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Examiner's Report
Principal Examiner Feedback

Summer 2018

Pearson Edexcel GCE
In Further Pure Mathematics FP3 (6669/01)

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Introduction

This paper proved to be a good test of student knowledge and understanding. It discriminated well between the different ability levels. There were many accessible marks available to students who were confident with topics such as hyperbolic and inverse hyperbolic functions, matrices, integration, vectors and conic sections.

Poor presentation was an issue for some students with sinh, cosh and tanh occasionally written as sin, cos and tan. A significant number of students also lost marks due to copying errors with matrix elements and vector components.

Question 1

Part (a) required students to prove the formula for tanh x in terms of e^{2x}. The question stated "Starting from the definitions of sinh x and cosh x in terms of exponentials" and so the full formulae for sinh x and cosh x needed to be seen. Those who used the correct formulae invariably proceeded to the given answer with no errors.

Part (b) saw more mixed results, although the first two marks of the three available were widely scored. The question required students to use the result from part (a) to obtain the logarithmic form of artanh θ. Most realised that the procedure to find the inverse of a function needed to be used and students were invariably successful in changing the subject of the formula. The last mark required their work to convey the printed answer in terms of θ and some students were unable to achieve this due to confusion with variables. A small number used the given result leading to "$\theta = \theta$". The last mark was only awarded in these cases if an appropriate conclusion was made. A small number of students merely replaced artanh θ with $\frac{\text{arsinh}\,\theta}{\text{arcosh}\,\theta}$ and gave the numerator and denominator in their logarithmic forms.

Question 2

Part (a) required the exact value of the x-axis intercept of the curve $y = 5\cosh x - 6\sinh x$ to be found and was a good source of marks for most students. Most used exponential forms to solve $5\cosh x - 6\sinh x = 0$ but finding x as artanh $\left(\frac{5}{6}\right)$ was an efficient method. It was pleasing that approaches that started by squaring both sides of $5\cosh x = 6\sinh x$ were not widely seen.

Part (b) proved more discriminating. It required $(5\cosh x - 6\sinh x)^2$ to be written in terms of cosh $2x$ and sinh $2x$. Errors were seen with the squaring, including $-30\cosh x\sinh x$ rather than $-60\cosh x\sinh x$ given as the middle term. Most students correctly replaced $2k\cosh x\sinh x$ with $k\sinh 2x$ but many chose not to directly substitute for $\cosh^2 x$ and $\sinh^2 x$ with the equivalent expressions in cosh $2x$. Some replaced $25\cosh^2 x + 25\sinh^2 x$ with $25\cosh 2x$ and ignored the remaining $11\sinh^2 x$. A surprising recurring error was to compute $36 - 25$ as 9. Incorrect "double angle" identities were often seen. Those who elected to convert to exponential forms and back again tended to be less successful.

Part (c) required the exact value of a volume of revolution to be found and most knew that $\pi \int y^2 \mathrm{d}x$ was needed, although 2π in place of π was a common pitfall. A small number used the formula for arc length or surface area. Most integrated successfully although a small number either failed to divide by 2 or multiplied by 2 when integrating sinh $2x$ and/or cosh $2x$. Slips were quite common with substitution of limits. Errors included computing sinh x and cosh x instead of sinh $2x$ and cosh $2x$, miscalculations of sinh(ln 11) and cosh(ln 11) and failure to use the answer from part (a). Particularly common was to assume that the lower limit of 0 gave a value of 0. As with part (b), students converting to exponentials had mixed success.

Question 3

Part (a) required students to use a given eigenvalue to find unknown elements in a 3 x 3 matrix. Those who immediately substituted $\lambda = 3$ into $|\mathbf{M} - \lambda\mathbf{I}| = 0$ tended to make light work of finding the value of k. Most students attempted the determinant of $\mathbf{M} - \lambda\mathbf{I}$ and substituted for λ at a later stage and were more prone to slips. An alternative method was to use $\mathbf{Mx} = 3\mathbf{x}$ but students choosing this route often got confused with the resulting three equations in four unknowns.

Part (b) required the other eigenvalues to be found. As in part (a), the method for processing a determinant was widely known and those who used their value of k from the start were more likely to succeed. Students who multiplied out their expression into a four term cubic were more vulnerable to mistakes. The rule of Sarrus was occasionally seen.

Part (c) asked students to obtain a corresponding eigenvector and although the standard method was well known, slips were seen in solving the simultaneous equations. The alternative of taking a vector product of two rows of $\mathbf{M} - 3\mathbf{I}$ was not common.

The unknown elements made this a somewhat more challenging question than those on previous papers. However, a significant number of well-presented and fully correct solutions were seen.

Question 4

This question on finding an exact arc length was a good source of marks for most. Part (a) required $\frac{\mathrm{d}y}{\mathrm{d}x}$ to be found for the curve $y = \text{arsinh } x + x\sqrt{x^2+1}$. It was very rare to see arsinh x differentiated incorrectly but slips with the product rule occasionally led to errors with the second term, most commonly a failure to use the chain rule correctly when differentiating $\sqrt{x^2+1}$. Those who obtained a fully correct expression for the derivative usually proceeded to the given answer convincingly via a variety of algebraic approaches.

In part (b), the correct formula for arc length was almost always used and the few errors noted largely arose from a missing “dx” and/or missing limits from the given answer.

Part (c) required the arc length to be determined from a given hyperbolic substitution. Most students managed to substitute completely, but errors were seen in simplifying the integrand, including processing of $\sqrt{5\sinh^2 u + 5}$ into $5\cosh u$ instead of $\sqrt{5}\cosh u$. Sign errors in the required identity for $\cosh^2 u$ were common but mistakes with integration were rare. It was unusual to see students reverting to the variable x although in some responses it was used as a surrogate for u. The correct upper limit of arsinh $\frac{2}{\sqrt{5}}$ or $\ln\sqrt{5}$ was widely used although errors in calculating $\sinh\left(2\operatorname{arsinh}\frac{2}{\sqrt{5}}\right)$ were seen. Use of exponential definitions was not common.

Question 5

Part (a) was a reduction formula proof and many completely correct responses were seen. Most students were able to use integration by parts in the correct direction with no slips but some then tried to use parts a second time. Most went on to write $(x+8)^{\frac{3}{2}}$ as $(x+8)(x+8)^{\frac{1}{2}}$ although some were unable to use this to obtain a right hand side involving I_n and I_{n-1}. Those who could, usually obtained the correct values of p and q. The "8" was lost by a small number of students. Other errors included replacing $x\sqrt{x+8}$ rather than $x^n\sqrt{x+8}$ with I_n and slips involving the denominator of 3 (often leading to values of p and q of $\frac{2}{3}$ and $\frac{16}{3}$ instead of 2 and 16).

Part (b) was generally a good source of marks although the final A mark proved challenging for many. Integration to obtain I_0 was usually correct although many students failed to realise that the substitution of the lower limit of 0 produced a non-zero value. The reduction formula was commonly used twice, usually correctly. Often a slip with bracketing precluded obtaining the correct answer. Those who chose to apply the limits at the end seemed more prone to error. A small number used the reduction formula once to obtain I_2 in terms of I_1 followed by a direct attempt at I_1 by parts. This tended to produce mixed results as with the very rare attempts that used integration by parts alone.

Question 6

This vector question was probably the most discriminating on the paper.

Part (a) required students to prove that two given lines were skew. Unfortunately, most believed that all they had to do was to show that the lines did not intersect. This was usually achieved successfully, although a few students were unable to correctly convert the cartesian equation of l_2 into the correct vector equation. General confusion between points and directions was not common. A small but significant number used the same parameter for both lines and solved equations in one variable. The minority who did attempt to show that the lines were not parallel often just referred to "different direction vectors" without identifying the vectors, rather than clearly demonstrating that one direction vector could not be a multiple of the other. Using vector or scalar products or finding the angle between the directions to show that the lines were not parallel was not widely seen and often included errors. Some misconceptions with the definition of skew were evident such as some students arguing that it meant "not perpendicular". Only a very small number of students delivered a proof that was sufficient to score all four marks.

Part (b) required the minimum distance between the skew lines to be found and it saw good scoring on the whole. Those who had remembered the formula correctly invariably proceeded to score all five marks. Most slips resulted from an incorrect calculation of the vector product or from copying errors with the vector components. Those who couldn't recall the formula correctly often attempted a vector product but generally made no further progress. The more cumbersome alternative approaches in the scheme - involving the use of a general chord between the lines - were only seen on occasion and were more likely to be incomplete or include slips.

Part (c) was particularly discriminating. It required a cartesian equation of a plane to be found and it relied on students finding another vector in that plane. Some drew a simple sketch to show the situation and this was usually of benefit. Once a second vector in the plane was identified, subsequent errors were not common, although the direction of l_2 rather than l_1 was sometimes used to obtain the normal vector. Many thought the normal to the plane had the same direction as the perpendicular to the lines from part (b). $\mathbf{r}.\mathbf{n} = \mathbf{a}.\mathbf{n}$ was widely used although some students left their answer in the form $\mathbf{r}.\mathbf{n} = p$. Alternative approaches were very rare.

Question 7

The final question on an ellipse was a good source of marks for most students, particularly parts (a), (b) and (d), although a small number had evidently not managed their time well and some rushed and/or incomplete responses were seen.

Part (a) required an ellipse equation to be found from given foci and directrices. Errors were not common – the correct equations in a and e and the correct eccentricity formula were widely seen so the full five marks were regularly awarded.

Part (b) required students to produce the equation of intersection of the ellipse and a straight line. Almost all substituted $y = mx + c$ into the ellipse equation and slips in proceeding to the given answer were unusual.

Part (c) expected students to use "discriminant = 0" to produce an equation in c and m for when the line was a tangent to the ellipse. Those who realised this immediately usually obtained the correct equation although some algebraic errors occurred, often when identifying a, b and c from $ax^2 + bx + c = 0$. Those who embarked upon methods using calculus gave themselves a lot of work to do and most who chose this route abandoned their attempt before reaching an equation in c and m.

Part (d) continued to see good scoring. The appropriate axes intercepts were usually obtained – sometimes in terms of m alone and sometimes in terms of both c and m – and the correct triangle method was widely used. Students who had reached area = $\frac{c^2}{m}$ could only score the final mark if they had obtained the correct equation in the previous part.

More of a mixed response was seen to the final part. However, many used a correct calculus method - usually differentiating by splitting the fraction rather than using the quotient rule - and proceeded to the correct minimum area. Some errors in differentiation were seen such as the derivative of $\frac{8}{m}$ given as 8 ln m rather than $-\frac{8}{m^2}$. Approaches by completing the square were rare. A small number of students attempted to differentiate the ellipse equation or thought that $m = 1$ rather than $m = \frac{4}{5}$ produced the minimum area.

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