Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE
In Mathematics (6665) Paper 1
Core Mathematics 3

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Question	Scheme		Marks
1	**Way 1** $\require{enclose}x^2+0x-4\enclose{longdiv}{4x^3-6x^2-18x+20}$ with quotient $4x-6$; $\underline{4x^3+0x^2-16x}$; $-6x^2-2x+20$; $\underline{-6x^2+0x+24}$; $-2x-4$	**Way 2** $x+2\enclose{longdiv}{4x^3-6x^2-18x+20}$ with quotient $4x^2-14x+10$; $\underline{4x^3+8x^2}$; $-14x^2-18x+20$; $\underline{-14x^2-28x}$; $10x+20$; $\underline{10x+20}$	M1 A1
	$\dfrac{4x^3-6x^2-18x+20}{x^2-4}$ $\equiv 4x-6+\dfrac{-2x-4}{(x+2)(x-2)}$	$x-2\enclose{longdiv}{4x^2-14x+20}$ with quotient $4x-6$; $\underline{4x^2-8x^2}$; $-6x+20$; $\underline{-6x+12}$; -2	M1
	$\equiv 4x-6-\dfrac{2}{(x-2)}$		A1
			(4)
Way 3	$4x^3-6x^2-18x+20\equiv(ax+b)(x^2-4)+c(x+2)$ o.e.		1st M1
	Either substitutes/ and or equates coefficients to find a value for *a*, *b* or *c*		2nd M1
	One of $a=4,\ b=-6,\ c=-2$		1st A1
	All of $a=4,\ b=-6,\ c=-2$		2nd A1
			(4)
			(4 marks)

Notes

Way 1

M1 Divides $4x^3-6x^2-18x+20$ by x^2-4 to get a linear quotient and a linear remainder.
To award this look for a minimum of the following

$$x^2(+0x)-4\enclose{longdiv}{4x^3-6x^2-18x+20}\quad \text{quotient } \mathbf{4x+A}$$
$$\underline{4x^3+0x^2-16x}$$
$$(Cx)+D$$

A1 Quotient = $4x-6$ and Remainder = $-2x-4$

M1 Writes their expression in the appropriate form. Allow a slip with sign if the intention is clear.
$\left(\dfrac{4x^3-6x^2-18x+20}{x^2-4}\right)\equiv \text{Their Linear Quotient}+\dfrac{\text{Their linear remainder}}{x^2-4}$ and factorises x^2-4 into $x-2\quad x+2$. This may be in one line ‘$\text{Their Linear Quotient}+\dfrac{\text{Their linear remainder}}{(x-2)(x+2)}$’

This may be seen as $\text{Their Linear Quotient}+\dfrac{\text{Their linear remainder}}{x^2-4}$ followed by writing separately $\dfrac{\text{Their linear remainder}}{(x-2)(x+2)}$

Question	Scheme	Marks

A1 All values correct $a = 4,\ b = -6$ and $c = -2$ or writes $4x - 6 - \dfrac{2}{(x-2)}$

Way 2

M1 Divides $4x^3 - 6x^2 - 18x + 20$ by $(x+2)$ to get a quadratic quotient and a constant remainder.

To award this look for a minimum of the following

$$\begin{array}{r l} & \boldsymbol{4x^2 + Ax + B} \\ x+2 \big) & 4x^3 - 6x^2 - 18x + 20 \\ & \underline{4x^3 + 0x^2 - 16x} \\ & \qquad \boldsymbol{D} \end{array}$$

A1 Quotient = $4x^2 - 14x + 10$ and Remainder = 0

M1 Divides their ‘ $4x^2 - 14x + 10$ ‘ by $(x-2)$ to get a linear quotient and a constant remainder.

To award this look for a minimum of the following

$$\begin{array}{r l} & 4\boldsymbol{x} + \boldsymbol{E} \\ \boldsymbol{x} - 2 \big) & 4\boldsymbol{x}^2 - 14\boldsymbol{x} + 20 \\ & \underline{4\boldsymbol{x}^2 - 8\boldsymbol{x}^2} \\ & \qquad \boldsymbol{F} \end{array}$$

A1 All values correct $a = 4,\ b = -6$ and $c = -2$ or writes $4x - 6 - \dfrac{2}{(x-2)}$

Way 3

1st M1 Forms the correct identity by muliplying through by $x^2 - 4$

2nd M1 Either equates coefficents and/or substitutes a value for x in an attempt to find a value for either a, b or c

1st A1 At least one correct value $a = 4,\ b = -6$ or $c = -2$

2nd A1 All values correct $a = 4,\ b = -6$ and $c = -2$ or writes $4x - 6 - \dfrac{2}{(x-2)}$

Question	Scheme	Marks
2(i)(a)	$y=\dfrac{(2x-1)^3}{(3x-2)} \Rightarrow \dfrac{dy}{dx}=\dfrac{\alpha(3x-2)(2x-1)^2-\beta(2x-1)^3}{(3x-2)^2}$ where $\alpha>0$ and $\beta>0$ OR $y=(2x-1)^3(3x-2)^{-1} \Rightarrow \dfrac{dy}{dx}=\alpha(2x-1)^2(3x-2)^{-1}-\beta(3x-2)^{-2}(2x-1)^3$ where $\alpha>0$ and $\beta>0$	M1
	$y=\dfrac{(2x-1)^3}{(3x-2)} \Rightarrow \dfrac{dy}{dx}=\dfrac{(3x-2)\times 6(2x-1)^2-(2x-1)^3\times 3}{(3x-2)^2}$ OR $y=(2x-1)^3(3x-2)^{-1} \Rightarrow \dfrac{dy}{dx}=3\times 2\times(2x-1)^2(3x-2)^{-1}-3(3x-2)^{-2}(2x-1)^3$	A1
	$\dfrac{dy}{dx}=\dfrac{(2x-1)^2(18x-12-6x+3)}{(3x-2)^2}$	M1
	$\dfrac{dy}{dx}=\dfrac{(2x-1)^2(12x-9)}{(3x-2)^2}$ or $\dfrac{3(2x-1)^2(4x-3)}{(3x-2)^2}$ or $(2x-1)^2(12x-9)(3x-2)^{-2}$ o.e	A1
		(4)
(i)(b)	$\dfrac{dy}{dx}\geqslant 0 \Rightarrow$ either their $(12x-9)\geqslant 0 \Rightarrow x\geqslant \ldots$ or $2x-1=0 \Rightarrow x=\ldots$	M1
	$x=\dfrac{1}{2},\ x\geqslant\dfrac{3}{4}$	
		(2)
(ii)	**<u>Way 1</u>** $y=\ln(1+\cos 2x) \Rightarrow \dfrac{dy}{dx}=\dfrac{\pm\lambda\sin 2x}{1+\cos 2x}$	M1
	$\dfrac{dy}{dx}=\dfrac{-2\sin 2x}{1+\cos 2x}$	A1
	$\dfrac{dy}{dx}=\dfrac{-4\sin x\cos x}{2\cos^2 x}$	M1
	$\dfrac{dy}{dx}=-2\tan x$	A1
		(4)
	<u>Way 2 Careful with the order of the Method marks</u> $y=\ln(1+\cos 2x) \Rightarrow y=\ln\left(2\cos^2 x\right)$	2nd M1
	$\Rightarrow \dfrac{dy}{dx}=\dfrac{-4\sin x\cos x}{2\cos^2 x}$	1st M1 1st A1
	$\Rightarrow \dfrac{dy}{dx}=-2\tan x$	2nd A1
		(4)
		(10 marks)

Question	Scheme	Marks

Notes

(i)(a)

M1 Uses either quotient rule or product rule to the achieve the correct form.

A1 Correct unsimplified or simplified expression for $\frac{dy}{dx}$

M1 Quotient Rule: Takes out a common factor of at least $(2x-1)^2$ from the numerator, allow numerical slips but not algebraic slips, as long as the intention is clear.
Product Rule: Combines as a single fraction with a correct numerator allow numerical slips in the denominator **AND** takes out a common factor of at least $(2x-1)^2$ from the numerator, allow numerical slips as long as the intention is clear.

A1 $\frac{dy}{dx}=\frac{(2x-1)^2(12x-9)}{(3x-2)^2}$ o.e. such as $\frac{dy}{dx}=(2x-1)^2(12x-9)(3x-2)^{-2}$

(i)(b)

M1 Sets their $\frac{dy}{dx}\geqslant 0$ or $\frac{dy}{dx}=0$ or $\frac{dy}{dx}>0$ and proceeds correctly to find a critical value for their $\frac{dy}{dx}=0$ provided that their numerator is at least a quadratic. Condone setting equal to 0 and cancelling a common factor first as long as a value for *x* is achieved.

A1 $x=\frac{1}{2}, x\geqslant\frac{3}{4}$

(ii)

Way 1

M1 Differentiates to a form $\frac{dy}{dx}=\frac{\pm\lambda\sin 2x}{1+\cos 2x}$

A1 A correct derivative $\frac{dy}{dx}=\frac{-2\sin 2x}{1+\cos 2x}$

M1 Uses the correct double angle identities sin2x = 2sinxcoss $\sin 2x=2\sin x\cos x$ and $\cos 2x=2\cos^2 x-1$
If uses $\cos 2x=\cos^2 x-\sin^2 x$ do not award this mark until either $1-\sin^2 x$ becomes $\cos^2 x$ or $1+\cos^2 x-\sin^2 x$ becomes $2\cos^2 x$ in the denominator

A1 Achieves $\frac{dy}{dx}=-2\tan x$ with no incorrect work seen

Way 2 Careful with the order of the Method marks

2nd M1 Replaces $1+\cos 2x$ with $2\cos^2 x$

1st M1 Differentiates $y=\ln(f(x))\Rightarrow\frac{dy}{dx}=\frac{f'(x)}{f(x)}$ with their $f(x)$

1st A1 Correct derivative $\frac{dy}{dx}=\frac{-4\sin x\cos x}{2\cos^2 x}$

2nd A1 Achieves $\frac{dy}{dx}=-2\tan x$ with no errors or omissions

Question	Scheme	Marks
3.(a)	$R=\sqrt{65}$	B1
	$\tan\alpha=\frac{8}{1}\Rightarrow\alpha=\text{awrt }82.87°$	M1A1
		(3)
(b)	$13+\frac{'R'}{10}=13.81(°\text{C})$	M1 A1
		(2)
(c)	$\cos(15t+82.87)°=-\frac{5}{\sqrt{65}}$	M1
	$15t+82.87=128.33\Rightarrow t=3.03$	A1
	$15t+82.87=(360-128.33)\Rightarrow t=...(9.92)$	dM1
	Both times correct $03:02$ and $09:55$	A1
		(4)
		(9 marks)

Notes

(a)

B1 Sight of $R=\sqrt{65}$. Condone $R=\pm\sqrt{65}$

(Do not allow decimals for this mark e.g. 8.06 but remember to isw after $\sqrt{65}$)

M1 For sight of $\tan\alpha=\pm\frac{8}{1}\Rightarrow\alpha=...$ or $\tan\alpha=\pm\frac{1}{8}\Rightarrow\alpha=...$

Condone either $\sin\alpha=8, \cos\alpha=1\Rightarrow\tan\alpha=8\Rightarrow\alpha=...$ or using θ instead of α

If R is found first accept only $\sin\alpha=\pm\frac{8}{R}, \cos\alpha=\pm\frac{1}{R}\Rightarrow\alpha=...$

A1 $\alpha=\text{awrt }82.87°$ Answer in radians (1.45) are A0

(b)

M1 For their $13+\frac{'R'}{10}$

A1 awrt $13.81(°\text{C})$

(c)

M1 Sets $13+\frac{'R'}{10}\cos(15t+'\alpha')=12.5$ with their values for R and α and rearranges to achieve $\cos(15t+'\alpha')=k$ where $-1<k<1$. Condone starting from $13+'R'\cos(15t+'\alpha')=12.5$

Allow for $13+\frac{'R'}{10}\cos(\theta+'\alpha')=12.5$ leading to $\cos(\theta+'\alpha')=k$

A1 For one correct value of t. Accept either awrt $t=3.03$ or $t=9.92$

dM1 Dependent on the first method mark. For the correct method to find a second value for t.

Look for $15t\pm"82.87"=(360-"128.33")\Rightarrow t=..$

A1 Both times correct $03:02$ and $09:55$

Note 1: Starting with $13+'R'\cos(15t+'\alpha')=12.5$ can score a maximum of M1 A0 dM1 A0. It should lead to $t=0.712$ and $t=12.238$

Note 2: Alpha in radians can score a maximum of M1 A0 dM1 A0. It should lead to $t=0.012$ and $t=0.213$

Question	Scheme	Marks
4(a)	$\text{ff}(x)=\dfrac{2\left(\dfrac{2x+5}{x-2}\right)+5}{\left(\dfrac{2x+5}{x-2}\right)-2}$	M1
	$\text{ff}(x)=\dfrac{2(2x+5)+5(x-2)}{(2x+5)-2(x-2)}=x$	dM1, A1
		(3)
(b)	Sets $\text{fg}(a)=\text{g}(a)\Rightarrow\dfrac{2\ln a+5}{\ln a-2}=\ln a$	M1
	$\Rightarrow(\ln a)^2-4\ln a-5=0$	A1
	$\Rightarrow(\ln a-5)(\ln a+1)=0\Rightarrow\ln a=5,-1$	dM1
	$\Rightarrow a=e^5$, ~~e^{-1}~~	A1
		(4)
		(7 marks)

Notes

(a)

M1 Attempts $\text{ff}(x)=\dfrac{2\left(\dfrac{2x+5}{x-2}\right)+5}{\left(\dfrac{2x+5}{x-2}\right)-2}$ condoning sign and numerical slips

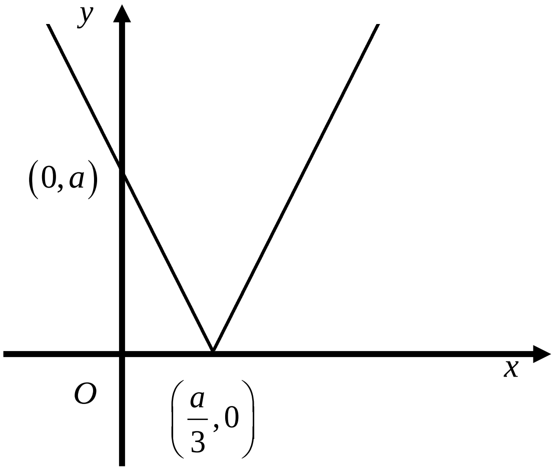
Alternatively writes $\text{f}(x)=2+\dfrac{9}{x-2}$ and hence $\text{ff}(x)=2+\dfrac{9}{2+\dfrac{9}{x-2}-2}$

dM1 Correct processing to obtain a single fraction of the form $\dfrac{p}{q}$. Achieved by either,

- multiplying all terms in both the numerator and the denominator by $(x-2)$
- attempting to write both the numerator and denominator as a single fraction followed by the multiplication of the numerator by an inverted denominator to obtain a single fraction of the form $\dfrac{p}{q}$
- attempting to write both the numerator and denominator as a single fraction followed by the cancelling of the same denominators to obtain a single fraction of the form $\dfrac{p}{q}$

A1 $\text{ff}(x)=x$

Question	Scheme	Marks
(b) **Condone the use of x instead of a for the first 3 marks** M1 Sets $\mathrm{fg}(a)=\mathrm{g}(a) \Rightarrow \frac{2\ln a+5}{\ln a-2}=\ln a$ A1 Correct simplified quadratic equation in ln*a*. Note $\ln^2 a-4\ln a-5=0$ is fine Condone poor notation $\ln a^2-4\ln a-5=0$ if an attempt to solve as a quadratic equation for ln*a* dM1 Correct attempt to find a value for ln*a* by solving 3TQ in ln*a* A1 $a=\mathrm{e}^5$ only, the solution $a=\mathrm{e}^{-1}$ must be rejected. A0 for $x=\mathrm{e}^5$		

Question	Scheme	Marks
5(a)		
	V shape on the +ve x axis	B1
	$(0,a)$ and $\left(\frac{a}{3},0\right)$	B1
		(2)
(b) Way 1	Substitutes $x=4$ into $\lvert 3x-a\rvert=\frac{1}{2}x+2 \Rightarrow \lvert 3\times 4-a\rvert=\frac{1}{2}\times 2+2$	M1
	Solves $12-a=\pm 4 \Rightarrow a=8,16$	dM1 A1
		(3)
Way 2	Substitutes x = 4, squares both sides and forms a 3TQ in a $(3x-a)^2=\left(\frac{1}{2}x+2\right)^2 \Rightarrow 4a^2-96a+512=0$	M1
	Solves 3TQ to find values for a	dM1
	$a=8,16$	A1
		(3)
	Sets $\pm(3x-a)=\frac{1}{2}x+2$ and substitutes $x=4$	M1
	Rearranges an equation to find a value for a	dM1
	$a=8,16$	A1
		(3)
(c)	Chooses larger value of 'a' solves $3x-a=\frac{1}{2}x+2 \Rightarrow x=...$	M1
	$x=\frac{36}{5}$ or 7.2	A1
		(7 marks)

Notes

(a)

B1 For a V shape on the positive x - axis in quadrants one and two. It must clearly pass through the y - axis

B1 Points $(0, a)$ and $\left(\frac{a}{3}, 0\right)$ both lie on the graph. Allow a on the y – axis and $\frac{a}{3}$ on the x - axis

Question	Scheme	Marks
(b) **<u>Way 1</u>** M1 Scored for setting $\|3x-a\|=\frac{1}{2}x+2$ and substituting in $x=4$ Implied by $\|12-a\|=4$, or $12-a=4$ or $12-a=-4$ dM1 An acceptable method of finding one value of a A1 Both $a=8,16$		
<u>Way 2</u> M1 Substitutes $x = 4$ and squared both sides in either order to form a 3TQ in a dM1 Solve their 3TQ to find a value for a A1 Both $a=8,16$		
<u>Way 3 (See if Way 1 is more relevant)</u> M1 Sets $3x-a=\frac{1}{2}x+2$ and either $-3x+a=\frac{1}{2}x+2$ or $3x-a=-\frac{1}{2}x-2$ and substitutes in $x=4$ dM1 Rearranges an equation to find a value for a **<u>Note:</u>** If they rearrange to find $a = \ldots$ then substitutes in $x = 4$ both M's awarded at this point. A1 Both $a=8,16$		
(c) M1 Chooses the larger value of 'their a' and solves $3x-a=\frac{1}{2}x+2 \Rightarrow x=..$ A1 $x=\frac{36}{5}$ or 7.2 **<u>Note:</u>** If they use both values of their a then M1 and/or A1 is awarded when the largest value of x following their values of a is selected.		

Question	Scheme	Marks
6.(a)	$f'(x)=\dfrac{d\left(x^2-x-12\right)}{dx}\times\ln\left(x+3\right)+\dfrac{d\left(\ln\left(x+3\right)\right)}{dx}\times\left(x^2-x-12\right)$	M1
	$f'(x)=\ln(x+3)\left(2x-1\right)+\left(x^2-x-12\right)\times\dfrac{1}{x+3}$ or $f'(x)=\ln(x+3)\left(2x-1\right)+x-4$	A1
		(2)
(b)	$\ln(x+3)\left(2x-1\right)+\left(x-4\right)(x+3)\times\dfrac{1}{x+3}=0$	M1
	$\Rightarrow\ln(x+3)\left(2x-1\right)+\left(x-4\right)=0$	
	$\Rightarrow 2x\ln(x+3)+x=4+\ln(x+3)$	dM1
	$\Rightarrow x\left(2\ln(x+3)+1\right)=4+\ln(x+3)\Rightarrow x=\dfrac{\ln(x+3)+4}{2\ln(x+3)+1}$ *	A1 *
		(3)
(c)	Substitutes $x_0=1$ in $x=\dfrac{4+\ln(x+3)}{2\ln(x+3)+1}\Rightarrow x_1=\dfrac{4+\ln(4)}{2\ln(4)+1}=\text{awrt }1.428$	M1, A1
	$x_2=\text{awrt }1.38\left(0\right),\quad x_3=\text{awrt }1.385$	A1
		(3)
(d)	$k=\pm2\times f(0)\Rightarrow k=24\ln3$	M1 A1
		(2)
		(10 marks)

Notes

(a)

M1 Applies correctly the product rule to $f(x)=\left(x^2-x-12\right)\ln(x+3)$. If they state $u=...\Rightarrow u'=...$ and $v=...\Rightarrow v'=...$ follow through on their $u'=...\quad v'=...$ as long and u and v are correct.

A1 $f'(x)=\ln(x+3)\left(2x-1\right)+\left(x^2-x-12\right)\times\dfrac{1}{x+3}$ correct un-simplified or simplified.
Award as soon as a correct version is seen, isw
Must have correct notation e. g. $\ln x+3$ is A0

(b)

M1 Sets $f'(x)=0$ and attempts to factorise $\left(x^2-x-12\right)$ which may have already been done in part (a)

dM1 Rearranges to an equation of the form $\pm..\boldsymbol{x}\ln(x+3)\pm..\boldsymbol{x}=\pm..\pm..\ln(x+3)$

A1* Factorises and divides to form the given equation with no errors or omissions or poor notation

Question	Scheme	Marks
(c) M1 Substitutes $x_0 = 1$ in $x = \frac{4+\ln(x+3)}{2\ln(x+3)+1}$. Implied by $x_1 = \frac{4+\ln(4)}{2\ln(4)+1}$ or awrt 1.4 A1 awrt 1.428 A1 $x_2 =$ awrt $1.38(0)$, $x_3 =$ awrt 1.385 (d) M1 $k = \pm 2 \times \mathrm{f}(0)$ This mark can be implied by seeing k = awrt 26.4 with no working seen A1 $k = 24\ln 3$		

Question	Scheme	Marks
7(a) **Way 1**	$2\cos(A-30°)\sec A \equiv 2\left(\cos A\cos 30° + \sin A\sin 30°\right)\times \sec A$ $\dfrac{2\left(\cos A\cos 30° + \sin A\sin 30°\right)}{\cos A} \Rightarrow ... \Rightarrow \tan x + k$ $\equiv \tan A + \sqrt{3}$ **cso**	M1 dM1 A1*
Way 2	$2\cos(A-30°)\sec A \equiv \tan A + k \Rightarrow 2\cos(A-30°) \equiv \sin A + k\cos A$ $\Rightarrow 2\left(\cos A\cos 30° + \sin A\sin 30°\right) \equiv \sin A + k\cos A$ $\Rightarrow \sqrt{3}\cos A + \sin A \equiv \sin A + k\cos A$ Hence true and $k = \sqrt{3}$	M1 dM1 A1 **(3)**
(b)	$2\cos(x-30°) = \sec x$ and $2\cos(x-30°)\sec x \equiv \tan A + \sqrt{3}$	
Way 1	**For example** **1)** $2\cos(x-30°)\sec x = \sec^2 x \Rightarrow \tan x + '\sqrt{3}' = \sec^2 x$ OR **2)** $\dfrac{\tan x + '\sqrt{3}'}{\sec x} = \sec x \Rightarrow \tan x + '\sqrt{3}' = \sec^2 x$ OR **3)** $2\cos(x-30°) \equiv \left(\tan A + '\sqrt{3}'\right)\cos x \Rightarrow \sec x = \left(\tan A + '\sqrt{3}'\right)\cos x$ $\Rightarrow \tan x + '\sqrt{3}' = \sec^2 x$ OR **4)** $2\cos(x-30°) = \sec x \Rightarrow 2\left(\cos x\cos 30° + \sin x\sin 30°\right) = \sec x$ $\Rightarrow 2\cos^2 x\cos 30° + 2\sin x\cos x\sin 30° = 1$ $\Rightarrow \cos^2 x\sqrt{5} + \sin x\cos x = 1 \Rightarrow \sqrt{5} + \tan x = \sec^2 x$ $\Rightarrow \tan^2 x - \tan x + 1 - \sqrt{3} = 0$ $\tan x = \dfrac{1 \pm \sqrt{1 - 4\left(1-\sqrt{3}\right)}}{2} = \text{awrt } 1.49, \quad -0.49 \Rightarrow x = ...$ $x = \text{awrt } 56.2°, -26.2°$	M1 M1A1 M 1 A1 **(5)** **(8 marks)**
Alt (b)	$2\cos(x-30°) = \sec x \Rightarrow 2\left(\cos x\cos 30° + \sin x\sin 30°\right) = \sec x$ $\Rightarrow 2\cos^2 x\cos 30° + 2\sin x\cos x\sin 30° = 1$ $\Rightarrow \sqrt{3}\left(\cos 2A + 1\right) + 1\sin 2A = 2$ $\Rightarrow 2\cos\left(2A - 30°\right) = 2 - \sqrt{3}$ $x = \text{awrt } 56.2°, -26.2°$	M1 M1A1 M1, A1 **(5)**

Question	Scheme	Marks
Notes		

(a)

M1 Uses $\cos(A-30°) \equiv \cos A\cos 30° \pm \sin A\sin 30°$ (Condone a slip with a missing 2)

dM1 Uses $\sec A = \dfrac{1}{\cos A}$ (maybe implied) and divides with an intermediate line to reach $\tan A + k$

A1 Achieves $\tan A + \sqrt{3}$ with no errors or poor notation cso

(b)

M1 Uses part (a) and the equation correctly to reach the form $\tan x + "\sqrt{3}" = \sec^2 x$

M1 Uses $\sec^2 x = \pm 1 \pm \tan^2 x$ to form a quadratic in $\tan x$, does not need to be collected onto one side.

A1 $\tan^2 x - \tan x + 1 - \sqrt{3} = 0$ or equivalent including $\tan^2 x - \tan x - \text{awrt } 0.73 = 0$

M1 Solves quadratic and proceeds to one value for *x*.

A1 $x = \text{awrt } 56.2°, -26.2°$

Question	Scheme	Marks
8 (a)	Substitute $N_A = 7200$ in $N_A = 3000 + 600\mathrm{e}^{0.12t}$	B1
	$\Rightarrow e^{0.12t} = 7$	M1 A1
	$\Rightarrow t = \dfrac{\ln 7}{0.12} = 16.22$ years	M1, A1
		(5)
(b)	Differentiates to achieve $\dfrac{\mathrm{d}N_A}{\mathrm{d}t} = \beta \mathrm{e}^{0.12t}$ $\left[\dfrac{\mathrm{d}N_A}{\mathrm{d}t} = 600 \times 0.12\mathrm{e}^{0.12t}\right]$	M1
	Substitutes $\mathrm{e}^{0.12t} = \dfrac{N_A - 3000}{600}$ into $\dfrac{\mathrm{d}N_A}{\mathrm{d}t} = \beta \mathrm{e}^{0.12t}$ OR Substitutes $600\mathrm{e}^{0.12t} = N_A - 3000$ into $\dfrac{\mathrm{d}N_A}{\mathrm{d}t} = \alpha \times 600\mathrm{e}^{0.12t}$	dM1
	$\Rightarrow \dfrac{\mathrm{d}N_A}{\mathrm{d}t} = 0.12\left(N_A - 3000\right) = 0.12N_A - 360$ or $\dfrac{3}{25}N_A - 360$	A1
		(3)
(c)(i)	$200 = C\mathrm{e}^{k}$ and $500 = C\mathrm{e}^{2k}$	M1
	$\mathrm{e}^{k} = \dfrac{500}{200} \Rightarrow k = \ldots$ or $\mathrm{e}^{-k} = \dfrac{200}{500} \Rightarrow k = \ldots$	dM1
	$k = \ln\left(\dfrac{5}{2}\right)$ * cso	A1*
(ii)	80	B1
		(4)
		(12 marks)

Notes

(a)

B1 $7200 = 3000 + 600\mathrm{e}^{0.12t}$

M1 Moving from $N_A = 3000 + 600\mathrm{e}^{0.12t}$ with a numerical N_A and using the correct order of operations and algebra to achieve an equation $e^{0.12t} = \lambda$ where $\lambda > 0$

A1 Achieves $e^{0.12t} = 7$ from correct work

dM1 Moving from $e^{0.12t} = \lambda$ using lns to $t = \dfrac{\ln \lambda}{0.12}$ where $\lambda > 0$

A1 cso 16.22 years

Note: If they work with $2N_A$ do not award any marks until a value for N_A is substituted. If they achieve the correct answer then all marks can be awarded. If they do not achieve the correct answer B0 M1 A0 dM1 A0.

Special Case: A student who starts with $N_A = 6000$ and achieves $t = 13.41$ years can score a maximum B0, M1, A1 $e^{0.12t} = 5$, M1, A0 for 3 out of 5

Question	Scheme	Marks

(b)

M1 Differentiates to achieves the correct form $\frac{dN_A}{dt} = \beta e^{0.12t}$

dM1 Rearranges the equation $N_A = 3000 + 600e^{0.12t}$ to $e^{0.12t} = ...$ and substitutes into $\frac{dN_A}{dt} = \beta e^{0.12t}$

or rearranges to $600e^{0.12t} = ...$ and substitutes into $\frac{dN_A}{dt} = \alpha \times 600e^{0.12t}$

Alternatively writes $\frac{3}{25}\left(3000 + 600e^{0.12t}\right)$ and substitutes $\frac{dN_A}{dt} = \beta e^{0.12t}$ to achieve $\frac{dN_A}{dt} = \frac{3}{25}\left(3000 + 600e^{0.12t}\right) - \lambda$

A1 $\frac{dN_A}{dt} = 0.12N_A - 360$ or $\frac{3}{25}N_A - 360$ o.e.

(c)(i)

M1 States $200 = Ce^k$ and $500 = Ce^{2k}$

dM1 Uses a correct method to eliminate C from their equations to achieved either $e^k = "\frac{500}{200}" \Rightarrow k = ...$ or $e^{-k} = \frac{200}{500} \Rightarrow k = ...$

A1* Achieves $k = \ln\left(\frac{5}{2}\right)$ with no errors or omissions or poor notation. cso

(c)(ii)

B1 80

Question	Scheme	Marks
9(a) Way 1	$x=\dfrac{1}{1+\cot y}\Rightarrow\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{-1}{(1+\cot y)^2}\times\ldots$ OR $\dfrac{\ldots}{(1+\cot y)^2}$ OR $\ldots(1+\cot y)^{-2}$	M1
	$x=\dfrac{1}{1+\cot y}\Rightarrow\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{-1}{(1+\cot y)^2}\times-\mathrm{cosec}^2 y$ OR $\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{-1\times-\mathrm{cosec}^2 y}{(1+\cot y)^2}$	A1
	$\cot y=\dfrac{1}{x}-1$ or $\cot y=\dfrac{1-x}{x}$	B1
	Uses $\mathrm{cosec}^2 y=1+\cot^2 y$ and $\cot y=\dfrac{1}{x}-1$ to eliminate y and achieve $\dfrac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{f}(x)$	M1
	$\dfrac{\mathrm{d}x}{\mathrm{d}y}=2x^2-2x+1$ *	A1*
		(5)
Way 2	$x=\dfrac{1}{1+\cot y}\Rightarrow\cot y=\dfrac{1}{x}-1$ or $\cot y=\dfrac{1-x}{x}$	B1
	Differentiates $\Rightarrow\ldots\times\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{1}{x^2}$	M1
	$-\mathrm{cosec}^2 y\times\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{1}{x^2}$	A1
	Uses $\mathrm{cosec}^2 y=1+\cot^2 y$ and $\cot y=\dfrac{1}{x}-1$ to eliminate y and achieve $\dfrac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{f}(x)$	M1
	$\dfrac{\mathrm{d}x}{\mathrm{d}y}=2x^2-2x+1$*	A1*
		(5)
Way 3	$x=\dfrac{1}{1+\cot y}\Rightarrow\cot y=\dfrac{1}{x}-1\Rightarrow\tan y=\dfrac{x}{1-x}$	B1
	Differentiates $\Rightarrow\ldots\times\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{(1-x)^2}$	M1
	$\sec^2 y\times\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{(1-x)^2}$	A1
	Uses $\sec^2 y=1+\tan^2 y$ to eliminate y and achieve $\dfrac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{f}(x)$	M1
	$\dfrac{\mathrm{d}x}{\mathrm{d}y}=2x^2-2x+1$*	A1*
		(5)
(b)	$x=\dfrac{1}{1+3}=\dfrac{1}{4}$	B1
		(1)

Question	Scheme	Marks
(c)	Subs their $x=\frac{1}{4}$ into their $\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y}=2x^2-2x+1$ and INVERTS $\frac{\mathrm{d}y}{\mathrm{d}x}=1.6$ or $\frac{8}{5}$	M1 A1 **(2)** **(8 marks)**

(a)

Way 1

M1 Attempts to differentiate using the chain rule to reach a form on the rhs $\frac{1}{(1+\cot y)^2}\times ...$ or quotient rule to reach the form $\frac{...}{(1+\cot y)^2}$ or $...(1+\cot y)^{-2}$

A1 Correct differentiation $\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{-1}{\left(1+\cot y\right)^2}\times -\mathrm{cosec}^2 y$ or $\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{-1\times -\mathrm{cosec}^2 y}{\left(1+\cot y\right)^2}$

B1 States $\cot y=\frac{1}{x}-1$ or $\cot y=\frac{1-x}{x}$

M1 Attempts to use $\mathrm{cosec}^2 y=\pm 1\pm \cot^2 y$ with $\cot y=\frac{1}{x}\pm 1$ to eliminate y and find an expression for $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x only e.g. $\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{1+\left(\frac{1}{x}-1\right)^2}{\left(\frac{1}{x}\right)^2}$

Note: $\cot^2 y=\left(\frac{1}{x}-1\right)^2$ $\mathrm{cosec}^2 y=1+\left(\frac{1}{x}-1\right)^2$ $1+\cot y=\frac{1}{x}$

A1* Achieves the printed answer $\frac{\mathrm{d}x}{\mathrm{d}y}==2x^2-2x+1$ with no errors or omissions

Way 2

B1 States $\cot y=\frac{1}{x}-1$ or $\cot y=\frac{1-x}{x}$

M1 Using implicit differentiation to reach $\Rightarrow ...\times\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{x^2}$

A1 Correct differentiation $-\mathrm{cosec}^2 y\times\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{x^2}$

M1 Attempts to use $\mathrm{cosec}^2 y=\pm 1\pm \cot^2 y$ with $\cot y=\frac{1}{x}\pm 1$ to eliminate y and find an expression for $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x only e.g. $\frac{\mathrm{d}x}{\mathrm{d}y}=-\left(1+\left(\frac{1}{x}-1\right)^2\right)\times -x^2$

A1* Achieves the printed answer $\frac{\mathrm{d}x}{\mathrm{d}y}==2x^2-2x+1$ with no errors or omissions

Question	Scheme	Marks
	Way 3 B1 $\tan y = \dfrac{x}{1-x}$ M1 Using implicit differentiation to reach $\Rightarrow ...\times\dfrac{dy}{dx} = \dfrac{1}{(1-x)^2}$ A1 Correct differentiation $\sec^2 y\times\dfrac{dy}{dx} = \dfrac{1}{(1-x)^2}$ M1 Attempts to use $\sec^2 y = \pm1\pm\tan^2 y$ with $\tan y = \dfrac{x}{1\pm x}$ to eliminate *y* and find an expression for $\dfrac{dx}{dy}$ in terms of *x* only $\dfrac{dx}{dy} = \left(1+\left(\dfrac{x}{x-1}\right)^2\right)\times(1-x)^2$ A1* Achieves the printed answer $\dfrac{dx}{dy} == 2x^2-2x+1$ with no errors or omissions	
	(b) B1 States $x=\dfrac{1}{4}$ o.e. (c) M1 Subs their $x=\dfrac{1}{4}$ into $\Rightarrow\dfrac{dx}{dy} = 2x^2-2x+1$ and INVERTS **Note:** If they do not show you the substitution you will need to check their value of *x*. A1 $\dfrac{dy}{dx} = \dfrac{8}{5}$ or 1.6 cso **Note:** If part (b) $x=\dfrac{3}{4}$ this still leads to $\dfrac{dy}{dx}=\dfrac{8}{5}$ this scores M1 A0	

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