Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE Further Mathematics
Further Pure 3 Paper 6669/01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to $x=\ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to $x=\ldots$

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

<u>Use of a formula</u>

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

<u>Exact answers</u>

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Notes	Marks
1	$\coth^2 x + 5\text{cosech}x + 5 = 0$		
Way 1	$\text{cosech}^2 x + 1 + 5\text{cosech}x + 5$	Uses $\coth^2 x = \pm\ \text{cosech}^2 x \pm 1$	M1
	$\text{cosech}^2 x + 5\text{cosech}x + 6 = 0$	Correct quadratic	A1
	$(\text{cosech}x + 3)(\text{cosech}x + 2) = 0$	M1: Solves their 3 term quadratic	M1A1
	$\Rightarrow \text{cosech}x = -3,\ -2$	A1: Both correct values	
	$x = \ln\left(\dfrac{\sqrt{10}-1}{3}\right),\ \ln\left(\dfrac{\sqrt{5}-1}{2}\right)$	A1: One correct answer	A1, A1
		A1: Both answers correct with no errors.	
			(6)
			Total 6
Way 2	$\coth^2 x + 5\text{cosech}x + 5 = 0$ $\Rightarrow \cosh^2 x + 5\sinh^2 x + 5\sinh x = 0$	M1: $\times \sinh^2 x$ and uses $\cosh^2 x = \pm 1 \pm \sinh^2 x$	M1
	$\Rightarrow 6\sinh^2 x + 5\sinh x + 1 = 0$	A1: Correct quadratic	A1
	$(3\sinh x + 1)(2\sinh x + 1) = 0$	M1: Solves their 3 term quadratic	M1A1
	$\Rightarrow \sinh x = -\dfrac{1}{3},\ -\dfrac{1}{2}$	A1: Both correct values	
	$x = \ln\left(\dfrac{\sqrt{10}-1}{3}\right),\ \ln\left(\dfrac{\sqrt{5}-1}{2}\right)$	A1: One correct answer	A1A1
		A1: Both answers correct with no errors.	
Way 3	$\dfrac{(e^x + e^{-x})^2}{(e^x - e^{-x})^2} + \dfrac{5\times 2}{e^x - e^{-x}} + 5 = 0$	Substitutes the correct exponential forms	M1
	$3e^{4x} + 5e^{3x} - 4e^{2x} - 5e^x + 3 = 0$	Correct quartic in e^x	A1
	$(3e^{2x} + 2e^x - 3)(e^{2x} + e^x - 1) = 0$	M1: Solves their 3 term quadratic which must have come from their quartic.	M1A1
	$e^x = \dfrac{-2 \pm \sqrt{40}}{6},\ \dfrac{-1 \pm \sqrt{5}}{2}$	A1: Both correct values	
	$x = \ln\left(\dfrac{\sqrt{10}-1}{3}\right),\ \ln\left(\dfrac{\sqrt{5}-1}{2}\right)$	A1: One correct answer	A1A1
		A1: Both answers correct with no errors.	

Notes

If more than two solutions are found then final A0.

Question Number	Scheme	Notes	Marks
2	$y = e^{\text{arcosh}x}, \quad x > 1$		
(a) **Way 1**	$\frac{dy}{dx} = \frac{e^{\text{arcosh}\,x}}{\sqrt{x^2-1}}$	M1: $e^{\text{arcosh}\,x} \times \frac{d\left(e^{\text{arcosh}\,x}\right)}{dx}$ A1: Correct derivative	M1A1
	$\frac{d^2y}{dx^2} = \frac{\sqrt{x^2-1}\frac{e^{\text{arcosh}\,x}}{\sqrt{x^2-1}} - e^{\text{arcosh}\,x}.\frac{1}{2}\left(x^2-1\right)^{-\frac{1}{2}}.2x}{x^2-1}$ or $= e^{\text{arcosh}x}.[-x(x^2-1)^{-3/2}] + e^{\text{arcosh}x}.(x^2-1)^{-1}$	dM1: Correct use of quotient rule or product rule. A1: $\frac{e^{\text{arcosh}\,x}}{x^2-1}$ or $-\frac{x\,e^{\text{arcosh}\,x}}{\left(x^2-1\right)^{\frac{3}{2}}}$ A1: Correct second derivative	dM1A1A1
	$\left(= e^{\text{arcosh}x}\left(\frac{1}{x^2-1} - \frac{x}{\left(x^2-1\right)^{\frac{3}{2}}}\right)\right)$		
			(5)
(b)	$\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = e^{\text{arcosh}\,x} - \frac{x\,e^{\text{arcosh}\,x}}{\sqrt{x^2-1}} + \frac{x\,e^{\text{arcosh}\,x}}{\sqrt{x^2-1}} - e^{\text{arcosh}\,x} = 0$ M1: Substitutes into the given expression A1: Completes correctly with no errors		M1A1
			(2)
			Total 7
	Alternatives for (a)		
Way 2	$y = e^{\text{arcosh}x} \Rightarrow \ln y = \text{arcosh}\,x$		
	$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\sqrt{x^2-1}} \Rightarrow \frac{dy}{dx} = \frac{e^{\text{arcosh}\,x}}{\sqrt{x^2-1}}$	M1: Correct implicit attempt A1: Correct derivative	M1A1
	$\frac{1}{y}\frac{d^2y}{dx^2} - \frac{1}{y^2}\frac{dy}{dx} = -\frac{x}{\left(x^2-1\right)^{\frac{3}{2}}}$ or use Way 1	Attempt second derivative	dM1
	$\frac{d^2y}{dx^2} = y\left(\frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 - \frac{x}{\left(x^2-1\right)^{\frac{3}{2}}}\right)$		
	$\frac{d^2y}{dx^2} = e^{\text{arcosh}x}\left(\frac{1}{x^2-1} - \frac{x}{\left(x^2-1\right)^{\frac{3}{2}}}\right)$	A1: $\frac{e^{\text{arcosh}\,x}}{x^2-1}$ or $-\frac{x\,e^{\text{arcosh}\,x}}{\left(x^2-1\right)^{\frac{3}{2}}}$ A1: Correct second derivative	A1A1
	See appendix for way 3 and way 4		

Question Number	Scheme	Notes	Marks
3	$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$		
(a)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{b\cos\theta}{a\sin\theta}\ \left(=\dfrac{-b^2x}{a^2y}\right)$	Correct gradient	B1
	$y-b\sin\theta=-\dfrac{b\cos\theta}{a\sin\theta}(x-a\cos\theta)$	M1: Correct straight line method A1: Correct equation	M1A1
	$bx\cos\theta+ay\sin\theta=ab$ *	Correct completion to printed answer with at least one more line of working and no errors seen	A1*
			(4)
(b)	$Q: x=0\Rightarrow y=\dfrac{b}{\sin\theta}, R: y=0\Rightarrow x=\dfrac{a}{\cos\theta}$	Q and R correct	B1
	$M:\left(\dfrac{a}{2\cos\theta},\dfrac{b}{2\sin\theta}\right)$	Correct midpoint method	M1
	$\cos\theta=\dfrac{a}{2x},\sin\theta=\dfrac{b}{2y}\Rightarrow\left(\dfrac{a}{2x}\right)^2+\left(\dfrac{b}{2y}\right)^2=1$	Full attempt to eliminate θ	M1
	$y^2=\dfrac{x^2b^2}{4x^2-a^2},\quad\left(x>\dfrac{a}{2},y>\dfrac{b}{2}\right)$	Correct equation (oe)	A1
			(4)
(c)	$x=0$ in $y-b\sin\theta=\dfrac{a\sin\theta}{b\cos\theta}(x-a\cos\theta)$		M1
	$T:\left(0,\dfrac{(b^2-a^2)}{b}\sin\theta\right)$ or $y=\dfrac{(b^2-a^2)}{b}\sin\theta$	Allow unsimplified	A1
			(2)
(d) Way 1	$TQ=OQ+OT=\dfrac{6}{\sqrt{3}}+\dfrac{16-9}{3}.\dfrac{\sqrt{3}}{2}$	Correct method for TQ (May be implied by 2 triangles)	M1
	Area $MTQ=\dfrac{1}{2}\left(\dfrac{6}{\sqrt{3}}+\dfrac{7\sqrt{3}}{6}\right).4=\dfrac{19}{3}\sqrt{3}$	M1: Fully correct method for area A1: Correct area	M1A1
			(3)
(d) Way 2	$MQ=\sqrt{16+3},\quad PT=\sqrt{4+\left(\dfrac{16\sqrt{3}}{6}\right)^2}$	Correct method for MQ and PT	M1
	Area $MTQ=\dfrac{1}{2}\sqrt{19}.\sqrt{\dfrac{76}{3}}=\dfrac{19\sqrt{3}}{3}$	M1: Fully correct method for area A1: Correct area	M1A1
	NB: $a=4,\ b=3,\ \theta=\dfrac{\pi}{3},\ M\left(4,\ \sqrt{3}\right),\ P\left(2,\ \dfrac{3\sqrt{3}}{2}\right),\ T\left(0,\ -\dfrac{7\sqrt{3}}{6}\right),\ Q\left(0,\ 2\sqrt{3}\right)$		
			Total 13

Notes

(a) B1 : The gradient must have come from differentiation

Question Number	Scheme	Notes	Marks
4	Π: $2x - y + 2z = 7$, L: $\dfrac{x+4}{3} = \dfrac{y-1}{2} = \dfrac{z-2}{1}$		
(a)	$L:\begin{pmatrix}-4\\1\\2\end{pmatrix}+\lambda\begin{pmatrix}3\\2\\1\end{pmatrix} \Rightarrow 2(-4+3\lambda)-(1+2\lambda)+2(2+\lambda)=7$	Attempts parametric form for L and substitutes into Π Allow one slip.	M1
	$6\lambda = 12 \Rightarrow \lambda = 2 \rightarrow (2, 5, 4)$	M1: Solves for λ and substitutes into L A1: Correct coordinates. Need not be given as a point.	M1A1
			(3)
(a) Way 2	From L, $y=\frac{2}{3}(x+4)+1, z=\frac{1}{3}(x+4)+2$ $2x-\left(\frac{2}{3}(x+4)+1\right)+2\left(\frac{1}{3}(x+4)+2\right)=7$	Attempts 2 variables in terms of the third and substitutes into the plane	M1
	$x=2 \Rightarrow y=5, z=4$	M1: Solves for x and finds y and z A1: Correct coordinates. Need not be given as a point.	M1A1
(b)	$\begin{pmatrix}2\\-1\\2\end{pmatrix}\bullet\begin{pmatrix}3\\2\\1\end{pmatrix}=6-2+2=6$	Attempt scalar product between normal and direction vector. Allow one slip.	M1
	$6 = \sqrt{9}\sqrt{14}\cos\theta$ or $6 = \sqrt{9}\sqrt{14}\sin\alpha$	Applies the correct scalar product formula	M1
	$\cos\theta = \sin\alpha = 6/[\sqrt{9}\sqrt{14}]$	Correct $\cos\theta$ or correct $\sin\alpha$	A1
	$(\theta = 57.7°) \Rightarrow \alpha = 32.3°$	M1: $\alpha = 90 - \theta$ Implied if θ is not found. A1: Correct α	M1A1
			(5)
(c) Way 1	$A=(2, 5, 4)$ and B is required point $AB\cos\theta = 3 \Rightarrow AB = \dfrac{3}{\cos\theta} = \dfrac{3\sqrt{14}}{2}$	M1: Correct attempt at AB A1: Correct numerical expression for AB	M1A1
	$\left\lvert\begin{matrix}3\\2\\1\end{matrix}\right\rvert=\sqrt{14} \Rightarrow B \text{ is at } \begin{pmatrix}2\\5\\4\end{pmatrix}\pm\frac{3}{2}\begin{pmatrix}3\\2\\1\end{pmatrix}$	Attempt $\begin{pmatrix}2\\5\\4\end{pmatrix}\pm k\begin{pmatrix}3\\2\\1\end{pmatrix}$ from their $k\sqrt{14}$	M1
	$(6.5, 8, 5.5), (-2.5, 2, 2.5)$	A1: One correct point A1: Both correct points Allow position vectors for one or both points.	A1A1
			(5)

(c) Way 2	$\dfrac{\lvert 2(-4+3\lambda)-(1+2\lambda)+2(2+\lambda)-7\rvert}{\sqrt{2^2+1^2+2^2}}$	M1: Uses a correct distance formula for *L* and *Π* A1: Correct expression	M1A1
	$\dfrac{\lvert 2(-4+3\lambda)-(1+2\lambda)+2(2+\lambda)-7\rvert}{\sqrt{2^2+1^2+2^2}}=3$ $\Rightarrow -8+6\lambda-1-2\lambda+4+2\lambda-7=\pm 9 \Rightarrow \lambda=...$	Sets distance = 3 and solves to find at least one value for λ	M1
	$\lambda=\dfrac{1}{2}, 3.5 \to (6.5,\ 8,\ 5.5),\ (-2.5,\ 2,\ 2.5)$	A1: One correct point A1: Both correct points Allow position vectors for one or both points.	A1A1
			(5)
			Total 13

Notes

(a) Way 2 : From L , $x = \dfrac{3y-11}{2}$, $x = 3z - 10$; $y = \dfrac{2x+11}{3}$, $y = 2z - 3$;

$z = \dfrac{x+10}{3}$, $z = \dfrac{y+3}{2}$.

(b) M1 : Attempt cross product between the normal and direction vector. Allow one slip.

$$n \times d = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \\ 7 \end{pmatrix}.$$

M1A1 : Complete method to find sin θ

$$|\, n \times d \,| = |n|.|d|.\sin\theta \Rightarrow \sqrt{90} = \sqrt{9}\sqrt{14}\sin\theta$$

M1A1 : As in the main scheme.

Question Number	Scheme	Notes	Marks
5(a)	$\int x\sin^n x\,dx = \int x\sin^{n-1}x\sin x\,dx$		B1
	$\left(u = x\sin^{n-1}x, \quad \frac{dv}{dx} = \sin x\right)$		
	$\frac{du}{dx} = \sin^{n-1}x + (n-1)x\sin^{n-2}x\cos x$		B1
	$I_n = \left[-x\sin^{n-1}x\cos x\right]_0^{\frac{\pi}{2}} - \int -\cos x\left(\sin^{n-1}x + (n-1)x\sin^{n-2}x\cos x\right)dx$		M1A1
	$I_n = \int \cos x\sin^{n-1}x\,dx + (n-1)\int x\cos^2 x\sin^{n-2}x\,dx$		
	$= \left[\frac{\sin^n x}{n}\right] + \ldots$		**d**M1
	$= \ldots + (n-1)\int x\sin^{n-2}x\,dx - (n-1)\int x\sin^n x\,dx$		**d**M1
	$I_n = \frac{1}{n} + (n-1)I_{n-2} - (n-1)I_n \Rightarrow I_n = \frac{1}{n^2} + \frac{(n-1)}{n}I_{n-2}$ *		A1*
			(7)
(b)	$I_4 = \frac{1}{4^2} + \frac{3}{4}I_2$		M1
	$I_4 = \frac{1}{4^2} + \frac{3}{4}\left(\frac{1}{2^2} + \frac{1}{2}I_0\right) = \frac{1}{4^2} + \frac{3}{4}\left(\frac{1}{2^2} + \frac{1}{2}\left(\frac{\pi^2}{8}\right)\right)$		M1
	$= \frac{1}{4} + \frac{3}{64}\pi^2$		A1
			(3)
			Total 10

Notes

Part (a)

B1 : Identifies parts with correct choice for u and dv/dx.

B1 : differentiates u by the product rule or integrates dv/dx by parts

M1A1 : correct application of parts dependent on first B1.

dM1 : integrating a term of the form $\sin^k x \cos x$

dM1 : expressing the integral in terms of I_n and I_{n-2}

A1 : correct final answer. **You can condone the occasional missing x, dx and limits along the way.**

Note also that I_2 may be found by direct integration.

Part (a) Way 2

$\int x \sin^n x \, dx = \int x \sin x \, . \, \sin^{n-1} x \, dx$ $\quad (u = \sin^{n-1} x , \; dv/dx = x \sin x)$ B1

$\therefore \; v = - x \cos x + \sin x$ B1

$I_n = [-x \cos x \sin^{n-1} x + \sin^n x] - \int (-x \cos x + \sin x)(n - 1)\sin^{n-2} x \cos x \, dx$ M1 A1

$= 1 - (n - 1)\int -x \sin^{n-2} x \cos^2 x + \sin^{n-1} x \cos x \, dx$

$= 1 - (n - 1) [-I_{n-2} + I_n] - (n - 1) \left[\frac{\sin^n x}{n}\right]$ dM1 dM1

$= 1 - (n - 1) [I_n - I_{n-2}] - (n - 1)(1/n)$

$= 1 - (n - 1) [I_n - I_{n-2}] - 1 + 1/n$

$= 1/n - (n - 1) [I_n - I_{n-2}] \Rightarrow I_n = 1/n^2 + \left(\frac{n-1}{n}\right) I_{n-2}.$ A1*

Part (b)

M1 : Expresses I_4 in terms of I_2

M1 : Expresses I_2 in terms of I_0 where I_0 has been evaluated using integration

A1 : correct final answer.

Question Number	Scheme	Notes	Marks
6	$x = t\cos t, \quad y = t\sin t, \quad 0 \le t \le 2\pi$		
(a)	$\dfrac{dx}{dt} = \cos t - t\sin t$	Correct derivative	B1
	$\dfrac{dy}{dt} = t\cos t + \sin t$	Correct derivative	B1
	$S = \int \sqrt{(\cos t - t\sin t)^2 + (t\cos t + \sin t)^2}\ dt$	Use of a correct formula with their derivatives	M1
	$= \int_0^{2\pi} \sqrt{1+t^2}\,dt^*$	M1: Expands to give at least 4 terms and collects terms A1: Obtains printed answer with no errors seen	M1A1
			(5)
(b)	$t = \sinh\theta \Rightarrow S = \int \sqrt{1+\sinh^2\theta}\cosh\theta\, d\theta$	Full substitution	M1
	$= \int \cosh^2\theta\ d\theta$	Correct expression	A1
	$= \dfrac{1}{2}\int (1+\cosh 2\theta)\, d\theta$	Use of cosh 2θ = 2 cosh²θ ± 1 or use of the exponential definition of cosh θ	M1
	$= \dfrac{1}{2}\left[\dfrac{1}{2}\sinh 2\theta + \theta\right](+c)$	Correct integration	A1
	$= \dfrac{1}{2}\left[\dfrac{1}{2}\sinh\left(2\operatorname{arsinh}(2\pi)\right) + \operatorname{arsinh}(2\pi) - 0\right]$	Correct use of correct limits. Dependent on both previous method marks.	ddM1
	$= \pi\sqrt{1+4\pi^2} + \dfrac{1}{2}\ln\left(2\pi + \sqrt{1+4\pi^2}\right)$	A1: $\pi\sqrt{1+4\pi^2}$ A1: $\dfrac{1}{2}\ln\left(2\pi + \sqrt{1+4\pi^2}\right)$	A1A1
			(7)
			Total 12

Notes

Part (a) : 2nd M1 : No need to see middle terms of the expansions.

A1 : Must have limits and the "dt" term

Part (b) : 2nd A1 : $= (1/4)\left[\tfrac{1}{2}e^{2\theta} - \tfrac{1}{2}e^{-2\theta} + 2\theta\right] (+ c)$

Question Number	Scheme	Notes	Marks
7	$\mathbf{A}=\begin{pmatrix}-2 & -4 & 2\\ -2 & 1 & 2\\ 4 & 2 & 5\end{pmatrix}$		
(a)	$\det(\mathbf{A}-\lambda\mathbf{I})=\begin{vmatrix}-2-\lambda & -4 & 2\\ -2 & 1-\lambda & 2\\ 4 & 2 & 5-\lambda\end{vmatrix}=0$	$\det(\mathbf{A}-\lambda\mathbf{I})=0$	M1
	$\mathrm{f}(\lambda)=(-2-\lambda)\left[(1-\lambda)(5-\lambda)-4\right]+4\left[-2(5-\lambda)-8\right]+2\left[-4-4(1-\lambda)\right]=0$ M1: Correct characteristic equation attempt		M1
	$\mathrm{f}(6)=-8-24+32=0$ so **6 is an eigenvalue**	Shows 6 is an eigenvalue – can score from factorisation of CE	B1
	$\mathrm{f}(\lambda)=\lambda^3-4\lambda^2-27\lambda+90$	Correct cubic	A1
	$(\lambda-6)(\lambda-3)(\lambda+5)=0\Rightarrow\lambda=3,-5$	A1: $\lambda=3$ or $\lambda=-5$ A1: $\lambda=3$ and $\lambda=-5$	A1A1
			(6)
(b)	$\begin{pmatrix}-2 & -4 & 2\\ -2 & 1 & 2\\ 4 & 2 & 5\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=6\begin{pmatrix}x\\ y\\ z\end{pmatrix}$ or $\begin{pmatrix}-8 & -4 & 2\\ -2 & -5 & 2\\ 4 & 2 & -1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$ Correct statement for eigenvalue 6. Allow equivalent statements.		M1
	$-2x-4y+2z=6x$ $-2x+y+2z=6y$ $4x+2y+5z=6z$	At least 2 correct equations	A1
	$\begin{pmatrix}1\\ 6\\ 16\end{pmatrix}$	Correct eigenvector (any multiple)	A1
			(3)
(c)	$\begin{pmatrix}-2 & -4 & 2\\ -2 & 1 & 2\\ 4 & 2 & 5\end{pmatrix}\begin{pmatrix}2+5\lambda\\ 1+3\lambda\\ -1+2\lambda\end{pmatrix}=\ldots$	Multiplies parametric form or position and direction by **A** Allow one slip	M1
	$\begin{pmatrix}-10-18\lambda\\ -5-3\lambda\\ 5+36\lambda\end{pmatrix}$	A1: $-10\mathbf{i}-5\mathbf{j}+5\mathbf{k}$ oe A1: $-18\mathbf{i}-3\mathbf{j}+36\mathbf{k}$ oe	A1A1
	$\dfrac{x+10}{-18}=\dfrac{y+5}{-3}=\dfrac{z-5}{36}$	M1: Correct cartesian form for their vectors A1: cao (oe)	M1A1
			(5)
			Total 14

Notes

Part (a) : do not need to see " = 0 " in any of the working if the final roots are obtained. Otherwise it needs to seen on the characteristic equation.

2nd M1 : May expand along any row or any column.

Appendix

Question 2(a)

Way 3	$y=\mathrm{e}^{\operatorname{arcosh}x}\Rightarrow\cosh(\ln y)=x$		
	$\frac{\sinh(\ln y)}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=1$	Correct implicit attempt	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y}{\sinh(\ln y)}=\frac{\mathrm{e}^{\operatorname{arcosh}x}}{\sinh(\operatorname{arcosh}x)}$	Correct derivative	A1
	$\frac{\cosh(\ln y)}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+\sinh(\ln y)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}y}{\mathrm{d}x}$		M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{\sinh(\ln y)}\left(\frac{y}{\sinh(\ln y)}-\frac{\cosh(\ln y)}{y}\frac{y^2}{\sinh^2(\ln y)}\right)$		
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{y}{\sinh^2(\ln y)}(1-\coth(\ln y))=\frac{\mathrm{e}^{\operatorname{arcosh}x}}{\sinh^2(\operatorname{arcosh}x)}(1-\coth(\operatorname{arcosh}x))$		
	$=\frac{\mathrm{e}^{\operatorname{arcosh}x}}{x^2-1}\left(1-\frac{x}{\sqrt{x^2-1}}\right)$	A1: $\frac{\mathrm{e}^{\operatorname{arcosh}x}}{x^2-1}$ or $-\frac{x\,\mathrm{e}^{\operatorname{arcosh}x}}{(x^2-1)^{\frac{3}{2}}}$ A1: Correct second derivative	A1A1

Way 4	y = $\mathrm{e}^{\operatorname{arcosh}x}$ = x + $\sqrt{(x^2-1)}\Rightarrow\frac{\mathrm{dy}}{\mathrm{dx}}=$		M1
	$\frac{\mathrm{dy}}{\mathrm{dx}}=1+x.(x^2-1)^{-1/2}$	Correct derivative	A1
	$\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}^2}=x.(-x)(x^2-1)^{-3/2}+(x^2-1)^{-1/2}$	Correct use of product rule or quotient rule	M1
	$=\frac{-x^2}{(x^2-1)^{3/2}}+(x^2-1)^{-1/2}$	A1: $\frac{-x^2}{(x^2-1)^{3/2}}$ or $(x^2-1)^{-1/2}$ A1 : Correct second derivative	A1A1

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