Examiner’s Report
Principal Examiner Feedback

Summer 2018

Pearson Edexcel International A Level
In Further Pure Mathematics F1 (WFM01/01)

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Summer 2018
Publications Code: WFM01_01_1806_ER

General Introduction

This paper was generally accessible and there were plenty of opportunities for a typical E grade student to gain marks across all the questions. There were some testing questions involving coordinate geometry, complex numbers and mathematical induction that allowed the paper to discriminate well between the higher grades.

In summary, Q1, Q2, Q4(a), Q5, Q6 and Q7 were a good source of marks for the average student, mainly testing standard ideas and techniques, whereas Q3, Q4(b), Q8, Q9, Q10(a) and Q10(b) were discriminating at the higher grades. Q10(c) proved to be the most challenging question on the paper.

Question 1

This question proved accessible with most students scoring full marks.

Almost all students expanded the expression $r(r+3)$ and substituted the standard formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ into $\sum_{r=1}^{n}(r^2+3r)$. Some simplified $3\sum_{r=1}^{n} r$ to give $3n$ and others applied $\sum_{r=1}^{n} r(r+3)$ to give the incorrect $\frac{1}{6}n(n+1)(2n+1)+\frac{1}{2}n(n+1)$, having quoted the correct fomula for $\sum_{r=1}^{n} r$. Some students lost the final mark by not giving their answer in its fully simplified form as specified in the question, with $\frac{n}{6}(n+1)(2n+10)$ seen as their final answer.

Question 2

This was a well-answered question with some students losing a mark in part (b).

In part (a), most students used the matrix $\begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}$ with $\theta = 45°$ to write down a correct exact matrix for **P**.

In part (b), most students correctly noted that the matrix **Q** represented an enlargement, but some failed to include the centre of enlargement in their description.

In part (c), the most popular method was to find the matrix **PQ** in terms of k and apply $6\left(\det(\mathbf{PQ})\right) = 147$ to find the value of k. Some students, who found two values for k, did not proceed to reject their negative value. A few students applied the alternative method $6\left(\det(\mathbf{Q})\right)=147$ to find the correct value of k. A common error in this part was to apply $147\left(\det(\mathbf{PQ})\right)=6$ or $147\left(\det(\mathbf{Q})\right)=6$.

Question 3

This was a well-answered question, with only a minority of students struggling to cope with the problem-solving nature of part (c).

In part (a), almost all students found the correct coordinates of *S*. A few, who stated a correct $a=1.5$, did not attempt to write down the coordinates for *S*.

In part (b), most students used the focus-directrix property of a parabola to correctly write down the distance *SP*. A few students incorrectly stated that $SP=14-1.5-1.5=11$.

In part (c), students used a variety of methods to find the coordinates of *P* and there were many correct solutions. The most common was to apply Pythagoras on $(x-1.5)$, y and 14, where $y=\sqrt{6x}$, and solve the resulting quadratic equation to find x and use $y^2=6x$ to find y. Some used the more efficient method to find x by applying $x=14-1.5$ and used $y^2=6x$ to find y. A common error was to assume $x=11$ and find $y=\sqrt{66}$.

Question 4

This question was accessible to most students with the majority gaining full marks.

In part (a), most students applied a full method to correctly find the matrix $\mathbf{A}^{-1}$, although some made arithmetic errors when finding the determinant of $\mathbf{A}$ and others did not find the correct adjoint of $\mathbf{A}$. A few students multiplied the original matrix by the reciprocal of their determinant and some students made simplification errors when attempting to divide each element in their adjoint of $\mathbf{A}$ by pq.

In part (b), many students applied $\mathbf{BA}^{-1}$ to find the correct matrix $\mathbf{X}$, although a few applied the incorrect method of finding $\mathbf{A}^{-1}\mathbf{B}$. A few students used their $\mathbf{X}=\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$ in the equation $\mathbf{XA}=\mathbf{B}$ to find the elements a, b, c, d, e and f by solving simultaneous equations. Calculation errors and manipulation errors sometimes led to a loss of marks in this part.

Question 5

This question was accessible to most students with the majority gaining full marks.

In part (a), most students correctly found the values $a = -6$, $b = 25$. The method of long division was the most popular and successful method. Those who used the more time-efficient method of comparing coefficients were slightly more prone to making sign or manipulation errors.

In part (b), while many students correctly solved $z^2 + 9 = 0$ to give $z = \pm 3\text{i}$, some surprisingly gave solutions such as $z = \pm\sqrt{3}\,\text{i}$ or $z = \pm 3$. Some students found $z = 3\text{i}$ and made no reference to $z = -3\text{i}$. Most students either applied the quadratic formula or completed the square to find the roots of their $z^2 - 6z + 25 = 0$.

It was good to see that in part (c), many students used a ruler and an appropriate scale to plot all four of their complex roots on an Argand diagram. It was also pleasing that there were many Argand diagrams with roots plotted in the correct positions relative to each other and that the roots were symmetrical about the real axis. There were some students, however, who plotted 3i and -3i on the real axis.

Question 6

This question was accessible to most students with just over half of them gaining full marks.

Success in part (a) very much depended on the strategy that students used to differentiate $\mathrm{f}(x)$. Those who simplified $\mathrm{f}(x) = \dfrac{2(x^3+3)}{\sqrt{x}} - 9$ to give $\mathrm{f}(x) = 2x^{\frac{5}{2}} + 6x^{-\frac{1}{2}} - 9$ were more successful in finding a correct $\mathrm{f}'(x)$. Those who used the quotient rule or the product rule to find $\mathrm{f}'(x)$ sometimes made manipulation errors, indices errors or sign errors. The Newton-Raphson procedure was understood by most students and those with a correct $\mathrm{f}'(x)$ usually gave a correct second approximation for α as 0.476 to 3 decimal places. In some cases, a lack of working meant that it was difficult for examiners to determine whether the Newton-Raphson process was applied correctly. A small number of students wasted their time by performing a second Newton-Raphson iteration.

There were many correct attempts at part (b). Those students who drew a diagram showing the relevant information usually proceeded to apply a correct method. Many students used similar triangles to form a correct equation in β and proceeded to solve it correctly. Some students, however, formed an equation in β with one of their fractions the wrong way around, while others used an odd number of negative lengths in their working. Some students obtained an answer outside the interval $[1.2, 1.3]$, with many not realising that their answer should have been sufficient evidence that something was amiss. Occasionally, students went back to first principles and found the equation of the line joining the points $(1.2, -0.367...)$ and $(1.3, 0.116...)$, before proceeding to find where their line crossed the x-axis.

Question 7

This question was accessible to most students with just over half of them gaining full marks. A few students found and applied $\alpha, \beta = \frac{2+\sqrt{11}\text{i}}{5}, \frac{2-\sqrt{11}\text{i}}{5}$, and so lost a considerable number of marks by not obeying the instruction 'Without solving the (quadratic) equation'.

In part (a), many students correctly manipulated $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ to give $\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$, with most writing down the correct identity $\alpha^2+\beta^2 \equiv (\alpha+\beta)^2 - 2\alpha\beta$. Most deduced that the sum and product of roots in the given quadratic equation $5x^2-4x+3=0$ were $\frac{4}{5}$ and $\frac{3}{5}$ respectively, and applied these to $\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$ to achieve the correct value of $-\frac{14}{9}$.

The complete method in part (b) was understood by most students, but a considerable number failed to score full marks because they made manipulation or substitution errors. Many students correctly deduced that the sum of the given roots was 3 times their answer to part (a), although a few proceeded to repeat what they had done in part (a) to find the correct sum. Many students correctly deduced that the product of the given roots was $\frac{9}{(\text{their } \alpha\beta)^2}$. At this stage, most students proceeded to use a correct method to form the quadratic equation described in the question. The three main errors in establishing the required quadratic equation were: applying the incorrect method of $x^2 + (\text{sum})x + (\text{product}) = 0$; the omission of "$=0$"; and the failure to give integer coefficients.

Question 8

This question discriminated well with just over half of the students gaining full marks.

Some students, who just wrote, for example, 'for $n=1$, LHS = RHS' lost the first mark by not explicitly demonstrating that the general statement is true for $n=1$. Many students wrote down the correct matrix multiplication, but some lost marks for moving from $\begin{pmatrix} a^k & 0 \\ \frac{a^k-b^k}{a-b} & b^k \end{pmatrix}\begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}$ directly to $\begin{pmatrix} a^{k+1} & 0 \\ \frac{a^{k+1}-b^{k+1}}{a-b} & b^{k+1} \end{pmatrix}$ with no intermediate working. Some students did not bring all strands of their proof together to give a fully correct proof. A minimal acceptable proof, following on from completely correct work, would incorporate the following parts: assuming the general result is true for $n = k$; then showing the general result is true for $n = k + 1$; showing the general result is true for $n = 1$; and finally concluding that the general result is true for all positive integers.

Question 9

This question discriminated well between students of all abilities.

In part (a), most students started their solution by multiplying both sides of the printed equation by $(z + 3\text{i})$ to give $z - k\text{i} = \text{i}(z + 3\text{i})$. Some students gave up at this point or produced work that did not gain any credit. The majority expanded $(1+\text{i})(z+5-\text{i})$ and used a complete method to make z the subject of their equation. A minority replaced z by $x + y\text{i}$ and proceeded to equate the real and imaginary parts of both sides of their equation. A few students who tried to "rationalise" $\dfrac{z - k\text{i}}{z + 3\text{i}}$ by writing $\dfrac{(z - k\text{i})(z - 3\text{i})}{(z + 3\text{i})(z - 3\text{i})}$ scored no marks in this part.

In part (b)(i), most students applied Pythagoras' Theorem correctly to find the exact value for the modulus of z when $k = 4$. A few students subtracted rather than added the two squared terms, while some did not proceed beyond writing a correct $z = -\dfrac{7}{2} + \dfrac{1}{2}\text{i}$.

In part (b)(ii), most students used $k = 1$ correctly to find $z = -2 - \text{i}$. A significant number of students did not realise that the required answer was in the fourth quadrant and many of them found an angle in the first quadrant. Most students worked in radians, but a few gave their answer in degrees.

Question 10

Part (a) proved to be accessible to students of all abilities. Part (b) and Part (c) discriminated well across higher ability students with part (b) more successfully answered than part (c).

Students were generally familiar with the required method in part (a) and most gained full marks in this part. Students used a variety of methods to find $\dfrac{\text{d}y}{\text{d}x}$, with the most common being to make y the subject in order to find $\dfrac{\text{d}y}{\text{d}x} = -\dfrac{144}{x^2}$. Most students used the coordinates of P to obtain an expression for $\dfrac{\text{d}y}{\text{d}x}$ in terms of p, which was followed by a correct method for finding the gradient of the normal in terms of p. Many students who progressed this far applied a correct straight-line method and arrived at the given equation $y = p^2x + \dfrac{12}{p} - 12p^3$.

Part (b) proved more challenging for some students, although there was a significant number of many fully correct solutions. Most students used the equation of the normal in a complete method of setting $x = 0$ to find *y* and setting $y = 0$ to find *x*. A common error was to multiply the correct $y_R = \frac{12}{p} - 12p^3$ by -1 to give the incorrect $y_R = 12p^3 - \frac{12}{p}$. Some students lost the final mark in this part for failing to give their answers as coordinates.

Part (c) proved to be the most challenging question on the paper, and only the most able students were able to produce fully correct solutions. Most students used their coordinates for *R* and *Q* and attempted to find the area of triangle *OQR* by applying the formula $\frac{1}{2}$(base)(height). Many students, who did not realise that they needed to take the modulus of the *y*-coordinate for *R*, wrote the incorrect equation $\frac{1}{2}\left(12p - \frac{12}{p^3}\right)\left(\frac{12}{p} - 12p^3\right) = 512$. Other students were unable to manipulate their area equation into a 3-term quadratic in p^4 or a 3-term quadratic in p^2. Of those that did, many provided a correct method to solve their 3-term quadratic, with some finding $p = \pm\sqrt{3}$ and $p = \pm\frac{1}{\sqrt{3}}$. Only the most able students were able to ascertain that $p = \sqrt{3}$ and $p = -\frac{1}{\sqrt{3}}$ were the only answers to part (c).