Examiners' Report
Principal Examiner Feedback

January 2019

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F1 (WFM01/01)

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General Introduction

This paper was accessible and there were plenty of opportunities for a typical E grade student to gain some marks across all the questions. There were some testing questions involving matrices, complex numbers and mathematical induction that allowed some discrimination between the higher grades.

It is pleasing to report that students seem to be well-prepared in answering questions that test the topic "Roots of quadratic equations". Most students followed the advice given in Q6 and answered this question without finding the explicit roots of the quadratic equation $12x^2 - 3x + 4 = 0$.

In summary, Q1, Q2, Q3(a), Q4, Q5, Q6, Q7(a), Q9(a), Q9(b), Q9(c) were a good source of marks for the average student, mainly testing standard ideas and techniques and Q3(b), Q7(b), Q8 and Q9(d) were discriminating at the higher grades. Q7(c) proved to be the most challenging question on the paper.

Question 1

This question proved accessible with most students scoring full marks.

In part (a), most students used the correct focus $S(3,\ 0)$ to find a correct equation of l which joined $S(3,\ 0)$ to $A(12,\ 12)$. Most students applied $y - y_1 = m(x - x_1)$ or $y = mx + c$ to find an equation for l. Some students used the points $S(3,\ 0)$ and $A(12,\ 12)$ to write down two linear equations and solved their simultaneous equations to find the gradient and y-intercept of l. A few students applied an incorrect method of differentiating the parabola equation $y^2 = 12x$ and using $A(12,\ 12)$ to find the gradient of l.

In part (b), most students substituted the correct equation $x = -3$ for the directrix of the parabola into the equation for l and stated the correct coordinates for B. Some students incorrectly evaluated $\frac{4}{3}(-3) - 4$ as either 8 or 16. A few students applied an incorrect method of finding the coordinates where l intersects the parabola. A clearly drawn diagram would have undoubtedly helped such students in visualising what was being asked for.

Question 2

This question was accessible to most students with the majority gaining full marks.

In part (a), nearly all students showed sufficient working to prove the result $\mathrm{f}(2) = 0$.

In part (b), most students used a method of long division to factorise $z^3 - 2z^2 + 16z - 32$ to give $(z-2)(z^2+16)$. Those who used the more time-efficient method of comparing coefficients were slightly more prone to making sign or manipulation errors. Only a few students wrote $z^2(z-2)+16(z-2)$ which led to $(z-2)(z^2+16)$. While many students solved $(z-2)(z^2+16)=0$ in part (b) to give $z=2, \pm 4\mathrm{i}$, some gave incorrect solutions such as $z=\pm 4$ or $z=\pm 2\mathrm{i}$ while others omitted the solution $z=2$. A minority of students who did not answer the question "Use algebra to solve $\mathrm{f}(z)=0$ completely", left their final answer to part (b) as either $(z-2)(z^2+16)$ or $(z-2)(z-4\mathrm{i})(z+4\mathrm{i})$ and so did not gain the marks for solving $\mathrm{f}(z)=0$.

In part (c), many students used a ruler and an appropriate scale to plot all three roots on an Argand diagram. It was pleasing to see many Argand diagrams with roots plotted in the correct positions relative to each other and that the roots $4\mathrm{i}$ and $-4\mathrm{i}$ were plotted symmetrically about real axis. There were some students, however, who plotted $4\mathrm{i}$ and $-4\mathrm{i}$ on the real axis.

Question 3

This question was accessible to most students with just over half of them gaining full marks. Part (a) was well-answered by most students. A significant minority of students failed to give a correct method for answering part (b).

In part (a), most students expanded the expression $(2r+5)^2$ and correctly substituted the standard formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ into $\sum_{r=1}^{n}(4r^2+20r+25)$. Some incorrectly expanded $(2r+5)^2$ to give $4r^2+10r+25$, while others simplified $\sum_{r=1}^{n} 25$ to give 25 or 1. Most students obtained a correct $\frac{n}{3}(4n^2+36n+107)$ and used a method of completing the square to give a correct $\frac{n}{3}\left[(2n+9)^2+26\right]$.

In part (b), most students substituted $n=100$ into their part (a) answer with many obtaining 1456900. The majority applied a correct method of adding 25 to 1456900 to give the correct answer 1456925. Incorrect methods included either giving their 1456900 as their final answer or adding 5 to their 1456900. A few students who wrote $\frac{(100)}{3}\left[(2(100)+9)^2+26\right]-\frac{(-1)}{3}\left[(2(-1)+9)^2+26\right]=1456925$ received no credit for this incorrect method.

Question 4

This question was accessible to most students with part (b) more successfully answered than part (a).

In part (a), most students gave a fully correct method of interval bisection with a significant number presenting their method in a table. A few students did not give sufficient values of the function in their solution. In part (a), values for at least one of either $f(-2)$ or $f(-1)$ and values for both $f(-1.5)$ and $f(-1.75)$ are required to justify the final interval. Some students, however, did not give a correctly stated final interval of $[-2,-1.75]$, with some giving no interval at all while others stated a single value for α or an incorrect $-1.75 \le \alpha \le -2$. Students should be made aware that, if interval notation is used, the smaller number should be written first.

In part (b), many students differentiated $f(x)$ correctly and applied the Newton-Raphson procedure correctly to give a second approximation for β as 0.6504. A few students differentiated $2x^3 - \frac{7}{x^2} + 16$ incorrectly to give either $6x^2 - \frac{14}{x^3}$, $6x^2 + \frac{14}{x}$ or $6x^2 + \frac{14}{x^3} + 16x$. In a few cases, a lack of working did mean that it was sometimes difficult for examiners to determine whether the procedure was applied correctly.

Question 5

This question proved accessible with most students scoring full marks.

Students were familiar with what was required in part (a) and most gained full marks in this part. Students used a variety of methods to find $\frac{dy}{dx}$ with the most common being to make y the subject in order to find $\frac{dy}{dx} = -\frac{16}{x^2}$. Most students used the coordinates of P to obtain an expression for $\frac{dy}{dx}$ in terms of p. Many students who progressed this far usually applied a correct straight-line method and achieved the given equation $x + p^2 y = 8p$.

In part (b), many students substituted $x = 7$, $y = 1$ (with a few substituting $x = 1$, $y = 7$) into the given equation. Most solved the resulting quadratic equation, usually by factorisation or by simply stating the two roots. Some students stopped after finding $p = 7, 1$ and so they did not use these values of p to find the coordinates of the two possible positions of P.

Question 6

The majority of students gained full marks in this question. Few students found and applied $\alpha, \beta = \frac{3+\sqrt{183}\,\mathrm{i}}{24}, \frac{3-\sqrt{183}\,\mathrm{i}}{24}$ in this question. These students lost a considerable number of marks because they did not obey the instruction 'Without solving the (quadratic) equation' which was stated in this question.

In part (a), many students correctly manipulated $\frac{2}{\alpha}+\frac{2}{\beta}$ to give $\frac{2(\beta+\alpha)}{\alpha\beta}$. Most deduced that the sum and product of roots in the given quadratic equation $12x^2-3x+4=0$ were $\frac{1}{4}$ and $\frac{1}{3}$ respectively and substituted these into $\frac{2(\beta+\alpha)}{\alpha\beta}$ to give a correct value $\frac{3}{2}$.

The complete method in part (b) was understand by most students, but a considerable number failed to score full marks because they made manipulation or substitution errors. Some students applied $\left(x-\left(\frac{2}{\alpha}-\beta\right)\right)\left(x-\left(\frac{2}{\beta}-\alpha\right)\right)=0$, while most found the values for the sum and product of $\left(\frac{2}{\alpha}-\beta\right)$ and $\left(\frac{2}{\beta}-\alpha\right)$. At this stage, most students proceeded to use a correct method to form the quadratic equation described in the question. The three main errors in establishing the required quadratic equation were: applying the incorrect method of x^2 + (sum)x + (product)$=0$; the omission of "$=0$ "; and the failure to give integer coefficients.

Question 7

Part (a) proved to be accessible to students of all abilities. Part (b) and Part (c) discriminated well across higher ability students with part (b) more successfully answered than part (c).

In part (a), most students achieved the correct matrix $\mathbf{P}^2$ which was followed by a correct method for finding the matrix $\mathbf{P}^3$. A few students who showed that $\mathbf{P}^3=\begin{pmatrix}8 & 0\\ 0 & 8\end{pmatrix}$ did not conclude that this result was equal to 8$\mathbf{I}$.

Many students struggled to gain full marks in part (b), although most of them recognised that **P** represented both an enlargement and a rotation. Some students incorrectly stated the scale factor of enlargement as $\sqrt{3}$ or $\sqrt{2}$ while others stated an incorrect angle of rotation or an incorrect sense of rotation. Some students omitted to state either the centre of enlargement or the centre of rotation.

Many students struggled to access part (c) with a significant number making no attempt. Those students who were successful used their answers to part (a) and part (b) to give a variety of correct solutions. E.g.

- $\mathbf{P}^{35} = (\mathbf{P}^3)^{11} \times \mathbf{P}^2 = (8\mathbf{I})^{11} \times \begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix} = 2^{34}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$
- $\mathbf{P}^{35} = \mathbf{P}^{33} \times \mathbf{P}^2 = (2\mathbf{I})^{33} \times \begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix} = 2^{34}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$
- $\mathbf{P}^{35} = (2)^{35}\begin{pmatrix} \cos 4200 & -\sin 4200 \\ \sin 4200 & \cos 4200 \end{pmatrix} = 2^{34}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$

Some students found a correct $\mathbf{P}^4 = 2^3\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$ and used this result to deduce an incorrect $\mathbf{P}^{35} = 2^{34}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$.

Question 8

Students are well-prepared for this discriminating question with part (i) more successfully answered than part (ii).

In part (i), some students, who just wrote for example "for $n=1$, LHS = RHS" did not gain credit for demonstrating that the general result is true for $n=1$. Many students wrote down the correct matrix multiplication, but some students lost marks for moving from $\begin{pmatrix} 1+4k & -8k \\ 2k & 1-4k \end{pmatrix}\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$ directly to $\begin{pmatrix} 1+4(k+1) & -8(k+1) \\ 2(k+1) & 1-4(k+1) \end{pmatrix}$ with no intermediate working. Some students made arithmetic or sign slips when multiplying out their matrices and a small number failed to express the elements of their multiplied out matrix in terms of $k+1$. There were many very well-prepared students who gave a fully correct concluding statement either at the end of their proof or as a narrative in their proof. In part (i), a minimal acceptable proof, following on from completely correct work, would incorporate the following: assuming the general result is true for $n=k$; then showing the general result is true for $n=k+1$; showing the general result is true for $n=1$; and finally concluding that the general result is true for all positive integers.

In part (ii), some students failed to demonstrate that the general result was true for both $n=1$ and $n=2$. Many students substituted $u_{k+1} = 6^{k+1} + 2^{k+1}$ and $u_k = 6^k + 2^k$ into $u_{k+2} = 8u_{k+1} - 12u_k$ and manipulated their expression to give the result $u_{k+2} = 6^{k+2} + 2^{k+2}$, although some students fudged this correct result from incorrect intermediate work. Some students did not bring all strands of their proof together to give a fully correct proof. In part (ii), a minimal acceptable proof, following on from completely correct work, would incorporate the following: assuming the general result is true for $n=k$ and $n=k+1$; then showing the general result is true for $n=k+2$;

showing the general result is true for $n=1$ and $n=2$; and finally concluding that the general result is true for all positive integers.

Question 9

Parts (a), (b) and (c) proved to be accessible to students of all abilities. Part (d) provided good discrimination for the more able students.

In part (a), most students used z_1 and z_2 to find a correct $z_1 - z_2 = -4+3\mathrm{i}$. Some students did not realise that the required answer was in the second quadrant and many of them found an angle in either the first or third quadrants. Most students worked in radians, but a few gave their answer in degrees.

Part (b) was well attempted with many correct solutions. Common errors included mutipliying out $(3-4\mathrm{i})(3+4\mathrm{i})$ incorrectly to give $9+4=13$, or giving their final answer as $\dfrac{1-7\mathrm{i}}{25}$ which is not in the required form $a+\mathrm{i}b$.

Those students who had answered part (b) correctly usually found the correct exact answer for the modulus in part (c). Those who made mistakes in part (b) usually used a correct Pythagoras method for finding the modulus in part (c).

In part (d), most students multiplied both sides of the printed equation by $(p-\mathrm{i}q-8z_2)$ and applied z_1 and z_2 to give $p+\mathrm{i}q-8(-1-\mathrm{i}) = 3\mathrm{i}(p-\mathrm{i}q-8(3-4\mathrm{i}))$. Some students gave up at this point or produced work that did not gain any credit. Other students applied a complete method of equating the real and imaginary parts of both sides of their equation followed by solving their simultaneous equations to find the values for p and q. Sign errors, bracketing errors and incorrectly multiplying by the complex number i prevented some students from obtaining a correct $p=43$, $q=49$. A few students who tried to "rationalise" $\dfrac{p+\mathrm{i}q-8z_1}{p-\mathrm{i}q-8z_2}$ by writing $\dfrac{((p+8)+\mathrm{i}(q+8))((p-24)-\mathrm{i}(-q+32))}{((p-24)+\mathrm{i}(-q+32))((p-24)-\mathrm{i}(-q+32))}$ scored no marks in this part.