Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE Further Mathematics
Further Pure 2 Paper 6668/01

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Summer 2019
Publications Code 6668_01_1906_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- □ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1.	$m^2+6m+9=0$	
	$(m+3)^2=0,\ m=-3$	M1,A1
	$(\text{CF}=)\ \ (A+Bx)e^{-3x}$	A1
	$\text{PI}:\ y=\lambda e^{2x}$	
	$y'=2\lambda e^{2x}\quad y''=4\lambda e^{2x}$	
	$4\lambda e^{2x}+6\times 2\lambda e^{2x}+9\lambda e^{2x}=e^{2x}$	M1
	$\lambda=\dfrac{1}{25}$	A1
	$(\text{GS:})\ \ y=(A+Bx)e^{-3x}+\dfrac{1}{25}e^{2x}$	A1ft **[6]**

Notes

M1	Form aux equation and attempt to solve (any valid method). Equation need not be shown if CF is correct or complete solution $(m=-3\ \text{twice})$ is shown
A1	Correct solution Value may be seen only in the CF.
A1	Correct CF $y=..$ not needed.
M1	PI of the form shown, differentiated twice and substituted in the equation
A1	$\lambda=\dfrac{1}{25}$
A1ft	A complete solution, follow through their CF and PI. Both M marks must have been earned. Must start $y=...$

Question Number	Scheme	Marks
NB	Question states "Using algebra..." so purely graphical solutions (using calculator?) score 0/6. A sketch and some algebra to find CVs or intersection points can score according to the method used. Where a calculator has been used to solve a cubic (or quadratic), give the marks **only** where the roots are correct.	
2	$6x^2(x-1) < (x+6)x(x-1)^2$	M1
	$x(x-1)\left[6x-(x+6)(x-1)\right] < 0$	
	$x(x-1)(3-x)(2+x) < 0$	M1
	CVs: $0, 1;\ 3, -2$	B1; A1
	$x < -2 \quad 0 < x < 1 \quad x > 3$	A1A1
		[6]

Notes

	The first 4 marks can be awarded if = used	
M1	Multiply through by $x^2(x-1)^2$	
M1	Collect terms and attempt to factorise to obtain 4 linear factors	
B1	CVs 0 and 1 (seen anywhere)	
A1	CVs 3,-2 with no extras	
A1	A1 Any two correct intervals with inequalities (need not be strict)	
A1	A1 Third correct interval and no extras. Must have strict inequalities for all three intervals. Set notation may be used $\cup$ or "or" but not "and"	
ALT 1	$\frac{6}{x-1} - \frac{x+6}{x} < 0$	
	$\frac{6x-(x-1)(x+6)}{x(x-1)} < 0$	M1
	$\frac{(3-x)(2+x)}{x(x-1)} < 0$ etc	M1
M1 **M1**	Collect fractions and attempt the common denominator Factorise the numerator B1A1A1A1 As main scheme Also the first 4 marks can be awarded if = used	
ALT 2 **M1** **M1**	Multiply through by $x^2(x-1)^2$ Obtain a 4 term quartic equation $\pm\left(-x^4+2x^3+5x^2-6x=0\right)$ and attempt to factorise to obtain at least 2 non-zero values for x B1A1A1A1 as main scheme Also the first 4 marks can be awarded if = used	
ALT 3 **M1** **M1**	By drawing sketch of graphs of the 2 fractions (= y) Draw the sketch showing the area where they intersect. Graphs do not need to be labelled 2 vertical asymptotes and 2 intersection points needed. Only award if followed by some algebra to find the x coordinates of the points of intersection. Must obtain a quadratic Solve to $x = ...$ B1A1A1A1 as main scheme If the start is $6x(x-1) < (x+6)x(x-1)$ - no marks available unless separate cases for $x > 1$, $0 < x < 1$ and $x < 0$ are considered. Send to review.	

Question Number	Scheme	Marks
3 (a)	$\dfrac{2}{r(r+1)(r+2)}=\dfrac{1}{r}-\dfrac{2}{r+1}+\dfrac{1}{r+2}$	M1A1A1 (3)
(b)	$r=1 \quad 1-\dfrac{2}{2}+\dfrac{1}{3}$	
	$r=2 \quad \dfrac{1}{2}-\dfrac{2}{3}+\dfrac{1}{4}$	
	$r=3 \quad \dfrac{1}{3}-\dfrac{2}{4}+\dfrac{1}{5}$	
	$r=n-1 \quad \dfrac{1}{n-1}-\dfrac{2}{n}+\dfrac{1}{n+1}$	
	$r=n \quad \dfrac{1}{n}-\dfrac{2}{n+1}+\dfrac{1}{n+2}$	M1
	$\sum_{r=1}^{n}\left(\dfrac{1}{r}-\dfrac{2}{r+1}+\dfrac{1}{r+2}\right)=\left(1-\dfrac{2}{2}+\dfrac{1}{2}+\dfrac{1}{n+1}-\dfrac{2}{n+1}+\dfrac{1}{n+2}\right)$	A1
	$\sum_{r=1}^{n}\dfrac{1}{r(r+1)(r+2)}=\dfrac{1}{2}\times\left(\dfrac{1}{2}-\dfrac{1}{n+1}+\dfrac{1}{n+2}\right)=\dfrac{n(n+3)}{4(n+1)(n+2)}$	M1A1 (4)
		[7]

Notes

(a)	
M1	Attempt PFs by any valid method (by implication if 3 correct fractions seen)
A1A1	A1 any 2 fractions correct; A1 third fraction correct
(b)	
M1	Method of differences with at least 3 terms at start and 2 at end OR 2 at start and 3 at end. Last lines may be missing 1/(n - 1) - 2/n and 1/n as candidates know they cancel.
A1	Extract the remaining terms. 1 - 2/2 may be missing and 1/(n+1) - 2/(n + 1) may be combined
M1	Include the 1/2 and attempt a common denominator of the form $k(n+1)(n+2)$.
A1cso	$\dfrac{n(n+3)}{4(n+1)(n+2)}$
NB	May use $\left(\dfrac{1}{r}-\dfrac{1}{r+1}\right)-\left(\dfrac{1}{r+1}-\dfrac{1}{r+2}\right)$ oe M1 for 2 at start and 1 at end (or vv) for **both** differences

Question Number	Scheme	Marks
4		
	$u+\mathrm{i}v=\left(x+2\mathrm{i}\right)^2+4$	M1
	$u=x^2 \quad v=4x$	dM1
(i)	$u=\left(\frac{v}{4}\right)^2 \quad \therefore$ parabola	A1
	$v^2=16u$	M1
(ii)	focus: $\left(4,0\right)$	A1cao
(iii)	Directrix: $u+4=0$ oe	A1cao
		[6]
	Notes	
M1	Use $w=u+\mathrm{i}v, z=x+\mathrm{i}y$ and $y=2$ in the transformation equation	
dM1	Expand and equate real and imaginary parts	
(i)A1	Eliminate x between the two equations and deduce a parabola (must be stated)	
M1	Re-arrange to standard form with u, v (x, y scores M0). (Can be given by implication on basis of the answers shown).	
(ii)A1 cao	Deduce required coordinates (both must be shown but coordinate brackets not needed)	
(iii)A1 cao	Deduce the equation for the directrix. (x = -4 scores A0)	

Question Number	Scheme	Marks
5 (a)	$\frac{d^2y}{dx^2} = -\frac{5}{y}\left(\frac{dy}{dx}\right)^2 + 5$	M1
	$\frac{d^3y}{dx^3} = \frac{5}{y^2}\left(\frac{dy}{dx}\right)^3 - \frac{10}{y} \times \frac{d^2y}{dx^2} \times \frac{dy}{dx}$	M1A1A1 (4)
ALT	$\frac{dy}{dx}\frac{d^2y}{dx^2} + y\frac{d^3y}{dx^3} + 10\frac{dy}{dx} \times \frac{d^2y}{dx^2} - 5\frac{dy}{dx} = 0$	M1M1A1
	$\frac{d^3y}{dx^3} = -\frac{11}{y}\frac{dy}{dx} \times \frac{d^2y}{dx^2} + \frac{5}{y}\frac{dy}{dx}$ oe	A1
(b)	At $x = 0$ $\frac{d^2y}{dx^2} = -\frac{5}{4}\left(\frac{1}{2}\right)^2 + 5 = \frac{75}{16}$	B1
	$\frac{d^3y}{dx^3} = \frac{5}{16} \times \frac{1}{8} - \frac{10}{4} \times \frac{75}{16} \times \frac{1}{2} = -\frac{745}{128}$	M1
	$y = 4 + \frac{1}{2}x + \frac{75}{16}\frac{x^2}{2!} + \left(-\frac{745}{128}\right)\frac{x^3}{3!} + \ldots$	M1
	$y = 4 + \frac{1}{2}x + \frac{75}{32}x^2 - \frac{745}{768}x^3 + \ldots$	A1 (4)
		[8]

Notes

(a)	
M1	Re-arrange the equation
M1	Attempt the differentiation using product rule (ignore chain rule errors/omissions)
A1	One term correct
A1	Second term correct and no extras
ALT	
M1	Attempt differentiation of $y\frac{d^2y}{dx^2}$ using product rule
M1	Attempt differentiation of $\left(\frac{dy}{dx}\right)^2$ using chain rule
A1	Correct derivatives
A1	Rearrange to correct expression for $\frac{d^3y}{dx^3}$ Not necessarily simplified to 2 terms.
(b)	
B1	Correct value for $\frac{d^2y}{dx^2}$
M1	Use their expression from (a) to obtain a value for $\frac{d^3y}{dx^3}$
M1	Taylor's series formed using their values for the derivatives (2! or 2 and 3! or 6)
A1	Correct series, must start (or end) y =... but accept f(x) provided y = f(x) defined somewhere.

Question Number	Scheme	Marks
6(a)	$v = y^{-3} \quad \frac{\mathrm{d}v}{\mathrm{d}y} = -3y^{-4}$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}v} \times \frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{y^4}{3}\frac{\mathrm{d}v}{\mathrm{d}x}$	M1A1
	$-x\left(\frac{y^4}{3}\frac{\mathrm{d}v}{\mathrm{d}x}\right) + 2y = 3x^4y^4$	
	$\frac{x}{3}\frac{\mathrm{d}v}{\mathrm{d}x} - \frac{2}{y^3} = -3x^4$	
	$\frac{\mathrm{d}v}{\mathrm{d}x} - \frac{6v}{x} = -9x^3$	dM1A1cso (5)
ALT 1	$y = v^{-\frac{1}{3}} \quad \frac{\mathrm{d}y}{\mathrm{d}v} = -\frac{1}{3}v^{-\frac{4}{3}}$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}v} \times \frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{1}{3}v^{-\frac{4}{3}}\frac{\mathrm{d}v}{\mathrm{d}x}$	M1A1
	$-\frac{1}{3}v^{-\frac{4}{3}}\frac{\mathrm{d}v}{\mathrm{d}x} \times x + 2v^{-\frac{1}{3}} = 3x^4v^{-\frac{4}{3}}$	dM1
	$-\frac{1}{3}\frac{\mathrm{d}v}{\mathrm{d}x} \times x + 2v = 3x^4$	
	$\frac{\mathrm{d}v}{\mathrm{d}x} - 6\frac{v}{x} = -9x^3$	A1cso (5)
ALT 2	$v = y^{-3} \quad \frac{\mathrm{d}v}{\mathrm{d}y} = -3y^{-4}$	B1
	$\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{\mathrm{d}v}{\mathrm{d}y} \times \frac{\mathrm{d}y}{\mathrm{d}x} = -3y^{-4}\frac{\mathrm{d}y}{\mathrm{d}x}$	M1A1
	$-3y^{-4}\frac{\mathrm{d}y}{\mathrm{d}x} - 6\frac{y^{-3}}{x} = -9x^3$	dM1
	$x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 3x^4y^4$	A1cso (5)

Notes on (a)

	All Methods:
B1	Correct derivative. Need not be shown explicitly.
M1	Attempt $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}v}{\mathrm{d}x}$ using the chain rule
A1	Correct derivative
dM1	Substitute in equation (I) to obtain an equation in v and x only OR in equation (II) to obtain an equation in x and y only (ALT 2)
A1cso	Correct completion with no errors seen

Question Number	Scheme	Marks
6(b)	IF: $e^{\int -\frac{6}{x}dx} = e^{-6\ln x} = \frac{1}{x^6}$	M1A1
	$\frac{v}{x^6} = \int -9x^{-3}dx = \frac{9}{2}x^{-2}\ (+c)$	dM1A1
	$v = \frac{9}{2}x^4 + cx^6$	A1 (5)
(c)	$y^{-3} = \frac{9}{2}x^4 + cx^6$	
	$y^3 = \dfrac{1}{\frac{9}{2}x^4 + cx^6}$ oe eg $y^3 = \dfrac{2}{9x^4 + c'x^6}$	B1ft (1)
		[11]

Notes on (b) & (c)

(b)	
M1	IF of form $e^{\int \pm\frac{6}{x}dx}$ and attempt the integration. $\frac{6}{x} \to k\ln x$
A1	Correct IF
dM1	Multiply through by their IF and attempt to integrate both sides of the equation.
A1	Correct integration with or without constant (NB not ft)
A1	Include the constant and multiply through by x^6 Must start v = ...
(c)	
B1ft	Any equivalent to that shown. (No need to change letter used for constant when rearranging) Must start y^3 = ... and must include a constant (single letter or letter x number)

Question Number	Scheme	Marks
7 **(a)**	$\left(\cos\theta+\mathrm{i}\sin\theta\right)^5=\cos 5\theta+\mathrm{i}\sin 5\theta$ or $\sin 5\theta=\mathrm{Im}\left(\left(\cos\theta+\mathrm{i}\sin\theta\right)^5\right)$	B1
	$\cos^5\theta+5\cos^4\theta\left(\mathrm{i}\sin\theta\right)+\frac{5\times 4}{2!}\cos^3\theta\left(\mathrm{i}\sin\theta\right)^2$ $+\frac{5\times 4\times 3}{3!}\cos^2\theta\left(\mathrm{i}\sin\theta\right)^3+\frac{5\times 4\times 3\times 2}{4!}\cos\theta\left(\mathrm{i}\sin\theta\right)^4+\left(\mathrm{i}\sin\theta\right)^5$	M1
	$=\cos^5\theta+5\mathrm{i}\cos^4\theta\sin\theta-10\cos^3\theta\sin^2\theta$ $-10\mathrm{i}\cos^2\theta\sin^3\theta+5\cos\theta\sin^4\theta+\mathrm{i}\sin^5\theta$	A1
	$\sin 5\theta=5\cos^4\theta\sin\theta-10\cos^2\theta\sin^3\theta+\sin^5\theta$ $=5\left(1-\sin^2\theta\right)^2\sin\theta-10\left(1-\sin^2\theta\right)\sin^3\theta+\sin^5\theta$	M1
	$\left(\sin 5\theta=\right)16\sin^5\theta-20\sin^3\theta+5\sin\theta$ $\sin 5\theta-5\sin\theta=16\sin^5\theta-20\sin^3\theta$ *	A1 cso (5)
ALT	$z-\frac{1}{z}=2\mathrm{i}\sin\theta$ or $z^5-\frac{1}{z^5}=2\mathrm{i}\sin 5\theta$ oe	B1
	Binomial expansion of $\left(z-\frac{1}{z}\right)^5$	M1A1
	$32\sin^5\theta=2\sin 5\theta-10\sin 3\theta+20\sin\theta$ Use double angle formulae etc to obtain $\sin 3\theta=3\sin\theta-4\sin^3\theta$ and then use it in their expansion	M1
	$\left(\sin 5\theta=\right)16\sin^5\theta-20\sin^3\theta+5\sin\theta$ $\sin 5\theta-5\sin\theta=16\sin^5\theta-20\sin^3\theta$ *	A1cso

Notes on (a)

B1	Correct use of de Moivre Give by implication if $\sin 5\theta=$ Im part of the expansion
M1	Attempt the expansion appropriate to their method. Main scheme method need only show imaginary parts; ignore errors in real parts
A1	Simplify the coefficients and powers of i to obtain an expansion with all imaginary terms correct OR pair the terms and obtain the given expression
M1	Equate imaginary parts and use $\cos^2\theta=1-\sin^2\theta$ to eliminate $\cos\theta$ from the expansion. May have an i in each term. OR Use double angle formulae etc to obtain $\sin 3\theta=3\sin\theta-4\sin^3\theta$ and then use it in their expansion.
A1cso	Correct result (as shown in the question unless rearranged at the start) with no errors seen

Question Number	Scheme	Marks
7(b)	Let $x=\sin\theta$ $\quad 16x^5-20x^3+5x=\frac{1}{2} \Rightarrow \sin 5\theta=\frac{1}{2}$	M1
	$5\theta=30,\ 150,\ 390,\ 510,\ 750,\ 870,1110,1230$	A1
	$5\theta=\frac{\pi}{6},\frac{5\pi}{6},\frac{13\pi}{6},\frac{17\pi}{6},\frac{25\pi}{6},\frac{29\pi}{6},\frac{37\pi}{6},\frac{41\pi}{6},$	
	Or $\quad 0.524,\ 2.617,6.806,\ 8.901,13.08,15.18,19.37,21.46$ (truncated, not rounded)	
	$\theta=6,30,78,(102),(150),(174),222,246$ or in radians (0.105, 0.5235. 1.134 etc)	
	$x=\sin\theta=0.105,\ 0.5,\ 0.978,\ -0.669,\ -0.914$	dM1A1A1 (5)
(c)(i)	$\int\left(8\sin^5\theta-10\sin^3\theta\right)\mathrm{d}\theta=\frac{1}{2}\int\left(\sin 5\theta-5\sin\theta\right)\mathrm{d}\theta$	M1
	$=\frac{1}{2}\left[-\frac{1}{5}\cos 5\theta+5\cos\theta\right]\quad(+c)$	A1
(ii)	$\frac{1}{2}\left[-\frac{1}{5}\cos\frac{5\pi}{3}+5\cos\frac{\pi}{3}-\left(-\frac{1}{5}+5\right)\right]$	
	$=\frac{1}{2}\left[-\frac{1}{5}\times\frac{1}{2}+\frac{5}{2}-4\frac{4}{5}\right]$	dM1
	$=-1\frac{1}{5}$	A1 (4)
ALT **(c)(i)**	$\int\left(8\sin^5\theta-10\sin^3\theta\right)\mathrm{d}\theta=\int\left(8\sin\theta\sin^4\theta-10\sin\theta\sin^2\theta\right)\mathrm{d}\theta$	
	and use $\sin^2\theta+\cos^2\theta=1$	M1
	$=2\cos\theta+2\cos^3\theta-\frac{8}{5}\cos^5\theta$	A1
(ii)	Sub limits (M1) Correct ans (A1)	**[14]**

Notes on (b) & (c)

(b)	
M1	Substitute $x=\sin\theta$ deduce that $\sin 5\theta=\pm\frac{1}{2}$
A1	Give a set of at least 5 results for 5θ or θ with no repeats in the set
dM1	At least 2 different values for x or $\sin\theta$ corresponding to values in their set (not nec correct)
A1	3 different correct values for x or $\sin\theta$
A1	2 further different correct values of x or $\sin\theta$ (0.5 or ½)
(c)	
M1	Use part (a) to change the integrand
(i)A1	Correct integration with or w/o limits (ignore any shown). NB: Not ft
dM1	Substitute the given limits and replace the trig functions with their numerical values
(ii)A1	Final answer correct in any form.

Question Number	Scheme	Marks
8 **(a)**	$y = r\sin\theta \Rightarrow y = a\sin 2\theta\sin\theta$	B1
	$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = a\left(2\cos 2\theta\sin\theta + \sin 2\theta\cos\theta\right)$	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = a\left(2\sin\theta\left(\cos^2\theta - \sin^2\theta\right) + 2\sin\theta\cos^2\theta\right) = 0$	M1
	$\tan^2\theta = 2 \Rightarrow \tan\varphi = \sqrt{2}$ * $\left(\text{Accept } \tan\theta = \sqrt{2}\right)$	A1cso (4)
Notes on (a)		
B1	State $y = a\sin 2\theta\sin\theta$ May be given by implication	
M1	Attempt to differentiate $y = r\sin\theta$ or $y = r\cos\theta$ Product rule must be used	
M1	Use double angle formulae to eliminate 2θ and equate derivative to 0 Formulae used must be correct	
A1cso	Complete to given answer with no errors seen. Can be done via sine cos or tan	
ALT **(a)**	$y = r\sin\theta \Rightarrow y = a\sin 2\theta\sin\theta$	B1
	$y = 2a\sin^2\theta\cos\theta$	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = 2a\left(2\sin\theta\cos^2\theta - \sin^3\theta\right) = 0$	M1
	$\tan^2\theta = 2 \Rightarrow \tan\phi = \sqrt{2}$ *	A1 cso (4)
Notes on ALT for (a)		
B1	State $y = a\sin 2\theta\sin\theta$ May be given by implication	
M1	Use double angle formulae to eliminate 2θ	
M1	Attempt to differentiate $y = r\sin\theta$ or $y = r\cos\theta$ (product rule must be used) and equate derivative to 0	
A1cso	Complete to given answer with no errors seen (No need to state $\sin/\cos \neq 0$ if dividing)	
(b)	$\tan\phi = \sqrt{2} \Rightarrow \sin\phi = \dfrac{\sqrt{2}}{\sqrt{3}}$, $\cos\phi = \dfrac{1}{\sqrt{3}}$	
	$R = 2a \times \dfrac{\sqrt{2}}{\sqrt{3}} \times \dfrac{1}{\sqrt{3}} = \dfrac{2\sqrt{2}}{3}a$	M1A1 (2)
Notes for (b)		
M1	Attempting values for $\sin\theta$ and $\cos\theta$ and using these to obtain a value for R	
A1	A correct, exact value for R, as shown or any equivalent.	

Question Number	Scheme	Marks
8(c)	$x = r\cos\theta = a\sin 2\theta\cos\theta$	
	$\frac{dx}{d\theta} = 2a\cos 2\theta\cos\theta - a\sin 2\theta\sin\theta$	M1
	$2a\cos\theta\left(\cos^2\theta - 2\sin^2\theta\right) = 0$	
	$\tan\theta_B = \frac{1}{\sqrt{2}}$	M1
	Area $= \frac{1}{2}\int r^2 d\theta = \frac{1}{2}a^2\int \sin^2 2\theta\, d\theta$	M1
	$= \left(\frac{1}{2}a^2\right)\int_{\arctan\left(\frac{1}{\sqrt{2}}\right)}^{\arctan\sqrt{2}} \frac{1}{2}(1-\cos 4\theta)\,d\theta$ *	M1A1 (5)
(d)	$= \left(\frac{1}{2}a^2\right)\frac{1}{2}\left[\theta - \frac{1}{4}\sin 4\theta\right]_{\arctan\left(\frac{1}{\sqrt{2}}\right)}^{\arctan\sqrt{2}}$	M1
	$= \frac{1}{4}a^2\left[\arctan\sqrt{2} - \frac{1}{4}\sin 4\left(\arctan\sqrt{2}\right) - \arctan\frac{1}{\sqrt{2}} + \frac{1}{4}\sin 4\left(\arctan\left(\frac{1}{\sqrt{2}}\right)\right)\right]$	dM1
	$\arctan\frac{1}{\sqrt{2}} = \frac{\pi}{2} - \arctan\sqrt{2}$	B1
	$\sin 4\left(\arctan\sqrt{2}\right) = 2\sin 2\phi\cos 2\phi = 2\sin 2\phi\left(\cos^2\phi - \sin^2\phi\right) = -\frac{4\sqrt{2}}{9}$	M1
	$\sin 4\left(\arctan\frac{1}{\sqrt{2}}\right) = 2\sin 2\theta_B\cos 2\theta_B = 2\sin 2\theta_B\left(\cos^2\theta_B - \sin^2\theta_B\right) = \frac{4\sqrt{2}}{9}$	A1
	Area $= \frac{1}{4}a^2\left(\frac{2\sqrt{2}}{9} - \frac{\pi}{2} + 2\arctan\sqrt{2}\right) = a^2\left(\frac{\sqrt{2}}{18} - \frac{\pi}{8} + \frac{1}{2}\arctan\sqrt{2}\right)$ *	A1cso (6) **[17]**

Notes on (c) & (d)

(c)M1	Attempt the differentiation of $x = r\cos\theta$ or $r\sin\theta$ Not the one differentiated in (a) Product rule must be used
M1	Use the double angle formula and equate the derivative to 0 and solve the equation to find the value of $\tan\theta$ at B If $\tan\theta = \sqrt{2}$ award M0
M1	Use area $= \frac{1}{2}\int r^2 d\theta$ for C
M1	Use the double angle formula to obtain $k\int \frac{1}{2}(1 \pm \cos 4\theta)\,d\theta$ Ignore any limits given
A1	Reach the given integral with no omissions or errors seen.
(d) M1	Attempt the integration $\cos 4\theta \to \pm\frac{1}{4}\sin 4\theta$
dM1	Substitute the limits
B1	Connect the two angles, explicitly or in the following work
M1	Attempt a numerical value for $\sin 4\theta$ or for $\sin\theta$ and $\cos\theta$
A1	Correct numerical values for **both** sine functions or $\sin\theta$ and $\cos\theta$
A1cso	Complete to the given answer with no errors seen.
	If argument based on $\frac{dy}{d\theta}$ being infinitely large is used, send to Review.

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