Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE In Statistics 3
Paper 6691/01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark
  ⌙

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td rowspan="3">1(a)</td><td rowspan="2"><table>
<tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td><td>I</td></tr>
<tr><td>N</td><td>4</td><td>3</td><td>1</td><td>2</td><td>6</td><td>9</td><td>8</td><td>7</td><td>5</td></tr>
<tr><td>G</td><td>3</td><td>4</td><td>2</td><td>1</td><td>8</td><td>6</td><td>9</td><td>5</td><td>7</td></tr>
<tr><td>d</td><td>1</td><td>-1</td><td>-1</td><td>1</td><td>-2</td><td>3</td><td>-1</td><td>2</td><td>-2</td></tr>
<tr><td>d^2</td><td>1</td><td>1</td><td>1</td><td>1</td><td>4</td><td>9</td><td>1</td><td>4</td><td>4</td></tr>
</table></td><td>Attempt to rank at least 1 row with at least 6 correct.</td><td>M1</td></tr>
<tr><td>Attempt at d^2 - can be implied by $\sum d^2 = 26$</td><td>M1</td></tr>
<tr><td></td><td>$\sum d^2 = 26$</td><td>Can be implied by correct answer for r_s.</td><td>A1</td></tr>
<tr><td rowspan="2"></td><td>$r_s = 1 - \frac{6(26)}{9(80)}$</td><td>Use of correct formula with their $\sum d^2$</td><td rowspan="2">M1</td></tr>
<tr><td colspan="2">If $n \neq 9$ in (a) score this M0A0 but allow ft for cv and hence upto all 4 in (b)</td></tr>
<tr><td></td><td>$r_s = 0.78\dot{3}$</td><td>$0.78\dot{3}$, $\frac{47}{60}$, awrt 0.783</td><td>A1</td></tr>
<tr><td></td><td></td><td></td><td>(5)</td></tr>
<tr><td>(b)</td><td>$H_0 : \rho = 0$, $H_1 : \rho > 0$</td><td>Both correct in terms of ρ or ρ_s. Must be compatible with their ranking.</td><td>B1</td></tr>
<tr><td rowspan="2"></td><td>cv 0.6 or cr $r_s \geq 0.6$</td><td>0.6 with sign compatible with their ranking.</td><td rowspan="2">B1</td></tr>
<tr><td colspan="2">For two-tailed test allow cv = 0.7 with correct sign</td></tr>
<tr><td></td><td>$r_s = 0.78\dot{3}$ lies in cr so reject H_0</td><td>Correct statement e.g. “reject H_0”, “in critical region”, “significant”, “positive correlation”. $\left</td><td>\text{test stat}\right</td><td> > 1$ award M0</td><td>M1</td></tr>
<tr><td rowspan="2"></td><td>Data does not support University F’s claim.</td><td>Correct conclusion in context. <u>Must mention</u> “University F’s claim”. Allow “university’s claim”</td><td rowspan="2">A1ft</td></tr>
<tr><td colspan="2">Allow the final A1 independently of the hypotheses</td></tr>
<tr><td></td><td></td><td></td><td>(4)</td></tr>
<tr><td></td><td></td><td></td><td>Total 9</td></tr>
</table>

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td>2</td><td>H_0 : Gender and choice of ice cream are independent (or not associated).
H_1 : Gender and choice of ice cream are dependent (or associated).</td><td>“gender” and “ice cream” mentioned at least once.</td><td>B1</td></tr>
<tr><td></td><td><table>
<tr><td>OBSERVED</td><td>Male</td><td>Female</td></tr>
<tr><td>Cone</td><td>50</td><td>80</td></tr>
<tr><td>Sandwich</td><td>25</td><td>20</td></tr>
<tr><td>Sundae</td><td>10</td><td>10</td></tr>
<tr><td>Float</td><td>15</td><td>10</td></tr>
</table></td><td>Attempt to convert fractions to observed frequencies at least 1 correct;
at least 6 correct</td><td>M1;
A1</td></tr>
<tr><td></td><td><table>
<tr><td>EXPECTED</td><td>Male</td><td>Female</td><td>Totals</td></tr>
<tr><td>Cone</td><td>59.090…</td><td>70.909…</td><td>130</td></tr>
<tr><td>Sandwich</td><td>20.454…</td><td>24.545…</td><td>45</td></tr>
<tr><td>Sundae</td><td>9.090…</td><td>10.909…</td><td>20</td></tr>
<tr><td>Float</td><td>11.363…</td><td>13.636…</td><td>25</td></tr>
<tr><td>Totals</td><td>100</td><td>120</td><td>220</td></tr>
</table></td><td>Attempt $\frac{\text{RT}\times\text{CT}}{\text{GT}}$
with at least 1 correct;
at least 4 correct to 3sf.</td><td>M1;
A1</td></tr>
<tr><td></td><td><table>
<tr><td>Observed</td><td>Expected</td><td>$\frac{(O-E)^2}{E}$</td><td>$\frac{O^2}{E}$</td></tr>
<tr><td>50</td><td>59.090…</td><td>1.3986…</td><td>42.3076…</td></tr>
<tr><td>80</td><td>70.909…</td><td>1.1655…</td><td>90.2564…</td></tr>
<tr><td>25</td><td>20.45</td><td>1.0101…</td><td>30.5555…</td></tr>
<tr><td>20</td><td>24.55</td><td>0.8417…</td><td>16.2962…</td></tr>
<tr><td>10</td><td>9.09</td><td>0.0909…</td><td>11</td></tr>
<tr><td>10</td><td>10.91</td><td>0.0757…</td><td>9.1666…</td></tr>
<tr><td>15</td><td>11.36</td><td>1.1636…</td><td>19.8</td></tr>
<tr><td>10</td><td>13.64</td><td>0.9696…</td><td>7.3333….</td></tr>
<tr><td colspan="2">Totals</td><td>6.716</td><td>226.716</td></tr>
</table></td><td>At least 2 correct terms for $\frac{(O-E)^2}{E}$ or $\frac{O^2}{E}$ with their values. Accept 2sf.</td><td>M1</td></tr>
<tr><td></td><td></td><td>At least 4 correct to 2dp or better. Allow truncation.</td><td>A1</td></tr>
<tr><td></td><td>$\frac{(O-E)^2}{E}$ or $\sum\frac{O^2}{E}-220=6.716$</td><td>awrt 6.7</td><td>A1</td></tr>
<tr><td></td><td>$\nu=(4-1)(2-1)=3, \chi^2_3(5\%)=7.815$</td><td>3, 7.815</td><td>B1B1ft</td></tr>
<tr><td></td><td>$\chi^2=6.7$ does not lie in cr so insufficient evidence to reject H_0</td><td>For correct statement linking their test statistic and their cv.</td><td>M1</td></tr>
<tr><td></td><td>Gender and choice of ice cream are independent or
There is no evidence of an association between gender and choice of ice cream or
Dave’s belief is supported.</td><td>Context required i.e. “gender” and “choice of ice cream” or “Dave”. Condone “connection” or “relationship” but not “correlation”.</td><td>A1ft</td></tr>
<tr><td></td><td></td><td></td><td>(12)</td></tr>
<tr><td></td><td></td><td></td><td>Total 12</td></tr>
</table>

Question Number	Scheme	Notes	Marks
3.			
(a)	$\frac{20}{\sigma} = 3.0902$ (Calc 3.0902323…)	awrt 3.09 seen.	B1
	$\sigma = \frac{20}{3.0902}$ [= 6.47207…]	$\frac{20}{\text{their } \lvert z \rvert}$ provided $\lvert z \rvert > 1$	M1
	$\sigma = 6.47$*	Cso (Must see 3.0902 or better)	A1*
			(3)
(b)	$H_0 : \mu = 500$ $H_1 : \mu < 500$	Both	B1
	$\overline{x} = \frac{5940}{12} = 495$	495 cao. May be implied below.	B1
	$z = \frac{495 - 500}{\frac{6.47}{\sqrt{12}}} = -2.677\ldots$ or $\frac{w - 500}{\frac{6.47}{\sqrt{12}}} = -1.6449$	Standardise using their mean and correct standard error. or $\frac{w - 500}{\frac{6.47}{\sqrt{12}}} = z$ value	M1
	$z = -2.677...$ or CR $w < 496.92..$	awrt -2.68 or $w <$ awrt 497	A1
	Test statistic of + 2.677 or + awrt 2.68 scores M1A0 but allow final A1		
	5% cv -1.6449 (Calc: $-1.6448536\ldots$)	-1.6449 or better seen. Compatible signs.	B1
***p*-value**	Allow in range [0.0035, 0.0038)		
	Reject H_0	For correct statement linking their test statistic and their cv dependent on first M1.	dM1
	There is evidence that the machine is filling packets with a mean weight of less than 500g.	Context required. Must mention "machine" and "packets" or "filling".	A1
			(7)
ALT	$X = \sum_1^{12} W_i, X \sim \text{N}(12\mu, 12 \times 6.47^2)$	May be implied below.	B1
	$z = \frac{5940 - 6000}{6.47\sqrt{12}} = -2.68$	Standardise using 5940, 6000 and $6.47\sqrt{12}$	M1
			Total 10

Question Number	Scheme	Notes	Marks
4(a)	Standard error $= \frac{\sqrt{38}}{\sqrt{50}} = 0.8717...$	awrt 0.872	B1
			(1)
(b)	$172 \pm 2.5758 \times 0.8717...$	2.5758 or better (Calc: 2.575829…)	B1
	$= 172 \pm 2.2455...$	$172 \pm (z \text{ value}) \times$ (their (a)) with $\lvert z \rvert > 2$	M1
	(169.8,174.2)	awrt (170,174)	A1A1
SC	98% CI (No z seen) gives (169.97, 174.03) and scores B0M1A1A0		
			(4)
(c)	$z = 1.96$ (Calc: 1.9599639…) (Use in an attempt at a CI is OK)	1.96 or better	B1
	$2 \times 1.96 \times \frac{\sqrt{38}}{\sqrt{n}} \leq 3$ Allow use of "=" here	$2 \times (z \text{ value}) \times \frac{\sqrt{38}}{\sqrt{n}}$ ($\lvert z \rvert > 1$) (Can ft their "$\sqrt{38}$" from (a))	M1
	$n \geq \frac{2^2 \times 1.96^2 \times 38}{3^2} = 64.88...$ (Must have correct use of inequalities to score this mark)	Form inequality in $\sqrt{n}$ attempt to solve achieving a value for n (Can ft their z value, $\lvert z \rvert > 1$) (Dep on 1st M1)	dM1
	Minimum sample size is 65.	65 cao (No use of inequality loses cannot score final M1A1)	A1
	(Missing 2 x (giving 17) scores max of B1 in (c)) Use of $z = 1.6449$ or better (giving 46) could score B0M1M1 A0)		
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
5.			
(a)	Label male 1-2520 and female 1-1680	Label both male and female. Can start at 0.	B1
	Use random numbers to select from each group.	Must mention "random numbers" or "process".	B1
	36 male and 24 female	Both	B1
	(Use of systematic sampling scores B1 B0B0 max)		
			(3)
(b)	$\overline{w} = \frac{2700}{60} = 45$	45kg	B1
	$s^2 = \frac{125500 - 60 \times "45"^2}{59}$	Correct expression with their $\overline{w}$	M1A1ft
	$s^2 = 67.7966...$ (Ans of $\frac{4000}{59}$ only is A0)	awrt 67.8	A1
			(4)
(c)	$H_0 : \mu_F - \mu_M = 1$	If not clear which is which from subscripts, then defintion is required for the mark. oe	B1
	$H_1 : \mu_F - \mu_M > 1$	oe	B1
SC	No labels but consistent use of e.g. $\mu_1 - \mu_2$ scores B0B1		
	$z = \frac{2.1 - 0.8 - 1}{\sqrt{\frac{1.2^2}{40} + \frac{0.9^2}{20}}}$	M1 for attempting standard error. Condone swapping 1.5 and 0.9. A1 the completely correct expression.	M1A1
	$z = 1.0846...$	1.085 or awrt 1.08	A1
	cv $z = 1.2816$ (Calc: 1.2815515…)	± 1.2816 or better	B1
***p*-value**	Allow a 3sf value in the range [0.1379, 0.1410] (calc gives awrt 0.139)		
	So not significant, insufficient evidence to reject H_0		
	Manager's claim is not supported or The mean weight loss of the female dogs is **not** greater than 1kg more than the mean weight loss of the male dogs.	Correct conclusion in context. Must mention "manager" or "weight loss", "female dogs" and "male dogs".	A1 (No ft)
			(7)
			Total 14

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td rowspan="2">6.</td><td>H_0 : Arrows hit the target following the model 60:90:90:120 / 2:3:3:4
H_1 : Arrows do not hit the target following the model 60:90:90:120 / 2:3:3:4 or
H_0 : Arrows land in a particular sector in proportion to area / angle of sector.
H_1 : Arrows do not land in a particular sector in proportion to area / angle of sector.</td><td>Both oe. Accept in terms of ‘fit’or Paula’s claim. Accept
H_0 : Continuous uniform distribution is a suitable model
H_1 : Continuous uniform distribution is not a suitable model</td><td rowspan="2">B1</td></tr>
<tr><td>Must mention arrows</td><td>Allow U[0, 360]</td></tr>
<tr><td></td><td>Expected: A: $\frac{60}{360}\times 100$ etc</td><td>All E correct to 2 sf.</td><td>B1</td></tr>
<tr><td rowspan="2"></td><td rowspan="2"><table>
<tr><th>Sector</th><th>O</th><th>E</th><th>$\frac{(O-E)^2}{E}$</th><th>$\frac{O^2}{E}$</th></tr>
<tr><td>A</td><td>13</td><td>$16.\dot{6}$</td><td>$0.80\dot{6}$</td><td>10.14</td></tr>
<tr><td>B</td><td>22</td><td>25</td><td>0.36</td><td>19.36</td></tr>
<tr><td>C</td><td>25</td><td>25</td><td>0</td><td>25</td></tr>
<tr><td>D</td><td>40</td><td>$33.\dot{3}$</td><td>$1.\dot{3}$</td><td>48</td></tr>
<tr><td></td><td></td><td></td><td>2.5</td><td>102.5</td></tr>
</table></td><td>M1 for attempting $\frac{(O-E)^2}{E}$ or $\frac{O^2}{E}$ with at least 2 correct expressions or values.</td><td>M1</td></tr>
<tr><td>All correct (2 dp)</td><td>A1</td></tr>
<tr><td></td><td>$\sum\frac{(O-E)^2}{E}=\sum\frac{O^2}{E}-100=2.5$</td><td>2.5 or awrt 2.50</td><td>A1</td></tr>
<tr><td></td><td>$\nu=4-1=3, \chi^2_3(1\%)=11.345$</td><td>3 can be implied by 11.345 seen</td><td>B1B1ft</td></tr>
<tr><td></td><td>Insufficient evidence to reject H_0</td><td>For correct statement linking their test statistic and their cv.</td><td>M1</td></tr>
<tr><td rowspan="2"></td><td>Paula’s claim is supported. (o.e.)</td><td>A correct comment suggesting support for Paula’s claim.
Condone: “Paula’s claim is correct”
Hypotheses wrong way around scores A0 here.</td><td rowspan="2">A1
(No ft)</td></tr>
<tr><td colspan="2">For an “o.e.” conclusion condone missing “arrows”</td></tr>
<tr><td></td><td></td><td></td><td>(9)</td></tr>
<tr><td></td><td></td><td></td><td>Total 9</td></tr>
</table>

Question Number	Scheme	Notes	Marks
7(a)	$M = 4X_1 - 3X_2 - \bar{Y}$		
	$\mathrm{E}(4X_1 - 3X_2 - \bar{Y}) = 0$	May be seen below e.g. N(0, .. (Otherwie need E(…) = 0)	B1
	$\mathrm{Var}(4X_1 - 3X_2 - \bar{Y}) = 16\sigma^2 + 9\sigma^2 + \frac{\sigma^2}{4}$	$\mathrm{Var}(-3X_2) = 9\sigma^2$, $\mathrm{Var}(-\bar{Y}) = \frac{\sigma^2}{4}$	M1, M1
	$\mathrm{Var}(M) = \frac{101\sigma^2}{4}$	oe	
	$\mathrm{N}\left(0, \frac{101\sigma^2}{4}\right)$	Normal and both correct.	A1
			(4)
(b)	$\mathrm{P}(4X_1 < 3X_2 + \bar{Y} + \sigma) = 0.579$		
	$\mathrm{P}(M < \sigma) = \mathrm{P}\left(Z < \frac{\sigma - 0}{\sqrt{\frac{101\sigma^2}{4}}}\right)$	Standardise using their part (a) and clearly cancel σ	M1
	$= \mathrm{P}(Z < 0.199...)$	awrt 0.199 (Allow value in [0.195, 0.20])	A1
	$= 0.5793*$ (Must see more than just 0.579 1st)	Calc 0.57887… awrt 0.579*	A1cso
			(3)
(c)	$T = 4W_1 - 3W_2 - \bar{W}$		
	$4W_1 - 3W_2 - \bar{W} = 4W_1 - 3W_2 - \frac{W_1 + W_2 + W_3 + W_4}{4} = \frac{15W_1 - 13W_2 - W_3 - W_4}{4}$	$\frac{15W_1 - 13W_2 - W_3 - W_4}{4}$	M1A1
	For M1 must "unpack" $\bar{W}$ and attempt to collect W_i terms		
	$\mathrm{E}(4W_1 - 3W_2 - \bar{W}) = 0$	May be seen below.	B1
	$\mathrm{Var}\left(\frac{15W_1 - 13W_2 - W_3 - W_4}{4}\right) = \frac{225\sigma^2 + 169\sigma^2 + \sigma^2 + \sigma^2}{16}$	$\mathrm{Var}(aW_i) = a^2\sigma^2$ For i = 1 or 2	dM1 (Dep on 1st M1)
	$= \frac{99\sigma^2}{4}$	oe	
	$\mathrm{N}\left(0, \frac{99\sigma^2}{4}\right)$	For distribution.	A1
			(5)
			Total 12

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