Examiner's Report
Principal Examiner Feedback

Summer 2018

Pearson Edexcel International A Level
In Further Pure Mathematics F2
(WFM02/01)

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Introduction

This paper proved to be a good test of student knowledge and understanding. It discriminated well between the different ability levels. There were many accessible marks available to students who were confident with topics such as inequalities, differential equations, Taylor series, complex numbers, series summations and polar coordinates.

Reports on Individual Questions

Question 1

This inequality question was a good source of marks for the majority of students. Responses were equally split between those who collected terms to one side and then combined the fractions and those who started by multiplying both sides by $x^2\,(x-2)^2$. The latter approach was more prone to error from careless cancelling. The three critical values were obtained by most and the correct solutions usually followed, although a small number gave the opposite regions. Open rather than closed intervals were given occasionally. A very small number of students considered regions before cross-multiplying and largely proceeded correctly. A minority purely cross-multiplied. Graphical attempts were rare and could only access the B mark if no algebra was used.

Question 2

This question on a first order differential equation was more discriminating, with most marks being lost as a result of integration errors. Most students successfully converted the differential equation into the form $\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{P}(x)y = \mathrm{Q}(x)$ although a few made $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject and invariably abandoned their attempt. The formula for the integrating factor was well known although the integration of $\frac{x}{1+x^2}$ was not always correct or errors simplifying $\mathrm{e}^{\frac{1}{2}\ln(1+x^2)}$ occurred. Students who proceeded directly to $yI = \int \mathrm{Q}I\,\mathrm{d}x$ tended to be more successful than those who needlessly tried to write the left hand side as the derivative of a product. The resulting integration of $\frac{x}{(1+x^2)^{\frac{1}{2}}}$ also caused problems with some doomed attempts at integration by parts seen. Unusually, this differential equation could be solved by separation of variables although sign errors in integration were common for those who chose this route.

The method mark for the required particular solution in part (b) was widely scored. The few who had an exponential constant from separation of variables often found themselves unable to determine the value of c.

Question 3

A substantial number of students scored full marks with little obvious difficulty with this question which required a Taylor series solution to a differential equation.

In part (a), most differentiated implicitly and dealt with the products correctly. A smaller number made $\frac{d^2y}{dx^2}$ the subject first but were more prone to slips. An "x" was occasionally missing from the final answer. A few students failed to differentiate the "1" at the start of their working.

Part (b) required a series solution for y and most attempted the values of the necessary derivatives at $x = 2$, although slips were seen with less organised work. A correct Taylor expansion for their values usually resulted although a small number produced a series in powers of x rather than $x - 2$ or had 2 in place of f(2).

In part (c) the method mark was widely scored but the accuracy mark was often lost due to miscalculation. The final answer was required to the correct number of decimal places.

Question 4

This question involved the transformation of a circle using complex numbers and it discriminated widely.

Part (a) required the locus of $|z + \mathrm{i}| = 1$ to be sketched and although almost all students drew a circle it was often incorrectly centred. A few students just gave the equation of the locus in terms of x and y.

A range of approaches were viable in part (b) although those students who chose to make z the subject, added i and then applied $|z + \mathrm{i}| = 1$ tended to score all seven marks with little fuss. Many students made z the subject correctly but were then unsure how to proceed, with many neglecting to use the equation of the original circle or using $|z| = 1$ instead. Many introduced $w = u + \mathrm{i}v$ at an inappropriate time and when this was followed by an attempt to multiply numerator and denominator by the complex conjugate of the denominator, a correct equation was often lost in the resulting algebra. Those who obtained a correct equation this way were also vulnerable to applying Pythagoras incorrectly. Attempts that expressed w in terms of $z + \mathrm{i}$ were rare but usually led to an elegant solution. A common pitfall was to use $|z + \mathrm{i}| = 1$ before z has been isolated. This led to $|w| = |3\mathrm{i}z - 2|$ and usually no further progress.

Question 5

This question on the method of differences saw good scoring in part (a) but less marks were awarded for part (b).

Part (a) required a rational expression to be written in partial fractions and it was rare to see an inappropriate method such as the wrong form of fractions, although a few succumbed to sign or numerical slips.

It seemed that many students were poorly prepared to deal with applying the method of differences to groups of three fractions, with many unable to deduce how the cancelling took place. Some found terms for at least $r = 1$, 2 and 3 but neglected to find any terms at the upper end of the summation. Those who presented their work carefully were more likely to identify the six terms that remained and were less likely to miss any out. Those with appropriate terms usually scored the final method mark for combining their fractions but fully correct solutions were not widely seen. A few attempts involved rewriting the fractions from part (a) into groups of two and and then applying the method of differences separately – these produced mixed results.

Question 6

This question on solving a second order differential equation by first changing the variable also produced a range of mark profiles.

Part (a) required students to carry out a given transformation. Many excellent proofs were seen, usually from students who took care with their presentation and notation. Many were able to differentiate $x = e^t$ to obtain an appropriate expression but the second differentiation, which required both chain and product rules, had more mixed results. It was clear that some students were ill-prepared to handle the differentiation of expressions where a change of variable was involved. Those that obtained a correct first and second derivative usually proceeded correctly by substitution. A few who had obtained expressions for the derivatives of y with respect to t substituted into equation (II) to prove equation (I). Some students were clearly working backwards from the given answer and were unlikely to construct a convincing proof.

Part (b) required the solution of the transformed differential equation and the method was largely well known. It was rare to see an incorrect auxiliary equation or errors in solving it. Students should be aware that in questions on this topic which are not proofs, it is perfectly acceptable to write the auxiliary equation straight down. A correct complementary function usually followed although it was sometimes given in terms of x and not changed to t at any point. Use of a correct form of particular integral was also common although pe^{qt} was occasionally used needlessly instead of ke^{2t}. Differentiation of $y = ke^{2t}$ occasionally led to the incorrect $y' = 2kte^{2t}$. Slips in determining the correct value for k were fairly rare. The general solution was occasionally given as “GS =” rather than “y =”.

Those who had correctly answered part (b) generally had little difficulty stating the general solution to differential equation (I) in part (c). Although a polynomial in x was anticipated, answers with unsimplified logarithmic powers of e were accepted.

Question 7

Part (a) required the use of de Moivre’s theorem to prove the identity for $\cos 7\theta$ in terms of powers of $\cos\theta$ and was generally well-answered. The method of expanding $(\cos\theta + i\sin\theta)^7$ was well known and although occasional slips, usually with signs, were seen, most students were able to correctly express $\cos 7\theta$ in terms of $\cos\theta$ and $\sin\theta$. The majority proceeded to replace $\sin^2\theta$ with $1 - \cos^2\theta$ but many students did not explicitly show use of the expansions of $(1 - \cos^2\theta)^2$ or $(1 - \cos^2\theta)^3$. A small number wrote down the given answer when it was not consistent with their previous work. Attempts using $\left(z + \frac{1}{z}\right)^7$ were rare and usually ended abruptly.

Part (b) required the use of part (a) to solve a heptic equation in x. Some students got confused with the variables here or thought that $\cos 7\theta = 0$ or $+1$ rather than -1. Most who had the correct equation were able to obtain at least one correct solution for 7θ although $\pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi$... was occasionally seen instead of $\pm\pi, \pm 3\pi, \pm 5\pi, \pm 7\pi$. A very common error was the failure to calculate the cosine of the angles after division by seven. Many merely offered the decimal versions of their angles in radians.

Question 8

The final question on finding an area enclosed by two curves given in polar form was predictably discriminating.

Part (a) required students to find the polar coordinates of the points of intersection of the curves. Most students were able to find a correct value for θ, although a second value was sometimes incorrect or missing. Further values of θ were occasionally seen. The correct value of $r = 1$ was widely seen although a small number of students gave an additional value of r, usually -1, for the point Q.

Part (b) asked for the exact area enclosed between the circle and limacon. Those who considered the situation carefully at the start, often by annotating the diagram or drawing their own, were more successful in choosing a correct strategy to find the area of the region enclosed by the curves. The correct formula of $\frac{1}{2}\int r^2 \, d\theta$ was widely used although the $\frac{1}{2}$ was sometimes missing. Use of $\int r \, d\theta$ was very rare. Many were able to score 3 or 4 of the first four marks by obtaining an appropriate integrated expression for the area bounded by the limacon and the straight lines of the segments. Errors included poor squaring, occasional sign errors with the identity for $\sin^2\theta$ and sign and other errors made when integrating the trigonometric terms. Most then used their angles from part (a) as limits, although some chose to double the result of using their smaller angle with an upper limit of $\frac{\pi}{2}$. A wide variety of incorrect limits were seen. The next mark required a correct method to find the area of at least one segment. Most had the correct expression after integration but incorrect limits were again common. Use of the segment area formula $\frac{1}{2}r^2(\theta - \sin\theta)$ was quite rare. The final method mark required a fully correct method and many attempts did not combine the found areas appropriately. A few students used fully correct limits throughout but mixed up the curves. A very small number attempted $\int (\text{circle} - \text{limacon})^2 \, d\theta$. The final A mark was demanding but it was pleasing to see a good number of students reaching the correct exact value.