln(2x) - 4\sqrt{x}\right]_1^4\right\}$ $= (2\sqrt{4}\ln(8) - 4\sqrt{4}) - (2\sqrt{1}\ln(2) - 4\sqrt{1})$	**dependent on the first M mark** Some evidence of applying limits of 4 and 1 and subtracts the correct way round	dM1
	$\{= 4\ln 8 - 8 - 2\ln 2 + 4\} = -4 + 10\ln 2$	$-4 + 10\ln 2$	A1 **cso**
			(5)
			14

Question Number	Scheme	Notes	Marks
6. (ii)(a) **Way 2**	$\dfrac{4y^2+3y-4}{y(2y-1)} \equiv 2+\dfrac{5y-4}{y(2y-1)}$	Their constant term = 2	B1
	$\dfrac{5y-4}{y(2y-1)} \equiv \dfrac{B}{y}+\dfrac{C}{(2y-1)}$		
	$5y-4 \equiv B(2y-1)+Cy$	Forming a correct identity	B1
	Either • constant: $-4=-B \Rightarrow B=4$ • $y: 5=2B+C \Rightarrow 5=8+C \Rightarrow C=-3$ • $y=0 \Rightarrow -4=-B \Rightarrow B=4$ • $y=\dfrac{1}{2} \Rightarrow \dfrac{5}{2}-4=\dfrac{1}{2}C \Rightarrow C=-3$	Uses their identity in an attempt to find the value of at least one of either their *B* or their *C*	M1
	$\left\{\dfrac{4y^2+3y-4}{y(2y-1)} \equiv\right\} 2+\dfrac{4}{y}-\dfrac{3}{(2y-1)}$	Correct partial fractions. Can be seen anywhere in part (ii)	A1
			(4)

	Question 6 Notes	
6. (ii)(a)	**Note**	Give B1B1M1A1 writing down $2+\dfrac{4}{y}-\dfrac{3}{(2y-1)}$ from no working.
(iii)	**SC**	Give ***Special Case*** 1st M1 for writing down the correct "by parts" formula and using $u=\ln(2x),\ \dfrac{\mathrm{d}v}{\mathrm{d}x}=\dfrac{1}{\sqrt{x}},$ but making only one error in the application of the correct formula If the Special Case 1st M1 is given, then this allows access to any of the other two M marks in this part.

Question Number	Scheme	Notes	Marks
7.	$x=-3+6\sin\theta,\quad y=4\sqrt{3}\cos 2\theta,\quad -\frac{\pi}{2}<\theta<\frac{\pi}{2}$		
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-(2)(4)\sqrt{3}\sin 2\theta}{6\cos\theta}\left\{=\frac{-4\sqrt{3}\sin 2\theta}{3\cos\theta}=-\frac{8}{3}\sqrt{3}\sin\theta\right\}$	their $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ ÷ their $\frac{\mathrm{d}x}{\mathrm{d}\theta}$	M1
		Correct simplified or un-simplified result	A1 isw
			(2)
(b)	$\{x=0\Rightarrow\}\ 0=-3+6\sin\theta\Rightarrow\sin\theta=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{6}$	Sets $x=0$ to find θ or $\sin\theta$ and uses their θ or $\sin\theta$ to find y	M1
	$y_A=4\sqrt{3}\cos\left(\frac{\pi}{3}\right)=2\sqrt{3}\ \{\Rightarrow A(0,\,2\sqrt{3})\}$	$y_A=2\sqrt{3}$ or $\sqrt{12}$ or awrt 3.46	A1
	$m_T=\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-4\sqrt{3}\sin(2(\frac{\pi}{6}))}{3\cos(\frac{\pi}{6})}=\frac{-4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)}{3\left(\frac{\sqrt{3}}{2}\right)}=-\frac{4\sqrt{3}}{3}=-\frac{4}{\sqrt{3}}$	Substitutes $\theta=\frac{\pi}{6}$ or $30°$ or $\sin\theta=\frac{1}{2}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$. Can be implied.	M1
	So, $m_N=\frac{3}{4\sqrt{3}}$ or $\frac{\sqrt{3}}{4}$	Correctly applies $m_N=-\frac{1}{\text{their } m_T}$	M1
	• $y-2\sqrt{3}=\frac{\sqrt{3}}{4}(x-0)$ • $y=\frac{\sqrt{3}}{4}x+2\sqrt{3}$	$y-(\text{their } y_A)=(\text{their } m_N)(x-0)$ or $y=(\text{their } m_N)x+(\text{their } y_A)$ with a numerical $m_N\ (\neq m_T)$	M1
	$4y-8\sqrt{3}=\sqrt{3}x\Rightarrow\sqrt{3}x-4y+8\sqrt{3}=0$ *	Correct proof	A1*
			(6)
(c) **Way 1**	$\sqrt{3}(-3+6\sin\theta)-4(4\sqrt{3}\cos 2\theta)+8\sqrt{3}=0$	Substitutes $x=-3+6\sin\theta$ and $y=4\sqrt{3}\cos 2\theta$ into the normal equation to form an equation in θ only	M1
	$-3+6\sin\theta-16\cos 2\theta+8=0$		
	$-3+6\sin\theta-16(1-2\sin^2\theta)+8=0$	**dependent on the previous M mark** Applies $\cos 2\theta\equiv 1-2\sin^2\theta$	dM1
	$32\sin^2\theta+6\sin\theta-11=0$ or $32\sin^2\theta+6\sin\theta=11$	Correct 3TQ in $\sin\theta$ e.g. $32\sin^2\theta+6\sin\theta-11\{=0\}$	A1
	$(2\sin\theta-1)(16\sin\theta+11)=0\ \Rightarrow\ \sin\theta=...$	**dependent on the first M mark** Correct method (e.g. factorising, applying the quadratic formula, completing the square or calculator) of solving a 3TQ to give $\sin\theta=...$	dM1
	$\left\{\sin\theta=\frac{1}{2}\right\}\quad\sin\theta=-\frac{11}{16}$		
	So, $B(x,\,y)=B\left(-\frac{57}{8},\,\frac{7}{32}\sqrt{3}\right)$	Either x or y is correct	A1 o.e.
		Both x and y are correct	A1 o.e.
			(6)
			14

Question Number	Scheme	Notes	Marks
7.	$x = -3 + 6\sin\theta, \quad y = 4\sqrt{3}\cos 2\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$; **N**: $\sqrt{3}x - 4y + 8\sqrt{3} = 0$		
(c) **Way 2**	$y = 4\sqrt{3}\cos 2\theta = 4\sqrt{3}(1 - 2\sin^2\theta)$ $y = 4\sqrt{3} - 8\sqrt{3}\sin^2\theta$	Substitutes the normal equation into the Cartesian equation of C which is of the form $y = \lambda \pm \mu(x \pm 3)^2$; $\lambda,\ \mu \neq 0$ to give an equation in x only	M1
	$y = 4\sqrt{3} - 8\sqrt{3}\left(\frac{x+3}{6}\right)^2$	**dependent on the previous M mark** Applies $\cos 2\theta \equiv 1 - 2\sin^2\theta$	dM1
	$\frac{\sqrt{3}x + 8\sqrt{3}}{4} = 4\sqrt{3} - 8\sqrt{3}\left(\frac{x+3}{6}\right)^2$	Correct un-simplified or simplified equation in x	A1
	$36\left(\frac{\sqrt{3}x + 8\sqrt{3}}{4}\right) = 144\sqrt{3} - 8\sqrt{3}(x^2 + 6x + 9)$		
	$9\sqrt{3}x + 72\sqrt{3} = 144\sqrt{3} - 8\sqrt{3}x^2 + 48\sqrt{3}x - 72\sqrt{3}$		
	$8\sqrt{3}x^2 + 57\sqrt{3}x = 0 \Rightarrow x = ...$	**dependent on the first M mark** Correct method of solving their quadratic equation in x to give $x = ...$ **Note:** x could be cancelled out to give a linear equation in x	dM1
	So, $B(x,\ y) = B\left(-\frac{57}{8},\ \frac{7}{32}\sqrt{3}\right)$	Either x or y is correct	A1 o.e.
		Both x and y are correct	A1 o.e.
			(6)

	Question 7 Notes	
7. (a)	**Note**	Condone poor notation, when they are applying the method of parametric differentiation
(b)	**Note**	2nd M1 can be implied by writing down a correct result for their $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $\theta = \frac{\pi}{3}$ or $60°$
(c)	**Note**	Condone for $\theta = \frac{-6 \pm \sqrt{6^2 - 4(32)(-11)}}{2(32)}$ for the 3rd M1 mark
	Note	Allow 2nd A1 for any of $x = -\frac{57}{8}$ or awrt -7.13 or -7.125 or $y = \frac{7}{32}\sqrt{3}$ or awrt 0.379 or $\sqrt{\frac{147}{1024}}$

Question Number	Scheme	Notes	Marks
7.	$x=-3+6\sin\theta,\quad y=4\sqrt{3}\cos 2\theta,\quad -\frac{\pi}{2}<\theta<\frac{\pi}{2}$; **N**: $\sqrt{3}x-4y+8\sqrt{3}=0$		
(c) **Way 3**	$y=4\sqrt{3}\cos 2\theta=4\sqrt{3}(1-2\sin^2\theta)$ $\sqrt{3}(-3+6\sin\theta)-4y+8\sqrt{3}=0$	Complete method of eliminating θ from the equations for C and N simultaneously to achieve an equation in y only	M1
	$\sin\theta=\frac{4y-5\sqrt{3}}{6\sqrt{3}}\Rightarrow\sin^2\theta=\frac{(4y-5\sqrt{3})^2}{108}$	**dependent on the previous M mark** Applies $\cos 2\theta\equiv 1-2\sin^2\theta$	dM1
	$y=4\sqrt{3}\left(1-\frac{2(4y-5\sqrt{3})^2}{108}\right)$	Correct un-simplified or simplified equation in y	A1
	$\frac{\sqrt{3}y}{12}=\frac{108-2(4y-5\sqrt{3})^2}{108}$ $9\sqrt{3}y=108-32y^2+80\sqrt{3}y-150$ $32y^2-71\sqrt{3}y+42=0$		
	$(32y-7\sqrt{3})(y-2\sqrt{3})=0\Rightarrow y=\ldots$	**dependent on the first M mark** Correct method (e.g. factorising, applying the quadratic formula, completing the square or calculator) of solving a 3TQ to give $y=\ldots$	dM1
	So, $B(x,\,y)=B\left(-\frac{57}{8},\,\frac{7}{32}\sqrt{3}\right)$	Either x or y is correct	A1 o.e.
		Both x and y are correct	A1 o.e.
			(6)
(c) **Way 4**	$y=4\sqrt{3}\cos 2\theta=4\sqrt{3}(1-2\sin^2\theta)$	Complete method of eliminating θ from the equations for C and N simultaneously to achieve an equation in y only	M1
	$\sin^2\theta=\frac{4\sqrt{3}-y}{8\sqrt{3}}\Rightarrow\sin\theta=\sqrt{\frac{4\sqrt{3}-y}{8\sqrt{3}}}$	**dependent on the previous M mark** Applies $\cos 2\theta\equiv 1-2\sin^2\theta$	dM1
	$\sqrt{3}\left(-3+6\sqrt{\frac{4\sqrt{3}-y}{8\sqrt{3}}}\right)-4y+8\sqrt{3}=0$	Correct un-simplified or simplified equation in y	A1
	$\left(-3+6\sqrt{\frac{4\sqrt{3}-y}{8\sqrt{3}}}\right)=\frac{4y-8\sqrt{3}}{\sqrt{3}}\Rightarrow 6\sqrt{\frac{4\sqrt{3}-y}{8\sqrt{3}}}=\frac{4y}{\sqrt{3}}-5$		
	$\sqrt{\frac{4\sqrt{3}-y}{8\sqrt{3}}}=\frac{2y}{3\sqrt{3}}-\frac{5}{6}\Rightarrow\frac{4\sqrt{3}-y}{8\sqrt{3}}=\frac{4}{27}y^2-\frac{10}{9\sqrt{3}}y+\frac{25}{36}$		
	$\frac{1}{2}-\frac{y}{8\sqrt{3}}=\frac{4}{27}y^2-\frac{10}{9\sqrt{3}}y+\frac{25}{36}\Rightarrow\frac{4}{27}y^2-\frac{71}{72\sqrt{3}}y+\frac{7}{36}=0$		
	$32y^2-71\sqrt{3}y+42=0$		
	$(32y-7\sqrt{3})(y-2\sqrt{3})=0\Rightarrow y=\ldots$	**dependent on the first M mark** Correct method (e.g. factorising, applying the quadratic formula, completing the square or calculator) of solving a 3TQ to give $y=\ldots$	dM1
	So, $B(x,\,y)=B\left(-\frac{57}{8},\,\frac{7}{32}\sqrt{3}\right)$	Either x or y is correct	A1 o.e.
		Both x and y are correct	A1 o.e.
			(6)

Question Number	Scheme	Notes	Marks
8.	$l_1 : \mathbf{r} = \begin{pmatrix} -6 \\ 13 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$; direction $\mathbf{d} = \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$, $\overrightarrow{OA} = \begin{pmatrix} 6 \\ -1 \\ 7 \end{pmatrix}$ and $\overrightarrow{OP} = \begin{pmatrix} 2 \\ c \\ d \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix}$		
(a)	$-6 + 4\mu = 2 \Rightarrow \mu = 2$	Uses the **i** component to find μ and substitutes μ into l_1 to find the point P	M1
	$\overrightarrow{OP} = \begin{pmatrix} -6 \\ 13 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix}$	Correct vector for $\overrightarrow{OP}$ **or** $c = 3$ and $d = 7$	A1
	$\overrightarrow{PA} = \overrightarrow{OA} - \overrightarrow{OP} = \begin{pmatrix} 6 \\ -1 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ 0 \end{pmatrix}$		
	$AP = \sqrt{(4)^2 + (-4)^2 + (0)^2} = \sqrt{32} = 4\sqrt{2}$	Full method for finding PA	M1
		$4\sqrt{2}$	A1
			(4)
(b)	$\{l_2 :\}\ \mathbf{r} = \begin{pmatrix} 6 \\ -1 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$	$\mathbf{a} + \lambda\mathbf{d}$ or $\mathbf{a} + \mu\mathbf{d}$, $\mathbf{a} \neq 0$, $\mathbf{d} \neq 0$ with either $\mathbf{a} = 6\mathbf{i} - \mathbf{j} + 7\mathbf{k}$ or $\mathbf{d} = 4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$ or $\mathbf{d}$ = multiple of $4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$	M1
		Correct vector equation using $\mathbf{r} =$ or $l =$ or $l_2 =$ or $L =$	A1
	Do not allow $l_2 :$ or $l_2 \rightarrow$ or $l_1 =$ for the A1 mark		**(2)**
(c)	$\overrightarrow{PA} = \begin{pmatrix} 4 \\ -4 \\ 0 \end{pmatrix}$ and $\mathbf{d}_2 = \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} 4 \\ -4 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$	Realisation that the dot product is required between their $\overrightarrow{AP}$ or $\overrightarrow{PA}$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	M1
	$\left\{\cos\theta = \dfrac{\overrightarrow{PA} \bullet \mathbf{d}_2}{\lvert \overrightarrow{PA} \rvert \lvert \mathbf{d}_2 \rvert} = \right\} \dfrac{\pm\left(\begin{pmatrix} 4 \\ -4 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}\right)}{\sqrt{(4)^2 + (-4)^2 + (0)^2}\sqrt{(4)^2 + (-5)^2 + (3)^2}}$	**dependent on the previous M mark** Applies the dot product formula between their $\overrightarrow{AP}$ or $\overrightarrow{PA}$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	dM1
	$\{\cos\theta = \}\ \dfrac{16 + 20 + 0}{\sqrt{32}\sqrt{50}} = \dfrac{9}{10}$ *	$\{\cos\theta\} = \dfrac{9}{10}$ *	A1
			(3)
(d) **Way 1**	$\left\{\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix} 6 + 4\lambda \\ -1 - 5\lambda \\ 7 + 3\lambda \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix}\right\} \Rightarrow \overrightarrow{PB} = \begin{pmatrix} 4 + 4\lambda \\ -4 - 5\lambda \\ 3\lambda \end{pmatrix}$	Correct method for finding $\overrightarrow{PB}$ in terms of λ. (Can be implied)	M1
	$(4 + 4\lambda)^2 + (-4 - 5\lambda)^2 + (3\lambda)^2 = (4\sqrt{2})^2$	**dependent on the previous M mark** Uses Pythagoras correctly to form an equation in terms of λ for PB = their $4\sqrt{2}$ or PB^2 = (their $4\sqrt{2})^2$	dM1
	$\{16 + 32\lambda + 16\lambda^2 + 16 + 40\lambda + 25\lambda^2 + 9\lambda^2 = 32\}$		
	$\{\Rightarrow 50\lambda^2 + 72\lambda = 0 \Rightarrow \lambda(50\lambda + 72) = 0\} \Rightarrow \lambda = -\dfrac{72}{50}$	$\lambda = -\dfrac{72}{50}$ or $-\dfrac{36}{25}$ or -1.44	A1
	$\overrightarrow{OB} = \begin{pmatrix} 6 \\ -1 \\ 7 \end{pmatrix} - \dfrac{36}{25}\begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{6}{25} \\ \frac{31}{5} \\ \frac{67}{25} \end{pmatrix}$ or $\begin{pmatrix} 0.24 \\ 6.2 \\ 2.68 \end{pmatrix}$	**dependent on the previous M mark** Substitutes one of their non-zero value(s) of λ into l_2	dM1
	$\Rightarrow B(0.24,\ 6.2,\ 2.68)$	Correct coordinates of B. Condone B expressed as a position vector.	A1 o.e.
			(5)
			14

Question Number	Scheme	Notes	Marks
8. (d) **Way 2**	$AB = 4\sqrt{2}\left(\frac{9}{10}\right)(2)\quad \left\{= \frac{36}{5}\sqrt{2}\right\}$	$AB = (\text{their "}4\sqrt{2}\text{"})(0.9)$	M1
	$\pm\sqrt{(4\lambda)^2 + (-5\lambda)^2 + (3\lambda)^2} = 4\sqrt{2}\left(\frac{9}{10}\right)(2)$ or $\frac{36}{5}\sqrt{2}$ $(4\lambda)^2 + (-5\lambda)^2 + (3\lambda)^2 = \left(4\sqrt{2}\left(\frac{9}{10}\right)(2)\right)^2$ or $\frac{2592}{25}$	**dependent on the previous M mark** Forms a correct equation in terms of λ for their AB or their AB^2	dM1
	$\left\{\pm\sqrt{50\lambda^2} = \frac{36}{5}\sqrt{2} \Rightarrow \lambda = \pm\frac{36\sqrt{2}}{5\sqrt{50}}\right\} \Rightarrow \lambda = -\frac{36}{25}$	$\lambda = -\frac{72}{50}$ or $-\frac{36}{25}$ or -1.44 (**Ignore** $\lambda = \frac{72}{50}$ or $\frac{36}{25}$ or 1.44)	A1
	$\overrightarrow{OB} = \begin{pmatrix} 6 \\ -1 \\ 7 \end{pmatrix} - \frac{36}{25}\begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{6}{25} \\ \frac{31}{5} \\ \frac{67}{25} \end{pmatrix}$ or $\begin{pmatrix} 0.24 \\ 6.2 \\ 2.68 \end{pmatrix}$	**dependent on the previous M mark** Substitutes one of their non-zero value(s) of λ into l_2	dM1
	$\Rightarrow B(0.24,\ 6.2,\ 2.68)$	Correct coordinates of B. Condone B expressed as a position vector.	A1 o.e.
			(5)
(d) **Way 3**	Let X be the midpoint of AB $\left\{\overrightarrow{PX} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix} 6+4\lambda \\ -1-5\lambda \\ 7+3\lambda \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix}\right\} \Rightarrow \overrightarrow{PX} = \begin{pmatrix} 4+4\lambda \\ -4-5\lambda \\ 3\lambda \end{pmatrix}$	Correct method for finding $\overrightarrow{PX}$ (or $\overrightarrow{PB}$) in terms of λ. (Can be implied)	M1
	$\overrightarrow{PX} \bullet \mathbf{d}_2 = 0 \Rightarrow \begin{pmatrix} 4+4\lambda \\ -4-5\lambda \\ 3\lambda \end{pmatrix} \bullet \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix} = 0$	**dependent on the previous M mark** Forms a correct equation in terms of λ for $(\text{their } \overrightarrow{PX}) \bullet \mathbf{d}_2 = 0$	dM1
	$\{16 + 16\lambda + 20 + 25\lambda + 9\lambda = 0\}$		
	$\{\Rightarrow 50\lambda + 36 = 0 \Rightarrow\} \Rightarrow \lambda_X = -\frac{36}{50}$	$\lambda_X = -\frac{36}{50}$ or $-\frac{18}{25}$ or -0.72	A1
	$\overrightarrow{OB} = \begin{pmatrix} 6 \\ -1 \\ 7 \end{pmatrix} - (2)\frac{36}{50}\begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{6}{25} \\ \frac{31}{5} \\ \frac{67}{25} \end{pmatrix}$ or $\begin{pmatrix} 0.24 \\ 6.2 \\ 2.68 \end{pmatrix}$	**dependent on the previous M mark** Substitutes one of their non-zero value(s) of 2λ into l_2 or a complete method to find B	dM1
	$\Rightarrow B(0.24,\ 6.2,\ 2.68)$	Correct coordinates of B. Condone B expressed as a position vector.	A1 o.e.
			(5)

		Question 8 Notes
8. (b)	**Note**	M1 can be implied for $\left\{\begin{pmatrix} 4 \\ -4 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}\right\} = 36$
(c)	**Note**	Give final A1 for using a correct method to find $\cos\theta = -\frac{9}{10}$ followed by $\cos\theta = \frac{9}{10}$
	Note	Give final A0 for finding $\cos\theta = -\frac{9}{10}$ by itself without reference to $\cos\theta = \frac{9}{10}$
(d)	**Note**	Give the final A0 for stating more than one set of coordinates for B
	Note	Send to review any obscure solutions leading to a correct answer $B(0.24,\ 6.2,\ 2.68)$

Question Number	Scheme		Notes	Marks
8. (b)	**<u>Vector Cross Product:</u>** Use this scheme if a vector cross product method is being applied			
	$\overrightarrow{PA}\times\mathbf{d}_2=\underline{\begin{pmatrix}4\\-4\\0\end{pmatrix}\times\begin{pmatrix}4\\-5\\3\end{pmatrix}}=\left\{\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\4&-4&0\\4&-5&3\end{vmatrix}=-12\mathbf{i}-12\mathbf{j}-4\mathbf{k}\right\}$		Realisation that the vector cross product is required between their $\overrightarrow{AP}$ or $\overrightarrow{PA}$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	M1
	$\sin\theta=\dfrac{\sqrt{(-12)^2+(-12)^2+(-4)^2}}{\sqrt{(4)^2+(-4)^2+(0)^2}.\sqrt{(4)^2+(-5)^2+(3)^2}}$	Applies vector cross product formula between their $\overrightarrow{AP}$ or $\overrightarrow{PA}$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ or a multiple of these vectors		dM1
	$\left\{\sin\theta=\dfrac{\sqrt{304}}{\sqrt{32}.\sqrt{50}}=\sqrt{\dfrac{304}{1600}}=\sqrt{\dfrac{19}{100}}\right\}$ $\{\Rightarrow\cos\theta\}=\sqrt{\dfrac{100-19}{100}}=\underline{\underline{\sqrt{\dfrac{81}{100}}}}=\dfrac{9}{10}$	$\{\cos\theta\}=\underline{\dfrac{9}{10}}$ *		A1
				[3]

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