Pearson Edexcel

# Mark Scheme (Results)

Summer 2018

Pearson Edexcel International A Level in Further Pure Mathematics F2 (WFM02/01)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

**EDEXCEL GCE MATHEMATICS**

## General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Further Pure Mathematics Marking

*(But note that specific mark schemes may sometimes override these general principles).*

**<u>Method mark for solving 3 term quadratic:</u>**

### 1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

### 2. Formula

Attempt to use the correct formula (with values for a, b and c).

### 3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = …

**<u>Method marks for differentiation and integration:</u>**

### 1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

### 2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

## Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

## Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

## Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

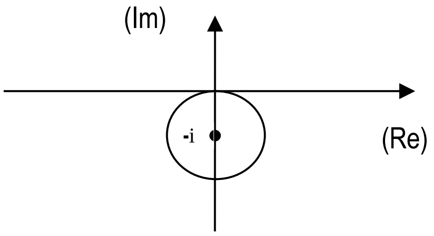
# June 2018
# WFM02 Further Pure Mathematics F2
# Mark Scheme

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| 1 | $\frac{1}{x-2} > \frac{2}{x}$ | | |
| | $\frac{1}{x-2} - \frac{2}{x} > 0 \Rightarrow \frac{4-x}{x(x-2)} > 0$ | Collect to one side and attempt common denominator of $x(x-2)$ | M1 |
| | $x = \underline{0,\ 2},\ \underline{\underline{4}}$ | B1 for 0 **and** 2, A1 for 4 | <u>B1</u>, <u>A1</u> |
| | $x<0,\ \ 2<x<4$<br>For their critical values $\alpha, \beta$ and $\gamma$ in ascending order, attempts $x<\alpha$ and $\beta<x<\gamma$ condoning the use of a mixture of open or closed inequalities<br>or<br>For **one of** $x<0$ **or** $2<x<4$ condoning the use of a mixture of open or closed inequalities | | M1 |
| | $x<0,\ \ 2<x<4$<br>$(-\infty,0)$ or $[-\infty,0),\ \ (2,\ 4)$ | Correct inequalities. Ignore what they have between their inequalities e.g. allow "or", "and", "," etc. but not $\cap$ | A1 |
| | | | **(5)** |
| | | | **Total 5** |
| | **Alternative 1:** $\times x^2(x-2)^2$ | | |
| | $x^2(x-2) > 2x(x-2)^2$ | | |
| | $x^2(x-2) - 2x(x-2)^2 > 0$ | | |
| | $x(x-2)(4-x) > 0$ | $\times x^2(x-2)^2$ and attempt to factorise by taking out a factor of $x(x-2)$ | M1 |
| | $x = \underline{0,\ 2},\ \underline{\underline{4}}$ | B1 for 0 **and** 2, A1 for 4 | <u>B1</u>, <u>A1</u> |
| | **Notes:**<br>$-x^3+6x^2-8x>0$ with no other working is M0<br>$-x^3+6x^2-8x>0 \Rightarrow x=0,\ 2$ is M1B1<br>$-x^3+6x^2-8x>0 \Rightarrow x=0,\ 2,\ 4$ is M1B1A1 | | |
| | $x<0,\ \ 2<x<4$<br>For their critical values $\alpha, \beta$ and $\gamma$ in ascending order, attempts $x<\alpha$ and $\beta<x<\gamma$ condoning the use of a mixture of open or closed inequalities<br>**or**<br>For **one of** $x<0$ **or** $2<x<4$ condoning the use of a mixture of open or closed inequalities | | <u>M1</u> |
| | $x<0,\ \ 2<x<4$<br>$(-\infty,0)$ or $[-\infty,0),\ \ (2,\ 4)$ | Correct inequalities. Ignore what they have between their inequalities e.g. allow "or", "and", "," etc. but not $\cap$ | A1 |

| | | | |
|---|---|---|---|
| | **Alternative 2 : Considers regions** | | |
| | Case 1<br>$x<0 \Rightarrow x-2<0, x<0 \Rightarrow x(x-2)>0$<br>$\Rightarrow x>2(x-2) \Rightarrow x<0$ | | |
| | Case 2<br>$0<x<2 \Rightarrow x-2<0, x>0 \Rightarrow x(x-2)<0$<br>$\Rightarrow x<2(x-2) \Rightarrow x>4 \Rightarrow$ Contradiction | | |
| | Case 3<br>$x>2 \Rightarrow x-2>0, x>0 \Rightarrow x(x-2)>0$<br>$\Rightarrow x>2(x-2) \Rightarrow x<4 \Rightarrow 2<x<4$ Contradiction | | |
| | **M1: Considers 3 regions as above**<br>**B1: *x* = 0 and 2 seen as critical values**<br>**A1: *x* = 4 seen as a critical value** | | |
| | $x<0,\ \ 2<x<4$<br>For their critical values $\alpha, \beta$ and $\gamma$ in ascending order, attempts $x<\alpha$ and $\beta<x<\gamma$ condoning the use of a mixture of open or closed inequalities<br>**or**<br>For **one of** $x<0$ **or** $2<x<4$ condoning the use of a mixture of open or closed inequalities | | M1 |
| | $x<0,\ \ 2<x<4$<br>$(-\infty,0)$ or $[-\infty,0)$, $(2,\ 4)$ | Correct inequalities. Ignore what they have between their inequalities e.g. allow “or”, “and”, “,” etc. but not $\cap$ | A1 |

| Question Number | Scheme | Notes | Marks |
| --- | --- | --- | --- |
| **2(a)** | $\left(x^2+1\right)\frac{\mathrm{d}y}{\mathrm{d}x}+xy-x=0$ | | |
| | $\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{xy}{\left(1+x^2\right)}=\frac{x}{\left(1+x^2\right)}$ | Correct form. | B1 |
| | $I=\mathrm{e}^{\int\frac{x}{1+x^2}dx}=\mathrm{e}^{\frac{1}{2}\ln\left(1+x^2\right)}=\left(1+x^2\right)^{\frac{1}{2}}$ | M1: $I=\mathrm{e}^{\int\frac{x}{1+x^2}dx}=\mathrm{e}^{k\ln\left(1+x^2\right)}$ where $k$ is a constant. (Condone missing brackets around the $x^2+1$)<br>A1: Correct integrating factor of $\left(1+x^2\right)^{\frac{1}{2}}$ | M1A1 |
| | $y\left(1+x^2\right)^{\frac{1}{2}}=\int\frac{x}{\left(1+x^2\right)^{\frac{1}{2}}}\mathrm{d}x$ | Uses their integration factor to reach the form $yI=\int Q\,\mathrm{I}\,\mathrm{d}x$ | M1 |
| | $=\left(1+x^2\right)^{\frac{1}{2}}(+c)$ | Correct integration (+ $c$ not needed) | A1 |
| | $y=1+c\left(1+x^2\right)^{-\frac{1}{2}}$ oe | Cao with the constant correctly placed. (The "$y=$" must appear at some point) | A1 |
| | | | **(6)** |
| **Way 2** | **Alternative by separation of variables:** | | |
| | $\int\frac{\mathrm{d}y}{1-y}=\int\frac{x}{x^2+1}\,\mathrm{d}x$ | Separates variables correctly | B1 |
| | $\int\frac{x}{x^2+1}\,\mathrm{d}x=\frac{1}{2}\ln\left(x^2+1\right)$ | M1: $\int\frac{x}{x^2+1}\,\mathrm{d}x=k\ln\left(x^2+1\right)$ where $k$ is a constant. (Condone missing brackets around the $x^2+1$)<br>A1: Correct integration $\frac{1}{2}\ln\left(x^2+1\right)$ | M1A1 |
| | $\int\frac{\mathrm{d}y}{1-y}=-\ln(1-y)$ | $\int\frac{\mathrm{d}y}{1-y}=k\ln(1-y)$ or e.g.<br>$\int\frac{\mathrm{d}y}{y-1}=k\ln(y-1)$ | M1 |
| | $-\ln(1-y)=\frac{1}{2}\ln\left(x^2+1\right)(+c)$ | Fully correct integration | A1 |
| | $y=1+c\left(1+x^2\right)^{-\frac{1}{2}}$ oe | Cao and isw if necessary. | A1 |
| | | | **(6)** |
| **(b)** | $2=1+c\left(1+3^2\right)^{-\frac{1}{2}}\Rightarrow c=...$ | Substitutes $x=3$ and $y=2$ and attempts to find a value for $c$. | M1 |
| | $(y=)1+\sqrt{10}\left(1+x^2\right)^{-\frac{1}{2}}$ oe | Cao. ("$y=$" not needed for this mark) and apply isw if necessary. | A1 |
| | | | **(2)** |
| | | | **Total 8** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **3** | $2\frac{d^2y}{dx^2}+\frac{dy}{dx}-xy=1$ | | |
| **(a)** | $2\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}-x\frac{dy}{dx}-y=0$ | B1: $2\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}$ or equivalent correct terms if they rearrange the given equation.<br>M1: Attempt product rule on *xy*. Allow sign errors only so need to see $\pm x\frac{dy}{dx}\pm y$ | B1M1 |
| | $2\frac{d^4y}{dx^4}+\frac{d^3y}{dx^3}-x\frac{d^2y}{dx^2}-\frac{dy}{dx}-\frac{dy}{dx}=0$ | Differentiates again to obtain an expression that contains the fourth derivative including product rule on $x\frac{dy}{dx}$ to give $\pm x\frac{d^2y}{dx^2}\pm\frac{dy}{dx}$.<br>(Allow terms to be "listed") | M1 |
| | $\frac{d^4y}{dx^4}=\frac{1}{2}\left(2\frac{dy}{dx}+x\frac{d^2y}{dx^2}-\frac{d^3y}{dx^3}\right)$ | If the "1" is not dealt with correctly e.g. if it "disappears" at the wrong time, this mark should be withheld. | A1 |
| | | | **(4)** |
| **(b)** | $y''(2)=1,\ y'''(2)=1,\ y''''(2)=\frac{3}{2}$ | M1: Attempt $y''(2)$, $y'''(2)$ and $y''''(2)$<br>A1: Correct values | M1A1 |
| | $y=f(2)+(x-2)f'(2)+\frac{(x-2)^2f''(2)}{2!}+\frac{(x-2)^3f'''(2)}{3!}+\frac{(x-2)^4f''''(2)}{4!}$<br>Attempt correct Taylor expansion **with their values.** Allow the terms to be "listed" for this mark. | | M1 |
| | $(y=)1+(x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{6}+\frac{(x-2)^4}{16}$ | Correct simplified expression. | A1 |
| | | | **(4)** |
| **(c)** | $x=2.1\Rightarrow y=1+(0.1)+\frac{(0.1)^2}{2}+\frac{(0.1)^3}{6}+\frac{(0.1)^4}{16}$ | Substitutes *x* = 2.1 into an expansion involving (*x* – 2) | M1 |
| | *y* = 1.105 **only**<br>Note this is **not** awrt. | Cao (Note that this mark must follow the final A1 in (b) i.e. 1.105 must come from a correct expansion).<br>Incorrect answer with no working scores M0. Correct answer following a correct expansion scores M1A1. | A1 |
| | | | **(2)** |
| | | | **Total 10** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **4(a)** |  | M1: A circle anywhere.<br>A1: A circle **correctly positioned** with centre –i or -1 marked in the correct place or (0, -1) or (-1, 0) or (0, -i) or (–i, 0) marked in the **correct place** and passing through (0, 0). The centre may be indicated away from the sketch but the sketch takes precedence. Ignore any shading. | M1A1 |
| | | | **(2)** |
| **(b)** **Way 1** | $w=\dfrac{3iz-2}{z+i}$ | | |
| | $z=\dfrac{wi+2}{3i-w}$ | M1: Attempt to make $z$ the subject<br>A1: Correct rearrangement oe | M1A1 |
| | $z+i=\dfrac{wi+2}{3i-w}+i=\dfrac{wi+2-3-wi}{3i-w}$ | Applies $z$ + i and finds common denominator | M1 |
| | $\left\|\dfrac{wi+2-3-wi}{3i-w}\right\|=1$ | M1: Sets $\|z+i\|=1$<br>A1: Correct equation, simplified or unsimplified | M1A1 |
| | Note if they work with $w = u + iv$ they should reach $\left\|\dfrac{2-v+ui-ui-(3-v)}{-u+(3-v)i}\right\|=1$ * | | |
| | $\left\|\dfrac{-1}{3i-w}\right\|=1\Rightarrow\|w-3i\|=1\Rightarrow\|u+iv-3i\|=1$<br>$\Rightarrow u^2+(3-v)^2=1$ or equivalent e.g. $u^2+(v-3)^2=1$, $u^2+v^2-6v+9=1$<br>**d**M1: Introduces $u$ and $v$ or $x$ and $y$ (may occur earlier *) and uses Pythagoras correctly to find a Cartesian form<br>**This mark is dependent on all the previous method marks**<br>A1: Correct equation (allow $u$, $v$ or $x$, $y$ or $a$, $b$) | | **d**M1A1 |
| | | | **(7)** |
| | | | |

**In part (b) apply the scheme that is most beneficial to the candidate.**

| | | | |
|---|---|---|---|
| **Way 2** | $z=\dfrac{w\text{i}+2}{3\text{i}-w}$ | M1: Attempt to make $z$ the subject<br>A1: Correct rearrangement oe | M1A1 |
| | $z=\dfrac{(u+\text{i}v)\text{i}+2}{3\text{i}-(u+\text{i}v)}=\dfrac{(2-v)+u\text{i}}{-u+(3-v)\text{i}}=\dfrac{(2-v)+u\text{i}}{-u+(3-v)\text{i}}\times\dfrac{-u-(3-v)\text{i}}{-u-(3-v)\text{i}}$<br>Introduces $u$ + i$v$ and multiplies numerator and denominator by the complex conjugate of the denominator | | M1 |
| | $z+\text{i}=\dfrac{u+(5v-6-u^2-v^2)\text{i}+(u^2+v^2+9-6v)\text{i}}{u^2+(3-v)^2}\left(=\dfrac{u+(3-v)\text{i}}{u^2+(3-v)^2}\right)$<br>M1: Applies $z$ + i and finds a common denominator<br>A1: Correct expression (simplified or unsimplified) but with no i's in the denominator | | M1A1 |
| | $\lvert z+\text{i}\rvert=1\Rightarrow\left\lvert\dfrac{u+(3-v)\text{i}}{u^2+(3-v)^2}\right\rvert=1\Rightarrow\dfrac{\sqrt{u^2+(3-v)^2}}{u^2+(3-v)^2}=1$ oe<br>**d**M1: Introduces $u$ and $v$ or $x$ and $y$ (may occur earlier *) and uses Pythagoras correctly to find a Cartesian form which may be unsimplified<br>**<u>This mark is dependent on all the previous method marks</u>**<br>A1: Correct equation (allow $u$, $v$ or $x$, $y$ or $a$, $b$) | | **d**M1A1 |
| | | | **(7)** |

| | | | |
|---|---|---|---|
| **Way 3** | $z=\dfrac{w\text{i}+2}{3\text{i}-w}$ | M1: Attempt to make $z$ the subject<br>A1: Correct rearrangement oe | M1A1 |
| | $z=\dfrac{(u+\text{i}v)\text{i}+2}{3\text{i}-(u+\text{i}v)}=\dfrac{(2-v)+u\text{i}}{-u+(3-v)\text{i}}=\dfrac{(2-v)+u\text{i}}{-u+(3-v)\text{i}}\times\dfrac{-u-(3-v)\text{i}}{-u-(3-v)\text{i}}$<br>Introduces $u$ + i$v$ and multiplies numerator and denominator by the complex conjugate of the denominator | | M1 |
| | $z=\dfrac{u+(-u^2-v^2+5v-6)\text{i}}{u^2+(3-v)^2}\Rightarrow x=\dfrac{u}{u^2+(3-v)^2}\quad y=-\dfrac{u^2+(v-3)^2+v-3}{u^2+(3-v)^2}$<br>M1: Obtains $x$ and $y$ in terms of $u$ and $v$<br>A1: Correct equations | | M1A1 |
| | $x^2+(y+1)^2=1\Rightarrow\dfrac{u^2+(v-3)^2}{\left(u^2+(v-3)^2\right)^2}=1$ oe | **d**M1: Uses $\lvert z+\text{i}\rvert=1$ to find an equation connecting $u$ and $v$<br>**<u>This mark is dependent on all the previous method marks</u>**<br>A1: Correct equation which may be unsimplified. | **d**M1A1 |
| | | | **(7)** |

| Way 4 | | | |
|---|---|---|---|
| | $$w=\frac{3\text{i}z-2}{z+\text{i}}$$ | | |
| | $z=\dfrac{w\text{i}+2}{3\text{i}-w}$ | M1: Attempt to make $z$ the subject<br>A1: Correct rearrangement oe | M1A1 |
| | $z+\text{i}=\dfrac{w\text{i}+2}{3\text{i}-w}+\text{i}=\dfrac{w\text{i}+2-3-w\text{i}}{3\text{i}-w}$ | Applies $z$ + i and finds common denominator | M1 |
| | $z+\text{i}=\dfrac{-1}{3\text{i}-u-\text{i}v}\times\dfrac{u-(v-3)\text{i}}{u-(v-3)\text{i}}=\dfrac{u-(v-3)\text{i}}{u^2+(v-3)^2}$<br>$\Rightarrow\left\lvert\dfrac{u-(v-3)\text{i}}{u^2+(v-3)^2}\right\rvert=1$<br>M1: Multiplies numerator and denominator by the complex conjugate of the denominator and sets = 1<br>A1: Correct equation with no i's in the denominator | | M1A1 |
| | $\dfrac{\sqrt{u^2+(3-v)^2}}{u^2+(3-v)^2}=1$ oe<br>**d**M1: Introduces $u$ and $v$ or $x$ and $y$ (may occur earlier *) and uses Pythagoras correctly to find a Cartesian form which may be unsimplified<br>**<u>This mark is dependent on all the previous method marks</u>**<br>A1: Correct equation (allow $u$, $v$ or $x$, $y$ or $a$, $b$) | | **d**M1A1 |
| | | | **(7)** |

| Way 5 | | | |
|---|---|---|---|
| | $$w=\frac{3\text{i}z-2}{z+\text{i}}$$ | | |
| | $u+\text{i}v=\dfrac{3\text{i}(x+\text{i}y)-2}{x+\text{i}y+\text{i}}=\dfrac{(3\text{i}x-3y-2)(x-(y+1)\text{i})}{x^2+(y+1)^2}$ | M1: Substitutes for $z$ and $\times\dfrac{x-(y+1)\text{i}}{x-(y+1)\text{i}}$<br>A1: Correct expression | M1A1 |
| | $=\dfrac{x+\left(3\left(x^2+(y+1)^2\right)-y-1\right)\text{i}}{x^2+(y+1)^2}$ | Express rhs in terms of $x^2+(y+1)^2$ | M1 |
| | $x^2+(y+1)^2=1\Rightarrow w=x+(2-y)\text{i}$ | M1: Use of $\lvert z+\text{i}\rvert=1$<br>A1: $w=x+(2-y)\text{i}$ | M1A1 |
| | $x^2+(y+1)^2=1\Rightarrow u^2+(v-3)^2=1$ | **d**M1: Attempts equation connecting $u$ and $v$<br>**<u>This mark is dependent on all the previous method marks</u>**<br>A1: $u^2+(v-3)^2=1$ oe | **d**M1A1 |
| | | | **(7)** |

| | | | |
|---|---|---|---|
| **Way 6** | $w=\dfrac{3\text{i}z-2}{z+\text{i}}=\dfrac{3\text{i}(z+\text{i})+1}{z+\text{i}}=3\text{i}+\dfrac{1}{z+\text{i}}$ | M1: Attempt rhs in terms of $z$ + i<br>A1: Correct rearrangement oe | M1A1 |
| | $w-3\text{i}=\dfrac{1}{z+\text{i}}$ | Isolates $z$ + i | M1 |
| | $\lvert w-3\text{i}\rvert=\left\lvert\dfrac{1}{z+\text{i}}\right\rvert=\dfrac{1}{\lvert z+\text{i}\rvert}=1$ | M1: Applies $\lvert z+\text{i}\rvert=1$<br>A1: Correct equation | M1A1 |
| | $\lvert w-3\text{i}\rvert=1\Rightarrow u^2+(v-3)^2=1$ | **d**M1: Introduces $u$ and $v$ or $x$ and $y$ and uses Pythagoras correctly to find a Cartesian form<br>**<u>This mark is dependent on all the previous method marks</u>**<br>A1: $u^2+(v-3)^2=1$ oe | **d**M1A1 |
| | | | **(7)** |

| | | | |
|---|---|---|---|
| **Way 7** | $z=\dfrac{w\text{i}+2}{3\text{i}-w}$ | M1: Attempt to make $z$ the subject<br>A1: Correct rearrangement oe | M1A1 |
| | $\lvert w\rvert=\left\lvert\dfrac{3\text{i}z-2}{z+\text{i}}\right\rvert=\lvert 3\text{i}z-2\rvert$ | Uses $\lvert w\rvert=\left\lvert\dfrac{3\text{i}z-2}{z+\text{i}}\right\rvert$ and $\lvert z+\text{i}\rvert=1$ | M1 |
| | $\lvert w\rvert=\left\lvert 3\text{i}\left(\dfrac{w\text{i}+2}{3\text{i}-w}\right)-2\right\rvert=\left\lvert\dfrac{-3w+6\text{i}-6\text{i}+2w}{3\text{i}-w}\right\rvert$ | M1: Attempts common denominator<br>A1: Correct equation | M1A1 |
| | $\lvert w-3\text{i}\rvert=1\Rightarrow u^2+(v-3)^2=1$ | **d**M1: Introduces $u$ and $v$ or $x$ and $y$ and uses Pythagoras correctly to find a Cartesian form<br>**<u>This mark is dependent on all the previous method marks</u>**<br>A1: $u^2+(v-3)^2=1$ oe | **d**M1A1 |
| | | | **(7)** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **5(a)** | $\dfrac{4r+2}{r(r+1)(r+2)}$ | | |
| | $\dfrac{1}{r}+\dfrac{2}{(r+1)}-\dfrac{3}{(r+2)}$ | M1: Correct partial fractions method e.g. substitution or compares coefficients to obtain one of *A*, *B* or *C* for $\dfrac{A}{r}, \dfrac{B}{(r+1)}, \dfrac{C}{(r+2)}$ <br> A1: 2 Correct fractions (or values) <br> A1: All correct (fractions or values) | M1A1 A1 |
| | Correct answer with no working scores full marks in (a) | | |
| | | | **(3)** |
| **(b)** | **Must have partial fractions of the form $\dfrac{A}{r}, \dfrac{B}{(r+1)}, \dfrac{C}{(r+2)}$ $A, B, C \neq 0$ to score the <u>first</u> M mark in (b)** | | |
| | $\sum_{r=1}^{n}=\left(\dfrac{1}{1}+\dfrac{2}{2}-\dfrac{3}{3}\right)+\left(\dfrac{1}{2}+\dfrac{2}{3}-\dfrac{3}{4}\right)+\ldots$ <br> $\ldots+\left(\dfrac{1}{n-1}+\dfrac{2}{n}-\dfrac{3}{n+1}\right)+\left(\dfrac{1}{n}+\dfrac{2}{n+1}-\dfrac{3}{n+2}\right)$ <br> Attempts at least the first 2 groups of terms and the last 2 groups of terms **which may be implied by their fractions identified below**. <br> Allow other letters for *n* (most likely to be *r*) except for the final mark – see below <br> If terms are found beyond the limits of the summation e.g. *r* = 0, *r* = *n* + 1, these can be ignored for this mark as long as at least the terms for *r* = 1, 2, *n* – 1 and *n* are seen | | M1 |
| | $=\dfrac{1}{1}+\dfrac{2}{2}+\dfrac{1}{2}-\dfrac{3}{n+1}+\dfrac{2}{n+1}-\dfrac{3}{n+2}$ | A1: $\dfrac{1}{1}+\dfrac{2}{2}+\dfrac{1}{2}\left(=\dfrac{5}{2}\right)$ identified as the only constant terms <br> A1: $-\dfrac{3}{n+1}+\dfrac{2}{n+1}-\dfrac{3}{n+2}$ oe e.g $-\dfrac{1}{n+1}-\dfrac{1}{n+2}-\dfrac{2}{n+2}$ identified as the only algebraic terms | A1 A1 |
| | $=\dfrac{5(n^2+3n+2)-2(n+2)-6(n+1)}{2(n+1)(n+2)}$ | Attempt common denominator from terms of the form $A, \dfrac{B}{n+1}, \dfrac{C}{n+2}$ only. <br> Must see (*n* + 1)(*n* + 2) in the denominator and an unsimplified polynomial of order 2 in the numerator. | M1 |
| | $\dfrac{n(5n+7)}{2(n+1)(n+2)}$ | Must be in terms of *n* for this mark. | A1 |
| | | | **(5)** |
| | | | **Total 8** |

| | | | |
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| | **Alternative for (b)** | | |
| | $\frac{1}{r}+\frac{2}{(r+1)}-\frac{3}{(r+2)}=\left(\frac{1}{r}-\frac{1}{r+2}\right)+2\left(\frac{1}{r+1}-\frac{1}{r+2}\right)$<br>$\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+2}\right)=\frac{1}{1}-\frac{1}{3}+\frac{1}{2}-\frac{1}{4}+...+\frac{1}{n-1}-\frac{1}{n+1}+\frac{1}{n}-\frac{1}{n+2}=1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}$<br>$2\sum_{r=1}^{n}\left(\frac{1}{r+1}-\frac{1}{r+2}\right)=\frac{1}{2}-\frac{1}{3}+...+\frac{1}{n}-\frac{1}{n+2}=\frac{1}{2}-\frac{1}{n+2}$<br>Re-writes their partial fractions correctly and attempts at least 2 groups of terms at start and end for first sum **and** 1 group at the start and end for the second sum | | M1 |
| | $\sum_{r=1}^{n}=\frac{5}{2}-\frac{1}{n+1}-\frac{3}{n+2}$ | A1: $\frac{1}{1}+\frac{2}{2}+\frac{1}{2}\left(=\frac{5}{2}\right)$ identified as the only constant terms<br>A1: A1: $-\frac{3}{n+1}+\frac{2}{n+1}-\frac{3}{n+2}$ oe e.g $-\frac{1}{n+1}-\frac{1}{n+2}-\frac{2}{n+2}$ identified as the only algebraic terms | A1A1 |
| | $=\frac{5\left(n^2+3n+2\right)-2\left(n+2\right)-6\left(n+1\right)}{2\left(n+1\right)\left(n+2\right)}$ | Attempt common denominator from terms of the form $A,\frac{B}{n+1},\frac{C}{n+2}$ only.<br>Must see $(n + 1)(n + 2)$ in the denominator and an unsimplified polynomial of order 2 in the numerator. | M1 |
| | $\frac{n\left(5n+7\right)}{2\left(n+1\right)\left(n+2\right)}$ | Must be in terms of $n$ for this mark. | A1 |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **6** | $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-3x\frac{\mathrm{d}y}{\mathrm{d}x}+3y=x^2$ | | |
| **(a)** | $x=\mathrm{e}^t\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{e}^t\frac{\mathrm{d}t}{\mathrm{d}y}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{-t}\frac{\mathrm{d}y}{\mathrm{d}t}$ | M1: Attempt first derivative using the chain rule to obtain $\frac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{e}^t\frac{\mathrm{d}t}{\mathrm{d}y}$<br>A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{-t}\frac{\mathrm{d}y}{\mathrm{d}t}$ oe | M1A1 |
| | $\frac{\mathrm{d}y}{\mathrm{d}x}=x^{-1}\frac{\mathrm{d}y}{\mathrm{d}t}\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-x^{-2}\frac{\mathrm{d}y}{\mathrm{d}t}+x^{-1}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}.\frac{\mathrm{d}t}{\mathrm{d}x}$ | dM1: Attempt product rule and chain rule. Dependent on the first method mark and must be a fully correct method with sign errors only<br>A1: Correct second derivative oe | dM1A1 |
| | $x^2\left(\frac{1}{x^2}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-\frac{1}{x^2}\frac{\mathrm{d}y}{\mathrm{d}t}\right)-3x\left(\frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}t}\right)+3y=\left(\mathrm{e}^t\right)^2$ | Substitutes their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $t$ into the differential equation | M1 |
| | $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-4\frac{\mathrm{d}y}{\mathrm{d}t}+3y=\mathrm{e}^{2t}$ | cso | A1 |
| | | | **(6)** |
| | Alternative | | |
| | $x=\mathrm{e}^t\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}t}=\mathrm{e}^t\frac{\mathrm{d}y}{\mathrm{d}x}=x\frac{\mathrm{d}y}{\mathrm{d}x}$ | M1: Attempt first derivative using $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}x}{\mathrm{d}t}\times\frac{\mathrm{d}y}{\mathrm{d}x}$<br>A1: $\frac{\mathrm{d}y}{\mathrm{d}t}=x\frac{\mathrm{d}y}{\mathrm{d}x}$ oe | M1A1 |
| | $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}=\frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}y}{\mathrm{d}x}+x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}.\frac{\mathrm{d}x}{\mathrm{d}t}=x\frac{\mathrm{d}y}{\mathrm{d}x}+x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ | dM1: Attempt product rule and chain rule. Dependent on the first method mark and must be a fully correct method with sign errors only<br>A1: Correct second derivative oe | dM1A1 |
| | $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-x\frac{\mathrm{d}y}{\mathrm{d}x}-3x\frac{\mathrm{d}y}{\mathrm{d}x}+3y=\mathrm{e}^{2t}$<br>$=\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-\frac{\mathrm{d}y}{\mathrm{d}t}-3\frac{\mathrm{d}y}{\mathrm{d}t}+3y=\mathrm{e}^{2t}$ | Substitutes their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $x\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $t$ into the differential equation | M1 |
| | $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-4\frac{\mathrm{d}y}{\mathrm{d}t}+3y=\mathrm{e}^{2t}$ | Cso | A1 |
| | | | |
| | | | **(6)** |

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| **(b)** | $m^2 - 4m + 3 = 0 \Rightarrow m = 1,\ 3$ | Solves (according to the General Guidance) the correct quadratic (so should be $m = \pm 1, \pm 3$) | M1 |
| | $(y =)\, A\mathrm{e}^{3t} + B\mathrm{e}^{t}$ | Correct CF in terms of *t* not *x*. (May be seen later in their GS) | A1 |
| | $y = k\mathrm{e}^{2t},\ y' = 2k\mathrm{e}^{2t},\ y'' = 4k\mathrm{e}^{2t}$ | Correct form for PI and differentiates twice to obtain multiples of $\mathrm{e}^{2t}$ each time but do not allow if they are clearly integrating. | M1 |
| | $4k\mathrm{e}^{2t} - 8k\mathrm{e}^{2t} + 3k\mathrm{e}^{2t} = \mathrm{e}^{2t} \Rightarrow k = \ldots$ | Substitutes their $y,\ y',\ y''$ that are of the form $\alpha\mathrm{e}^{2t}$ into the differential equation and sets = $\mathrm{e}^{2t}$ and proceeds to find their *k* | M1 |
| | $(y) = -\mathrm{e}^{2t}$ | Correct PI or $k = -1$ | A1 |
| | $y = A\mathrm{e}^{3t} + B\mathrm{e}^{t} - \mathrm{e}^{2t}$ | Correct ft GS **in terms of *t*** (their CF + their PI with non-zero PI).<br>**Must be *y* = …** | B1ft |
| | | | **(6)** |
| **(c)** | $(y =)\, Ax^3 + Bx - x^2$ | Allow equivalent expressions in terms of *x* e.g. $(y =)\, A\mathrm{e}^{3\ln x} + B\mathrm{e}^{\ln x} - \mathrm{e}^{2\ln x}$.<br>**Note that *y* = … is not needed here.** | B1 |
| | | | **(1)** |
| | | | **Total 13** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **7(a)** | $(\cos\theta+\mathrm{i}\sin\theta)^7=\cos^7\theta+\binom{7}{1}\cos^6\theta\mathrm{i}\sin\theta+\binom{7}{2}\cos^5\theta(\mathrm{i}\sin\theta)^2+...$<br>Attempts to expand $(\cos\theta+i\sin\theta)^7$ including a recognisable attempt at binomial coefficients<br>(May only see real terms) | | M1 |
| | $(\cos 7\theta=)c^7+{}^7\mathrm{C}_2c^5\mathrm{i}^2s^2+{}^7\mathrm{C}_4c^3\mathrm{i}^4s^4+{}^7\mathrm{C}_6c\mathrm{i}^6s^6$<br>Identifies real terms with $\cos 7\theta$ | | M1 |
| | $=c^7-21c^5s^2+35c^3s^4-7cs^6$ | Correct expression with coefficients evaluated and i's dealt with correctly | A1 |
| | $=c^7-21c^5(1-c^2)+35c^3(1-c^2)^2-7c(1-c^2)^3$ | Replaces $\sin^2\theta$ with $1-\cos^2\theta$ used anywhere in their expansion. | M1 |
| | $=22c^7-21c^5+35c^3(1-2c^2+c^4)-7c(1-3c^2+3c^4-c^6)$<br>Applies the expansions of $(1-\cos^2\theta)^2$ and $(1-\cos^2\theta)^3$ to their expression | | M1 |
| | $=64\cos^7\theta-112\cos^5\theta+56\cos^3\theta-7\cos\theta*$ | Correct expression obtained with no errors | A1 |
| | **Useful intermediate expression:**<br>$=22c^7-21c^5+35c^3-70c^5+35c^7-7c+21c^2-21c^5+7c^7$ | | |
| | | | **(6)** |

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| | **Alternative 1 for (a):** | |
| | $\left(z+\frac{1}{z}\right)^7=z^7+\binom{7}{1}z^6\frac{1}{z}+\binom{7}{2}z^5\frac{1}{z^2}+...$<br>Attempts to expand $\left(z+\frac{1}{z}\right)^7$ including binomial coefficients | M1 |
| | $\left(z+\frac{1}{z}\right)^7=z^7+\frac{1}{z^7}+7\left(z^5+\frac{1}{z^5}\right)+21\left(z^3+\frac{1}{z^3}\right)+35\left(z+\frac{1}{z}\right)$<br>$(2\cos\theta)^7=2\cos 7\theta+7(2\cos 5\theta)+21(2\cos 3\theta)+35(2\cos\theta)$<br>M1: Uses $z^n+\frac{1}{z^n}=2\cos n\theta$ at least once (including $n$ = 1)<br>A1: Correct expression in terms of cos | M1A1 |
| | $128\cos^7\theta=2\cos 7\theta+14(16\cos^5\theta-20\cos^3\theta+5\cos\theta)+42(4\cos^3\theta-3\cos\theta)+70\cos\theta$<br>M1: Correct method to find $\cos 5\theta$ in terms of $\cos\theta$ and applies this to their expression<br>M1: Correct method to find $\cos 3\theta$ in terms of $\cos\theta$ and applies this to their expression | M1M1 |
| | $\cos 7\theta=64\cos^7\theta-112\cos^5\theta+56\cos^3\theta-7\cos\theta*$ | A1 |
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| | **Alternative 2 for (a):** | |
| | $\left(z+\frac{1}{z}\right)^7 = z^7+\binom{7}{1}z^6\frac{1}{z}+\binom{7}{2}z^5\frac{1}{z^2}+...$<br>Attempts to expand $\left(z+\frac{1}{z}\right)^7$ including binomial coefficients | M1 |
| | $\left(z+\frac{1}{z}\right)^7 = z^7+\frac{1}{z^7}+7\left(z^5+\frac{1}{z^5}\right)+21\left(z^3+\frac{1}{z^3}\right)+35\left(z+\frac{1}{z}\right)$ | |
| | $z^7+\frac{1}{z^7}=2\cos 7\theta=\left(z+\frac{1}{z}\right)^7-7\left(z^5+\frac{1}{z^5}\right)-21\left(z^3+\frac{1}{z^3}\right)-35\left(z+\frac{1}{z}\right)$<br>M1: Identifies that $z^7+\frac{1}{z^7}=2\cos 7\theta$<br>A1: Correct expression for $2\cos 7\theta$ in terms of $z$ | M1A1 |
| | $2\cos 7\theta=128\cos^7\theta-7\left(z^5+\frac{1}{z^5}\right)-21\left(z^3+\frac{1}{z^3}\right)-35\left(z+\frac{1}{z}\right)$<br>Starts the process of replacing $\left(z+\frac{1}{z}\right)^n$ with $(2\cos\theta)^n$ | M1 |
| | $=128\cos^7\theta-7(2\cos\theta)^5+14\left(z^3+\frac{1}{z^3}\right)+35\left(z+\frac{1}{z}\right)$ | |
| | $=128\cos^7\theta-7(2\cos\theta)^5+14(2\cos\theta)^3-7\left(z+\frac{1}{z}\right)$ | |
| | $=128\cos^7\theta-7(2\cos\theta)^5+14(2\cos\theta)^3-14\cos\theta$<br>Reaches an expression in terms of cos only | M1 |
| | $\cos 7\theta=64\cos^7\theta-112\cos^5\theta+56\cos^3\theta-7\cos\theta$ | A1 |
| | | |

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| **(b)** | $\cos 7\theta+1=0\Rightarrow\cos 7\theta=-1$ | $\cos 7\theta=-1$ ($\cos 7x=-1$ is B0) | B1 |
| | $7\theta=\pm180, \pm540, \pm900, \pm1260,...$<br>or<br>$7\theta=\pm\pi, \pm3\pi, \pm5\pi, \pm7\pi,...$ | At least one correct value for $7\theta$. Condone the use of $7x$ here. | M1 |
| | $\theta=\pm\frac{180}{7}, \pm\frac{540}{7}, \pm\frac{900}{7}, \pm\frac{1260}{7},...\Rightarrow\cos\theta=...$<br>or<br>$\theta=\pm\frac{\pi}{7}, \pm\frac{3\pi}{7}, \pm\frac{5\pi}{7}, \pm\frac{7\pi}{7},...\Rightarrow\cos\theta=...$ | Divides by 7 and attempts at least one value for $\cos\theta$. Condone the use of $x$ for $\theta$ here. | M1 |
| | $x=\cos\theta=0.901, 0.223, -1, -0.623$ | A1: Awrt 2 correct values for $x$<br>A1: Awrt all 4 $x$ values correct and no extras | A1A1 |
| | | | **(5)** |
| | | | **Total 11** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **8(a)** | $2\sin\theta = 1.5 - \sin\theta \Rightarrow \theta = ...$<br>or<br>$\sin\theta = \frac{r}{2} \Rightarrow r = 1.5 - r \Rightarrow r = ...$ | Equate and attempt to solve for $\theta$<br>or<br>Eliminates $\sin\theta$ and solves for $r$ | M1 |
| | $P\left(1, \frac{\pi}{6}\right)$ | Correct coordinates. Allow the marks as soon as the correct values are seen and allow coordinates the wrong way round and allow awrt 0.524 for $\pi/6$ | A1 |
| | $Q\left(1, \frac{5\pi}{6}\right)$ | Correct coordinates. Allow the marks as soon as the correct values are seen and allow coordinates the wrong way round and allow awrt 2.62 for $5\pi/6$ | A1 |
| | | | **(3)** |

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| **(b)** | $\left(\frac{1}{2}\right)\int(1.5-\sin\theta)^2\mathrm{d}\theta$ or $\left(\frac{1}{2}\right)\int(2\sin\theta)^2\,\mathrm{d}\theta$<br>Attempts to use $...\int(\sin\theta)^2\mathrm{d}\theta$ or $...\int(1.5-\sin\theta)^2\mathrm{d}\theta$ | M1 |
| | $(1.5-\sin\theta)^2=2.25-3\sin\theta+\sin^2\theta=2.25-3\sin\theta+\frac{(1-\cos 2\theta)}{2}$<br>Expands (allow poor squaring e.g. $(1.5-\sin\theta)^2=2.25+\sin^2\theta$ **and** attempts to use $\sin^2\theta=\pm\frac{1}{2}\pm\frac{\cos 2\theta}{2}$ | M1 |
| | $\frac{1}{2}\int(1.5-\sin\theta)^2\,\mathrm{d}\theta=\frac{1}{2}\left[\frac{11}{4}\theta+3\cos\theta-\frac{1}{4}\sin 2\theta\right]$<br>M1: Attempt to integrate and reaches an expression of the form<br>$\alpha\theta+\beta\cos\theta+\gamma\sin 2\theta$<br>A1: Correct integration (with or without the ½) | M1A1 |
| | $\frac{1}{2}[\ ]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}=\frac{1}{2}\left\{\left(\frac{11}{4}.\frac{5\pi}{6}+3.\cos\frac{5\pi}{6}-\frac{1}{4}\sin 2.\frac{5\pi}{6}\right)-\left(\frac{11}{4}.\frac{\pi}{6}+3.\cos\frac{\pi}{6}-\frac{1}{4}\sin 2.\frac{\pi}{6}\right)\right\}$<br>This is a key step and must be the correct method for this part of the area e.g. uses their $\frac{\pi}{6}$ and their $\frac{5\pi}{6}$ (or twice limits of their $\frac{\pi}{6}$ and $\frac{\pi}{2}$) | M1 |
| | $\frac{1}{2}\int(2\sin\theta)^2\,\mathrm{d}\theta=\int(1-\cos 2\theta)\mathrm{d}\theta=\left[\theta-\frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{6}}=\left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right)(-0)$<br>Uses the limits 0 and their $\frac{\pi}{6}$ to find at least one segment.<br>If using integration, must have integrated to obtain $p\theta+q\sin 2\theta$ with correct use of limits<br>NB can be done as: $\frac{1}{2}(1)^2\left(\frac{\pi}{3}\right)-\frac{1}{2}(1)^2\sin\left(\frac{\pi}{3}\right)$ but must be correct work for their angles | M1 |
| | $\frac{11}{12}\pi-\frac{11\sqrt{3}}{8}+2\left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right)=\frac{5}{4}\pi-\frac{15}{8}\sqrt{3}$<br>**dd**M1: Adds their two areas to give a numerical value for the shaded area<br>Dependent on the previous 2 M marks and must be a completely correct strategy so needs to be an attempt at:<br>$\frac{1}{2}\int_{"\frac{\pi}{6}"}^{"\frac{5\pi}{6}"}(1.5-\sin\theta)^2\,\mathrm{d}\theta$ or $2\times\frac{1}{2}\int_{"\frac{\pi}{6}"}^{\frac{\pi}{2}}(1.5-\sin\theta)^2\,\mathrm{d}\theta$<br>$+$<br>$2\times\frac{1}{2}\int_0^{"\frac{\pi}{6}"}(2\sin\theta)^2\,\mathrm{d}\theta$ or $\left(\frac{1}{2}\int_0^{"\frac{\pi}{6}"}(2\sin\theta)^2\,\mathrm{d}\theta+\frac{1}{2}\int_{"\frac{5\pi}{6}"}^{\pi}(2\sin\theta)^2\,\mathrm{d}\theta\right)$<br>A1: Correct answer (allow equivalent fractions) | **dd**M1A1 |
| | | **(8)** |
| | | **Total 11** |

| | | |
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| | Note that attempts to use $\left(\frac{1}{2}\right)\int (C_1 - C_2)^2 \mathrm{d}\theta$ e.g. $\left(\frac{1}{2}\right)\int \left(2\sin\theta - (1.5 - \sin\theta)\right)^2 \mathrm{d}\theta$<br>Will probably only score a maximum of the first 3 marks<br>i.e.<br>M1 for $\left(\frac{1}{2}\right)\int \left(2\sin\theta - (1.5 - \sin\theta)\right)^2 \mathrm{d}\theta$<br>M1 for expanding **and** attempting to use $\sin^2\theta = \pm\frac{1}{2} \pm \frac{\cos 2\theta}{2}$<br>M1 for attempting to integrate and reaching an expression of the form<br>$\alpha\theta + \beta\cos\theta + \gamma\sin 2\theta$ | M1 |