Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE Further Mathematics
Further Pure 1 Paper 6667_01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q),\ \text{where}\ |pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q),\ \text{where}\ |pq| = |c|\ \text{and}\ |mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\quad q \neq 0$, leading to $x = \dots$

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

<u>Use of a formula</u>

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

<u>Exact answers</u>

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Notes	Marks
1	$f(z) = z^3 + az^2 + bz - 26$		
	Mark parts (a) and (b) together. Condone work in *x*		
(a)	–2 – 3i is also a root	Indicated as a root. "*z* = –2 – 3i" or –2 – 3i in a list of solutions is sufficient. Could be implied later e.g., by work for the first method mark.	B1
	Sum = (3i – 2) + (–2 – 3i) [= –4] **and** product = (3i – 2) (–2 – 3i) [=13] OR (*z* – (3i – 2))(*z* – (–2 – 3i)) OR $(z + 2)^2 = (\pm 3i)^2$ ⇒... $z^2 + 4z + 13$	M1: Correct method (Any of the statements is sufficient) and obtains quadratic with real coefficients A1: Correct quadratic	M1A1
	$f(z) = (z^2 + 4z + 13)(z - 2) \Rightarrow z = 2$	M1: A valid method which finds a real solution or linear factor. Can be implied e.g. $\left(z + \frac{-26}{c}\right)$ or (*z* =) $-\frac{-26}{c}$ from their quadratic, condoning sign errors. Do not accept (*z* + *a* – 4) or (*z* =) 4 – *a* (e.g., from long division) until a real value for *a* is substituted A1: (*z* =) 2. Note "*z* =" is not required but it must be clear that "2" is a root. "*z* = 2" or 2 in a list of solutions is sufficient	M1A1
	f(*z*) = (*z* – 2)(*z* – (3i – 2))(*z* – (–2 – 3i)) = 0 scores 11110 if the "2" is not "extracted"		
			(5)
Alt **Product of three roots**	–2 – 3i is also a root	Indicated as a root. "*z* = –2 – 3i" or –2 – 3i in a list of solutions is sufficient. Could be implied later.	B1
	$z_1 z_2 z_3 = (3i - 2)(-2 - 3i)\, z_3$ $\to 13z_3$	M1: Correct method and obtains kz_3 A1: $13z_3$	M1A1
	$13z_3 = -\frac{-26}{1} \to z_3 = 2$	M1: Sets their kz_3 equal to $-\frac{d}{a}$ and obtains root (or factor), condoning sign errors. A1: (*z* =) 2. Note "*z* =" is not required but it must be clear that "2" is a root. "*z* = 2" or 2 in a list of solutions is sufficient	M1A1

Question Number	Scheme	Notes	Marks
(b)	Obtains a computed value for *a* or *b* from appropriate work, e.g., $\left(z^2+4z+13\right)\left(z-2\right)=\left(z^3+\right)4z^2+13z-2z^2-8z\left(-26\right)$ Compares coefficients in z or z^2 and obtains a value for *a* or *b* Or uses $z = 4 - a$ to compare coefficients of at least the constant term and obtains a value for *a* or *b* [$(z^2 + 4z + 13)(z + (a - 4)) = z^3 + (4 + a - 4)z^2 + (13 + 4(a - 4))z + 13(a - 4)$] OR Uses long division of quadratic into cubic and sets one of their coefficients of their remainder of the form $cz + d$ (where *c* and *d* are real functions of *a* or *b* or both) equal to zero to obtain a value for *a* or *b* [Correct quotient is $z + a - 4$ and correct equations are $b - 13 - 4(a - 4) = 0$ and $-26 - 13(a - 4) = 0$] OR Substitutes and expands to find f(3i – 2) or f(–2 – 3i), sets equal to zero, equates real and imaginary parts to obtain 2 real equations in *a* and *b* and obtains a value for *a* or *b* [Correct equations are $5a + 2b = 20$ and $4a - b = 3$]		M1
	$a = 2$ or $b = 5$	Correct value for *a* or *b* (from correct work)	A1
	$a = 2$ and $b = 5$	Both values correct	A1
	Accept the embedded values from $z^3 + 2z + 5z - 26$		**(3)**
			Total 8

Question Number	Scheme	Notes	Marks
2(a)	$\begin{pmatrix} 9+k & -3 \\ 4-k & 2 \end{pmatrix}\begin{pmatrix} 2 & 7 & 7 \\ 1 & 1 & 12 \end{pmatrix} = \ldots$	Attempt to multiply all the coordinates appropriately. This statement oe is sufficient. Coordinate pairs could be multiplied separately	M1
	$\begin{pmatrix} 2(9+k)-3 & 7(9+k)-3 & 7(9+k)-36 \\ 2(4-k)+2 & 7(4-k)+2 & 7(4-k)+24 \end{pmatrix}$ or $\begin{pmatrix} 18+2k-3 & 63+7k-3 & 63+7k-36 \\ 8-2k+2 & 28-7k+2 & 28-7k+24 \end{pmatrix}$ or $\begin{pmatrix} 15+2k & 60+7k & 27+7k \\ 10-2k & 30-7k & 52-7k \end{pmatrix}$	Deduct 1 mark for each incorrect element up to a maximum of 2 marks. Accept all unsimplified and isw. No requirement to extract coordinates from the matrix and may be given individually as column vectors	A1A1
			(3)
(b)	Area of triangle $T = \frac{1}{2}(7-2)(12-1)$ or $\frac{1}{2}\lvert(2+84+7)-(7+7+24)\rvert$	Correct unsimplified expression for area for triangle *T* Could come from shoelace method	B1
	$\lvert\mathbf{M}\rvert = (9+k)\times 2-(4-k)\times -3$	Correct unsimplified expression for det**M.** Seen in part (b)	B1
	$"(9+k)\times 2-(4-k)\times -3"\times"\frac{55}{2}" = \pm 770$ $\Rightarrow k = \ldots$	Uses 770 correctly with their numerical area of *T* and their det**M** to give an equation in terms of *k* and attempts to solve. The $\pm$ sign is not required. |det**M**| replaced with (det**M**)^2 is M0 unless (det**M**)^2 = (their 28)^2	M1
	$30-k = \pm 28 \Rightarrow k = 2,\ 58$	Both values and no extra	A1
			(4)
			Total 7

Question Number	Scheme	Notes	Marks
3	$xy = 10$		
(a)	$xy = 10 \Rightarrow \frac{dy}{dx} = -10x^{-2}$ or $xy = 10 \Rightarrow x\frac{dy}{dx} + y = 0$ or $x = ct,\ y = \frac{c}{t} \Longrightarrow \frac{dy}{dx} = \frac{-\frac{c}{t^2}}{c}$	Correct differentiation	B1
	$m_T = -10(5)^{-2}$ or $m_T = -\frac{2}{5}$	Substitutes after differentiation and finds a numerical expression for the gradient at (5, 2). Note: $t = \frac{\sqrt{10}}{2}$	M1
	$m_N = -\frac{1}{m_T}\left(= \frac{5}{2}\right)$	Correct application of perpendicular gradient rule with a numerical gradient. Must follow attempt at differentiation.	M1
	$y - 2 = "\frac{5}{2}"(x - 5)$	Correct straight line method with a changed numerical gradient. Must follow differentiation. If $y = mx + c$ is used they must reach $c = ...$	M1
	$5x - 2y - 21 = 0$	Any integer multiple. The "=0" is required. Seen in part (a).	A1
	Condone a non-numerical gradient if (5, 2) is substituted later, provided the final answer is a straight line. Accept $a = 5$, $b = -2$, c = –21 or integer multiple		
			(5)
(b)	$5x - 2y - 21 = 0,\ xy = 10 \Rightarrow$ $5x - 2\frac{10}{x} - 21 = 0$ or $5\frac{10}{y} - 2y - 21 = 0$	Uses their normal and H to obtain an equation in one variable. [Note: In t: $5\sqrt{10}t - 2\frac{\sqrt{10}}{t} - 21 = 0$]	M1
	$5x^2 - 21x - 20 = 0$ or $2y^2 + 21y - 50 = 0$	Obtains 3TQ [$5\sqrt{10}t^2 - 21t - 2\sqrt{10} = 0$]	M1
	$(5x + 4)(x - 5) = 0 \Rightarrow x = ...$ $(2y + 25)(y - 2) = 0 \Rightarrow y = ...$	Solves 3TQ (usual rules) [$t = -\frac{2\sqrt{10}}{25}$]	M1
	$x = -\frac{4}{5}$ or $y = -\frac{25}{2}$	One correct coordinate. Must be from a correct normal	A1
	$x = -\frac{4}{5}$ and $y = -\frac{25}{2}$	Both coordinates correct. Allow exact equivalents	A1
	If coordinates are given in the wrong order score if identified individually elsewhere		
			(5)
(c) **Way 1** **2 triangles (*OPX* and *OQX* or *OPY* and *OQY*)**	$y = 0$ or $x = 0$, $5x - 2y - 21 = 0$ $\Longrightarrow x = \frac{21}{5}$ or $y = (-)\frac{21}{2}$	Attempt x- or y-intercept of normal	M1
	$\frac{1}{2} \times \frac{21}{5} \times 2$ or $\frac{1}{2} \times \frac{21}{5} \times \frac{25}{2}$ OR $\frac{1}{2} \times \frac{21}{2} \times \frac{4}{5}$ or $\frac{1}{2} \times \frac{21}{2} \times 5$	Correct method for one triangle with their values for intercepts	M1
	$\frac{1}{2} \times \frac{21}{5} \times 2 + \frac{1}{2} \times \frac{21}{5} \times \frac{25}{2}$ OR $\frac{1}{2} \times \frac{21}{2} \times \frac{4}{5} + \frac{1}{2} \times \frac{21}{2} \times 5$	Fully correct method with their values for intercepts	M1
	Area = $\frac{609}{20}$	$\frac{609}{20}$ or exact equivalent e.g., 30.45	A1
			(4)
			Total 14

(c) Way 2 **Shoelace**	Area = $\frac{1}{2}\begin{vmatrix} 0 & 5 & -\frac{4}{5} & 0 \\ 0 & 2 & -\frac{25}{2} & 0 \end{vmatrix}$	Correct ft determinant	M1
	$5\times"\frac{-25}{2}"$ or $2\times"-\frac{4}{5}"$	Correct method for one 'thread'	M1
	$\frac{1}{2}\left\lvert 5\times"\frac{-25}{2}"-2\times"-\frac{4}{5}"\right\rvert=\frac{609}{20}$	M1: Fully correct method A1: $\frac{609}{20}$ or exact equivalent e.g., 30.45 Condone sight of negative answer if subsequently corrected.	M1A1

(c) Way 3 **1 triangle**	$y=-\frac{2}{5}x,\ 5x-2y-21=0\Rightarrow\left(\frac{105}{29},-\frac{42}{29}\right)$	Attempt intersection of tangent through origin and normal obtaining coordinates of *R*	M1
	$OR=\sqrt{\left(\frac{105}{29}\right)^2+\left(\frac{42}{29}\right)^2}$ or $PQ=\sqrt{\left(5-\left(-\frac{4}{5}\right)\right)^2+\left(2-\left(-\frac{25}{2}\right)\right)^2}$	Correct use of Pythagoras for *OR* or OR^2 or *PQ* or PQ^2 with their values. Not scored as part of an attempt via Way 4	M1
	Area = $\frac{1}{2}\sqrt{\frac{441}{29}}\times\sqrt{\frac{24389}{100}}=\frac{609}{20}$	M1: Fully correct method A1: $\frac{609}{20}$ or exact equivalent e.g., 30.45	M1A1

(c) Way 4 **1 triangle using trig**	$OP=\sqrt{5^2+2^2}$ $OQ=\sqrt{\left(\frac{4}{5}\right)^2+\left(\frac{25}{2}\right)^2}$ $PQ=\sqrt{\left(5-\left(-\frac{4}{5}\right)\right)^2+\left(2-\left(-\frac{25}{2}\right)\right)^2}$	Attempts all three sides of their triangle *OPQ* by Pythagoras	M1
	e.g., angle *POQ* $=\arccos\left(\frac{\left(\frac{\sqrt{15689}}{10}\right)^2+\left(\sqrt{29}\right)^2-\left(\frac{29\sqrt{29}}{10}\right)^2}{2\times\frac{\sqrt{15689}}{10}\times\sqrt{29}}\right)$	Correct method to find an angle in triangle *OPQ*	M1
	Variation for the above two marks: M1: Attempts *OP* and *OQ* by Pythagoras and either angle *POX* or angle *QOY* M1: Correct method to find both angles [*POX* = arctan $\left(\frac{2}{5}\right)$, *POY* = arctan $\left(\frac{8}{125}\right)$] and then finds angle *POQ* = *POX* + *QOY* + 90°		
	Area = $\frac{1}{2}\times\frac{\sqrt{15689}}{10}\times\sqrt{29}\times\sin 115.4633...° = \frac{609}{20}$	M1: Fully correct method to obtain a numerical expression for the area. Note that sin *POQ* = 0.9028605... A1: $\frac{609}{20}$ or exact equivalent e.g., 30.45	M1A1

(c) **Way 5** **Rectangle or triangle minus triangles (or minus rectangle and triangles)**	rectangle = $\frac{29}{5} \times \frac{29}{2}$ or triangle = $\frac{1}{2} \times \frac{29}{5} \times \frac{29}{2}$	Attempts area of rectangle with sides parallel to the axes and opposite vertices *P* and *Q* or attempts area of right-angled triangle with hypotenuse *PQ* This mark is not scored if either of these are given as the area of *POQ*	M1
	e.g. for rectangle – triangles: $\frac{1}{2} \times \frac{29}{5} \times \frac{29}{2}$ and $\frac{1}{2} \times \frac{4}{5} \times \frac{29}{2}$ and $\frac{1}{2} \times 2 \times \frac{29}{5}$	Correct attempts at the areas of all the triangles (or rectangle and triangles) whose total area is to be subtracted	M1
	Area = $\frac{841}{10} - \frac{841}{20} - \frac{29}{5} - \frac{29}{5} = \frac{609}{20}$	M1: Fully correct method A1: $\frac{609}{20}$ or exact equivalent e.g., 30.45	M1A1

Question Number	Scheme	Notes	Marks
4	$f(x)=3x^2-\frac{1}{2\sqrt{x}}-5x, \quad g(\theta)=3\theta+6-\tan\left(\frac{\theta}{3}\right)$ Condone the use of other letters for α and β e.g., x_1		
(i)(a)	$f(1)=-2.5, \quad f(2)=1.646...$	Attempts both f(1) and f(2) and obtains either f(1) = –2.5 oe or f(2) = awrt 1.6 or awrt 1.65 or truncated 1.64	M1
	"sign change" oe $\Rightarrow$ root	Both f(1) and f(2) correct and conclusion. Allow $\frac{8-\sqrt{2}}{4}$ oe for 1.646...	A1
			(2)
(b)	$f'(x)=6x+\frac{1}{4}x^{-\frac{3}{2}}-5$	M1: Two terms correct A1: Fully correct derivative. Isw	M1A1
			(2)
(c)	$\alpha=2-\frac{f(2)}{f'(2)}=2-\frac{1.64644609...}{7.088388348...}$	Correct attempt at Newton-Raphson with "2" and their values	M1
	$\alpha=1.767726241...$	Awrt 1.77 cao. Ignore further iterations.	A1
			(2)
(ii)	$g(-2)=0.78684..., \quad g(-3)=-1.44259...$	Attempts both g(–2) and g(–3) in radians obtaining either g(–2) = awrt 0.79 or g(–3) = awrt –1.4	M1
	$\frac{\lvert g(-2)\rvert}{-2-\beta}=\frac{\lvert g(-3)\rvert}{\beta+3}=\frac{0.78684...}{-2-\beta}=\frac{1.4425...}{\beta+3}$	Correct interpolation statement with their values. No sign errors. Could equate gradient of line between end point and x-axis intercept and gradient of entire line segment, e.g., $\frac{1.4425\ldots}{\beta+3}=\frac{0.7864\ldots+1.4425\ldots}{-2-(-3)}$	M1
	$\beta=\frac{-3\times 0.7868...-2\times 1.4425...}{0.7868...+1.4425...}$	Makes "β" the subject. **Dependent on second M1.**	dM1
	$\beta=-2.353$	–2.353 only (not awrt)	A1
	Trial and improvement scores a maximum of 1000 Allow $\frac{B-2}{3-B}=\frac{0.7864\ldots}{1.4425\ldots}$ oe **if β = –B is used**. May also find γ where β= –2 – γ etc. Correct use of degrees leading to β = – 2.003886... scores 0110		
			(4)
			Total 10
(ii) Alt	$g(-2)=0.78684..., \quad g(-3)=-1.44259...$	Attempts both g(–2) and g(–3) obtaining either g(–2) = awrt 0.79 or g(–3) = awrt –1.4	M1
	e.g., m = 0.7868... + 1.4425... (x_1, y_1) = one end point so y_1 = 2.2294... $x_1 + c$ $\Rightarrow c$ = ... ($\Rightarrow y$ = 2.2294... x + 5.2457...)	Correctly finds the equation of the line joining their (–3, g(–3)) and (–2, g(–2))	M1
	$\beta=\frac{-5.2457\ldots}{2.2294\ldots}$	Puts y = 0 into their equation and solves. **Dependent on second M1.**	dM1
	$\beta=-2.353$	–2.353 only (not awrt)	A1

Question Number	Scheme	Notes	Marks
5(a)	$\sum_{r=1}^{n} r^2(4+r) = \frac{1}{12}n(n+1)(an^2+bn+c)$		
	$\sum_{r=1}^{n} r^2(4+r) = \sum_{r=1}^{n} 4r^2 + r^3 = \frac{4}{6}n(n+1)(2n+1) + \frac{1}{4}n^2(n+1)^2$ M1: Expands and substitutes at least one formula correctly A1: Fully correct expression		M1A1
	$= \frac{1}{12}n(n+1)\left[8(2n+1)+3n(n+1)\right]$	Takes out a factor of $\frac{1}{12}n(n+1)$ having obtained a sum of two expressions where both have $n(n + 1)$ as a factor	M1
	$= \frac{1}{12}n(n+1)(3n^2+19n+8)$	cao	A1
			(4)
(b)	$5^2 \times 9 + 6^2 \times 10 + 7^2 \times 11 + ...20^2 \times 24 = \sum_{r=5}^{20} r^2(4+r)$ $= \frac{1}{12}20(20+1)\left(3(20)^2+19(20)+8\right) - \frac{1}{12}4(4+1)\left(3(4)^2+19(4)+8\right)$ Uses their result from part (a) to find f(20) – f(4) [= 55360] Allow errors of one unit with one or both summation limits for this mark, i.e., f(20) – f(5) [= 55135] or f(19) – f(4) [= 45760] or f(19) – f(5) [= 45535]		M1
	$5^2+6^2+7^2+...+20^2 = \frac{1}{6}(20)(21)(41) - \frac{1}{6}(4)(5)(9)$ [= 2840] Attempts sum of squares (using subtraction) from 5 or 6 to 20 or 19 [5 to 19: 2440, 6 to 19: 2415, 6 to 20: 2815] Could be implied **by working**, e.g., $5\sum r^2$ seen as 5(2870 – 30) or 5 x 2840 or 14350 – 150 or 58450 Allow the summation to be computed as, e.g., 25 + 36 + ... + 400		M1
	$\sum_{r=5}^{20} r^2(4+r) + \sum_{r=5}^{20} r^2$ $= 55360 + 2840$	Adds their 2 sums. **Dependent on both previous method marks and must have used correct summation limits.**	ddM1
	= 58200	cao	A1
	If their $\frac{1}{12}$ *n*(*n* + 1)(*an*² + *bn* + *c*) in part (a) is not clearly used, only the second M mark is available.		
			(4)
			Total 8

Question Number	Scheme	Notes	Marks
6	$z = \dfrac{a+2\text{i}}{a+5\text{i}}$		
(a)	$z = \dfrac{a+2\text{i}}{a+5\text{i}} \times \dfrac{a-5\text{i}}{a-5\text{i}}$	Attempts to multiply numerator and denominator by $a - 5\text{i}$ This statement is sufficient	M1
	$\text{Re}(z) = \dfrac{a^2+10}{a^2+25}$	Correct real part. Award when seen as c in the form $(z =)$ $c + d\text{i}$	A1
	$\dfrac{a^2+10}{a^2+25} = \dfrac{13}{28} \Rightarrow a^2 = 3$	Sets their real part = 13/28 and reaches $a^2 = k$ where $k > 0$	M1
	$a = \sqrt{3}$	Allow awrt 1.73 and ignore the negative root if seen	A1
			(4)
(b)	$\text{Im}(z) = \dfrac{-3a}{a^2+25} = \dfrac{-3"\sqrt{3}"}{"\sqrt{3}"^2+25}$	Attempt Im(z) with their a substituted correctly at least once into an Im(z) of the form: $\dfrac{pa}{a^2+q}$ where $p, q \neq 0$ Award when seen as d in the form $(z =)$ $c + d\text{i}$ Condone a correct method repeating the first M mark in part (a) with their value of a provided a real value for Im(z) or $c + d\text{i}$ is obtained	M1
	$z = \dfrac{13}{28} - \dfrac{3\sqrt{3}}{28}\text{i} \Rightarrow \arg(z) = -\arctan\left(\dfrac{3\sqrt{3}}{13}\right)$	$z = c + d\text{i} \Longrightarrow \pm \arctan\left(\pm\dfrac{d}{c}\right)$ for their z Implied by $\pm$ a correct angle in radians	M1
	$= -0.38$	awrt -0.38 oe e.g. awrt 5.90 (including "5.9"). Mark the final answer. Requires a correct z	A1
			(3)
(c)	$zz^* = \left(\dfrac{13}{28}\right)^2 + \left(\dfrac{3\sqrt{3}}{28}\right)^2$	$\left(\dfrac{13}{28}\right)^2$ + (their imaginary part)^2 If (their imaginary part x i)^2 is used, $\text{i}^2 = -1$ must follow.	M1
	$= \dfrac{1}{4}$ or 0.25 only	Allow from $z = \frac{13}{28} + \frac{3\sqrt{3}}{28}\text{i}$ Ignore "+ 0i"	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
7(a)	Rotation 45° clockwise about the origin	B1: Rotation B1: 45° clockwise (or 315°/– 45° anticlockwise) about/around/at/from/with centre the origin or (0, 0) or *O* Allow just "315°/– 45° about *O*" Allow radian equivalents	B1B1
			(2)
(b)	$\mathbf{Q} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	Correct matrix	B1
	Condone straight lines used as matrix brackets throughout		
			(1)
(c)	$\mathbf{R} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$	Multiplies in the correct order with their $\mathbf{Q} \neq \mathbf{I}$. This statement is sufficient. May be implied by their **R**.	M1
	$\mathbf{R} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	Correct matrix. Ignore labelling, such as **T** = ... Allow exact equivalents for elements and isw. Allow $\begin{pmatrix} \text{awrt } 0.707 & \text{awrt } -0.707 \\ \text{awrt } -0.707 & \text{awrt } -0.707 \end{pmatrix}$	A1
			(2)
(d)	$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 1 \\ k \end{pmatrix} = \begin{pmatrix} 1 \\ k \end{pmatrix}$	Forms matrix equation correctly with their **R**. Implied by a correct ft linear equation. (1, *k*)**R** = (1, *k*) is M0	M1
	$\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}k = 1$ or $-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}k = k$	Either correct equation (or equivalent) from a correct **R**	A1
	$\frac{1}{\sqrt{2}}k = \frac{1}{\sqrt{2}} - 1 \Rightarrow k = ...$	Solves one of their equations to obtain a value for *k*. Allow obtaining and solving an equation from $\begin{pmatrix} 1 \\ k \end{pmatrix} = \mathbf{R}^{-1}\begin{pmatrix} 1 \\ k \end{pmatrix}$ **Dependent on previous method mark.**	dM1
	$k = 1 - \frac{2}{\sqrt{2}}$ or $\frac{\sqrt{2}-2}{\sqrt{2}}$ or $1-\sqrt{2}$ or $\frac{-1}{\sqrt{2}+1}$	Cao. Accept equivalent exact answers and isw. No extra non-equivalent solutions.	A1
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
8(i)	$\begin{pmatrix}-3 & -2\\ 8 & 5\end{pmatrix}^n = \begin{pmatrix}1-4n & -2n\\ 8n & 1+4n\end{pmatrix}$		
	When n = 1: lhs $= \begin{pmatrix}-3 & -2\\ 8 & 5\end{pmatrix}^1 = \begin{pmatrix}-3 & -2\\ 8 & 5\end{pmatrix}$ rhs $= \begin{pmatrix}1-4(1) & -2(1)\\ 8(1) & 1+4(1)\end{pmatrix} = \begin{pmatrix}-3 & -2\\ 8 & 5\end{pmatrix}$ (lhs = rhs so) true for n = 1 Checks n = 1 on both sides and "true for n = 1" oe (e.g., "shown", "QED", a tick, lhs=rhs) seen anywhere. Evaluation of the rhs only is not sufficient		B1
	Assume true for $n = k$ so $\begin{pmatrix}-3 & -2\\ 8 & 5\end{pmatrix}^k = \begin{pmatrix}1-4k & -2k\\ 8k & 1+4k\end{pmatrix}$		
	$\begin{pmatrix}-3 & -2\\ 8 & 5\end{pmatrix}^{k+1} =$ $\begin{pmatrix}1-4k & -2k\\ 8k & 1+4k\end{pmatrix}\begin{pmatrix}-3 & -2\\ 8 & 5\end{pmatrix}$ OR $\begin{pmatrix}-3 & -2\\ 8 & 5\end{pmatrix}\begin{pmatrix}1-4k & -2k\\ 8k & 1+4k\end{pmatrix}$	Attempts $\begin{pmatrix}-3 & -2\\ 8 & 5\end{pmatrix}^{k+1}$ either way round. Either statement is sufficient	M1
	$\begin{pmatrix}-3(1-4k)-16k & -2(1-4k)-10\text{k}\\ -24\text{k}+8(1+4k) & -16k+5(1+4k)\end{pmatrix}$ OR $\begin{pmatrix}-3(1-4k)-16k & 6k-2(1+4k)\\ 8(1-4k)+40k & -16k+5(1+4k)\end{pmatrix}$	At least 3 correct unsimplified elements. **Dependent on previous method mark.** Note that the simplified matrix is: $\begin{pmatrix}-3-4k & -2k-2\\ 8k+8 & 4k+5\end{pmatrix}$	dM1
	$= \begin{pmatrix}1-4(k+1) & -2(k+1)\\ 8(k+1) & 1+4(k+1)\end{pmatrix}$	Correct matrix in terms of $(k + 1)$ Allow proof to "meet in the middle"	A1
	If the statement is **true for *n* = *k*** then it has been shown to be **true for *n* = *k* + 1** and as the result is **true for *n* = 1**, it is **true for all *n* ∈ ℤ** All parts in bold (or equivalent statements) seen at some stage. Accept "all n", "all (positive) integers/values" or "n = 1, 2, 3..." Allow a correct proof in a different variable provided it is consistent.		A1 cso
			(5)

Question Number	Scheme	Notes	Marks
(ii)	$$\sum_{r=1}^{n}\frac{1}{r(r+1)(r+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$$		
	When n = 1: lhs = $\frac{1}{1(2)(3)}=\frac{1}{6}$ rhs = $\frac{1\times4}{4(2)(3)}=\frac{1}{6}$ (lhs = rhs so) true for n = 1 Checks n = 1 on both sides, achieving $\frac{1}{6}$, and "true for n = 1" oe (e.g., "shown", "QED", a tick, lhs=rhs) seen anywhere. Evaluation of the rhs only is not sufficient		B1
	Assume true for $n = k$ so $$\sum_{r=1}^{k}\frac{1}{r(r+1)(r+2)}=\frac{k(k+3)}{4(k+1)(k+2)}$$		
	$$\sum_{r=1}^{k+1}\frac{1}{r(r+1)(r+2)}=\frac{k(k+3)}{4(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$$ Adds $(k+1)^{\text{th}}$ term. Must be a recognisable attempt at g(k) + f(k + 1)		M1
	$$=\frac{(k+1)(k^2+5k+4)}{4(k+1)(k+2)(k+3)}$$	Attempts single fraction and factorises numerator. **Dependent on previous method mark.**	dM1
	$$=\frac{(k+1)(k+4)}{4(k+2)(k+3)}=\frac{(\underline{k+1})(\underline{k+1}+3)}{4(\underline{k+1}+1)(\underline{k+1}+2)}$$	Correct completion to expression in terms of k + 1 Allow proof to "meet in the middle"	A1
	If the statement is **true for *n* = *k*** then it has been shown to be **true for *n* = *k* + 1** and as the result is **true for *n* = 1**, it is **true for all *n* ∈ ℤ** All parts in bold (or equivalent statements) seen at some stage. Accept "all n", "all (positive) integers/values" or "n = 1, 2, 3..." Allow a correct proof in a different variable provided it is consistent.		A1 cso
			(5)
			Total 10

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