Examiners' Report

Principal Examiner Feedback

Summer 2017

Pearson Edexcel GCE Mathematics

Core Mathematics C4 (6666)

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Core Mathematics 4 (6666) – Principal Examiner's report

General introduction

This paper proved accessible to lower ability students, allowing a typical E grade student the opportunity to gain marks across all questions.

There were a number of testing questions mainly involving integration and vectors that allowed the paper to discriminate reasonably well between the higher ability levels. For example, in

- Q3(d), some students lost marks for not realising that they needed to split up $\frac{6}{u(u+2)}$ into partial fractions.
- Q5, a minority of students did not realise that they needed to expand $(e^{x} + 2e^{-x})^{2}$ before integrating.
- Q8(c), a significant number of students did not identify the correct methods for integrating both $\theta\sec^{2}\theta$ and $\tan\theta\sec^{2}\theta$ with respect to θ.
- Q6(d) and Q6(e), some students did not sketch a diagram to help them to understand the problems posed and so potentially lost a significant number of marks.

A number of students made basic sign, bracketing or manipulation errors in Q1, Q2, Q4 and Q5.

In summary, Q1, Q2, Q3(a), Q3(b), Q4, Q6(a), Q6(b) and Q6(c) were a good source of marks for the average student, mainly testing standard ideas and techniques; and Q3(c), Q3(d), Q5, Q6(d), Q6(e), Q7, Q8(b) and Q8(c) were discriminating at the higher grades. Q8(c) proved to be the most challenging question on the paper.

Question 1

This was a well-answered question with the majority of students scoring full marks.

In part (a), most students applied the correct method of parametric differentiation. A minority differentiated $y = 5 - \frac{6}{t}$ incorrectly to give $\frac{\mathrm{d}y}{\mathrm{d}t}$ as either $-6t^{-2}$ or $-6\ln t$. Some students who correctly obtained $\frac{\mathrm{d}y}{\mathrm{d}x}$ as $\frac{6t^{-2}}{3}$ simplified this expression incorrectly to give either $3t^{-2}$ or $2t^{2}$. A few students attempted to form a Cartesian equation for *C*, before differentiating to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of *x*. Only a small proportion of these students proceeded to write $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of *t*, as required by the question.

Part (b) proved straightforward for many students. The majority correctly substituted $t = \frac{1}{2}$ into the parametric equations to obtain $P\left(-\frac{5}{2}, -7\right)$. The most common error in this part arose from the substitution of the coordinates of *P* into the formula $y - y_1 = m_T(x - x_1)$. A small, but significant number, made basic arithmetic errors such as simplifying $y + 7 = 8x + 20$ to give $y = 8x + 27$. A few students found the equation of the normal instead of the tangent. Pleasingly, almost all solutions were given in the required form $y = px + q$.

In part (c), most students rearranged $x = 3t - 4$ to make *t* the subject and substituted the result into $y = 5 - \frac{6}{t}$, with many correctly obtaining the equation $y = 5 - \frac{6}{\left(\frac{x+4}{3}\right)}$. While a significant minority incorrectly manipulated this equation to give $y = 5 - \frac{6}{t}$, the majority succeeded in achieving the correct result in the required form. Less successful methods, which tended to be more prone to sign and manipulation errors, included substituting *t* as a function of *y* into $x = 3t - 4$ or rearranging both parametric equations to make *t* the subject followed by equating both results.

Question 2

Students improved their chance of success in this question by expanding $(2+kx)^{-3}$, in terms of k, to give $\frac{1}{8}\left(1+(-3)\left(\frac{kx}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{kx}{2}\right)^2+...\right)$ or a simplified $\frac{1}{8}-\frac{3}{16}kx+\frac{3}{16}k^2x^2+...$, before attempting to answer parts (a), (b) and (c). The $\frac{1}{8}$ outside the brackets was sometimes written incorrectly as either 2, 8 or $\frac{1}{2}$.

In part (a), most students correctly stated A as $\frac{1}{8}$. Some students stated A as 1 after writing down a binomial expansion with $\frac{1}{8}$ as their constant term. Other incorrect values for A included either 2, 8 or $\frac{1}{2}$.

In part (b), most students equated $\frac{243}{16}$ with $\left(\frac{1}{8}\right)\frac{(-3)(-4)}{2!}\left(\frac{k}{2}\right)^2$ and solved to give $k=9$, with some students losing the final mark for not rejecting $k=-9$. Common errors included equating $\frac{243}{16}$ with either $\frac{(-3)(-4)}{2!}\left(\frac{k}{2}\right)^2$, $\left(\frac{1}{8}\right)\frac{(-3)(-4)}{2!}k^2$, $\frac{(-3)(-4)}{2!}k^2$, $\left(\frac{1}{8}\right)\frac{(-3)(-2)}{2!}\left(\frac{k}{2}\right)^2$ $\left(\frac{1}{8}\right)\frac{(-3)(-4)}{2!}\left(\frac{k}{4}\right)$, etc.

In part (c), the majority of students substituted their k from part (b) into either $\frac{1}{8}(-3)\left(\frac{k}{2}\right)$ or a simplified $-\frac{3}{16}k$ in order to find the value of B. Students only achieved full marks in

part (c) if they found $B=-\frac{27}{16}$ from a correct $k=9$. Common errors in this part included substituting their k into either $\frac{3}{16}k$, $-\frac{3k}{2}$, $\frac{3k}{2}$, $-3k$ or $3k$. Some students stated $B=-\frac{27}{16}x$ instead of $-\frac{27}{16}$.

Question 3

The majority of students found parts (a) and (b) to be straightforward and many gained full marks. In part (a), it was extremely rare to see anything other than 1.86254 for the missing value. The application of the trapezium rule was generally correct and the final answer usually rounded to 4 decimal places. Errors seen included the use of an incorrect multiplier of $\frac{1}{6}$ or $\frac{1}{2}$ or a missing or an extra term in the bracket. Some miscalculated their answer from a correctly written expression for the approximate area. It was rare for students to find the approximate area as a sum of separate trapezia.

In part (c), a majority of the students clearly knew how to apply the substitution, but even so it was common for the "show that" mark to be lost. Some students did not show full working and a significant number did not start their proof by stating the area in terms of *x* as $\int_0^1 \frac{6}{e^x + 2}\,\mathrm{d}x$. Some students either missed out $\mathrm{d}x$ at the start of their proof or $\mathrm{d}u$ in the final line of their proof. Many students gained the mark for correct *u* limits of e and 1, though some gave an inexact value for the upper limit. A significant number did not work out the *u* limits until part (d), but the scheme did allow credit for this.

Part (d) proved to be discriminating with a clear divide between those students who appreciated that partial fractions were required, and those who did not. The latter, who sometimes integrated to give expressions such as $6\ln(u(u+2))$ or $\frac{6}{2u+2}\ln(u^2+2u)$, gained no marks in this part. Those who did use partial fractions were generally successful in finding the required fractions and then integrated correctly. Most students correctly applied their limits of e and 1 into an integrated expression in *u* or limits of 1 and 0 to a corresponding expression in *x*. Some students lost the final mark in this part by making sign errors or manipulation errors or by not simplifying terms such as $-3\ln 1$ or $3\ln \mathrm{e}$. A few students divided the correct answer of $3-3\ln(\mathrm{e}+2)+3\ln 3$ by 3 to give $1-\ln(\mathrm{e}+2)+\ln 3$ as their answer for the area of *R*, and so did not gain the final accuracy mark.

Question 4

Most students were familiar with implicit differentiation and generally scored well in this question.

In part (a), the 2^y term in the implicit equation caused problems for a significant number of students. These students differentiated 2^y with respect to x to give incorrect expressions such as $2^y \ln 2$, $2^y \ln y \frac{dy}{dx}$, $2^y \frac{dy}{dx}$, $y2^{y-1}\frac{dy}{dx}$ or $\frac{1}{y2^y}\frac{dy}{dx}$. Some students made sign errors when using the product rule to differentiate $-4xy$ with respect to x, with a significant number incorrectly obtaining $-4y+4x\frac{dy}{dx}$. Substituting $x=-2$, $y=4$ was almost always seen, and those who differentiated the implicit equation correctly usually proceeded to find the correct exact answer.

In part (b), the majority of students demonstrated the complete method for finding the equation of the normal at $(-2, 4)$ followed by substituting $x=0$ and solving for y. A significant number of students made sign or manipulation errors when working with a rational fraction containing $\ln 2$ or $\ln 4$, whilst others applied an incorrect method of finding the value of their normal gradient from their tangent gradient. Only a minority of students achieved the correct answer $y = \frac{13}{2} - \ln 2$.

Question 5

This question offered good discrimination across students of all abilities. Most students applied the volume formula $\pi\int y^2\,dx$ correctly to give a correct $\pi\int_0^{\ln 4}(e^x+2e^{-x})^2\,dx$, although a few used incorrect formulae such as $2\pi\int y^2\,dx$, $\int y^2\,dx$ or even $\int y\,dx$. Only a few students integrated $(e^x+2e^{-x})^2$ wrongly at this stage to give terms such as $\frac{(e^x+2e^{-x})^3}{3}$ or $\frac{(e^x+2e^{-x})^3}{3(e^x-2e^{-x})}$. The majority expanded $(e^x+2e^{-x})^2$ correctly to give $e^{2x}+4e^{-2x}+4$, although some obtained incorrect expressions such as $e^{2x}+4e^{-2x}$, $e^{2x}+2e^{-2x}+4$ or $e^{2x}+4e^{-2x}+4e$. Disappointingly, at this level, a few students expanded $(e^x+2e^{-x})^2$ to give $e^{x^2}+4e^{-x^2}+4$. Most students integrated their $e^{2x}+4e^{-2x}+4$ correctly with only a small number of students differentiating this expression. Most students applied the limits of $\ln 4$ and 0 to their integrated expression and found an exact answer. Some students at this stage made sign errors, manipulation errors and in some cases omitted π from their final exact answer.

Question 6

Most students scored full marks in parts (a), (b) and (c). Parts (d) and (e) offered good discrimination of the more able students.

Part (a) was well answered with most students gaining full marks. Many students, after finding values for both λ and μ, used the third equation to prove that the two lines intersected, not realising that this was not required. Errors included minor slips when substituting their λ or their μ into l_1 or l_2.

Part (b) was also well answered. The majority of students found the correct acute angle by taking the dot product between the direction vectors of l_1 and l_2. The dot product formula led to the obtuse angle and most realised that they needed to subtract this angle from 180°. A minority of students applied the scalar product formula to incorrect vectors such as $4\mathbf{i} + 28\mathbf{j} + 4\mathbf{k}$, $5\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ or their $\overrightarrow{OX}$.

In part (c), the majority of students found the difference between their $\overrightarrow{OX}$ and $\overrightarrow{OA}$, and applied Pythagoras to the result. Most students then gave their answer in the correct simplified surd form. Errors included minor slips in subtracting their vectors.

In part (d), students employed a variety of approaches to find the distance *AY*. Those, who drew a simple diagram to represent the situation, completed the question by multiplying their distance *AX* by the tangent of their answer from part (b), although some used the incorrect method of multiplying their distance *AX* by the sine or cosine of their angle from part (b). A significant number, who preferred a more algebraic approach, found a general expression in terms of μ for $\overrightarrow{AY}$ and applied the equation $\overrightarrow{AY} \bullet \mathbf{d}_1 = 0$ (where $\mathbf{d}_1$ is the direction vector of l_1) to find a value for μ They then substituted their μ into $\overrightarrow{AY}$ and used Pythagoras to find the distance *AY*. Many students found the algebraic manipulation cumbersome with this approach, and errors were common.

Many students struggled to make progress in part (e), although a minority took a number of different, and sometimes innovative, approaches. Those who drew a simple diagram to represent the situation usually applied a geometrical approach to find the possible position vectors of *B* by applying either $\overrightarrow{OA} \pm 0.5\overrightarrow{AX}$ or $\overrightarrow{OX} + 0.5\overrightarrow{XA}$ and $\overrightarrow{OX} + 1.5\overrightarrow{XA}$. A few students realised that one possible position of *B* is halfway between the point *A* and the point *X* and applied $0.5\left[\overrightarrow{OX} + \overrightarrow{OA}\right]$, with some using their answer to deduce the other position of *B*. A significant number of students preferred a more algebraic approach and used l_1 to find an expression for $\overrightarrow{AB}$ in terms of λ. They applied the equation $AX^2 = 4AB^2$ to form and solve a quadratic equation in λ and substituted their values for λ into l_1. Manipulation errors were common with this method with some students progressing only as far as forming the quadratic equation and then giving up.

Question 7

Only a minority of students answered part (a) correctly, with most students failing to understand the implication of the height of water decreasing in the cylinder over time. Many students used $\frac{\mathrm{d}h}{\mathrm{d}t} = k(h-9)^{\frac{1}{2}}$ to obtain $0.1 = k(130-9)^{\frac{1}{2}}$ and their resulting $k = 0.1$ inevitably led to a negative time answer in part (b), which was either ignored, switched or fudged to become a positive value.

In part (b), those students who separated the variables correctly, attempted to integrate both sides of $\int \frac{1}{\sqrt{(h-9)}}\,\mathrm{d}h = \int k\,\mathrm{d}t$, with some integrating $\frac{1}{\sqrt{(h-9)}}$ incorrectly to give expressions such as $\frac{1}{2}(h-9)^{\frac{1}{2}}$, $\frac{2}{3}(h-9)^{\frac{3}{2}}$ or $2\ln(h-9)^{\frac{1}{2}}$. Some students did not use a constant of integration and directly substituted $h = 50$ into their integrated equation to calculate the value of *t*. Other students, who did not read the question carefully, applied $t = 0, h = 130$ to their integrated equation, which contained a constant of integration. It was pleasing, however, to see a substantial number of students using a fully correct method of substituting $t = 0, h = 200$, immediately after integration, to find their constant of integration, followed by substituting $h = 50$ into the resulting equation to find the value of *t*.

Question 8

Fully correct solutions to part (a) were common. Parts (b) and (c) offered good discrimination of the more able students.

In part (a), the majority of students applied a full method of setting $y = 8$ to find θ and substituting their θ into $x = 3\theta\sin\theta$ to find the exact value of k. Common errors included working in degrees to find $k = 90\sqrt{3}$, deducing $k = \frac{\pi}{3}$ after correctly finding $\theta = \frac{\pi}{3}$ and not finding *k* as an exact value.

Responses were variable in part (b), and a significant number of students did not realise that they needed to apply $\int y\frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta$. Many students used the product rule of differentiation to correctly find $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ as $3\sin\theta + 3\theta\cos\theta$, with some incorrectly stating $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ as $3\theta\cos\theta$. Some students tried in vain to prove the result by either finding $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ and then $\frac{\mathrm{d}y}{\mathrm{d}x}$ by parametric differentiation; or by rewriting $\sec^3\theta$ as $(1+\tan^2\theta)\sec\theta$ and then using a variety of trigonometric identities. Those students who applied $\int y\frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta$ usually achieved the correct result, although some made a bracketing error in their proof. A small proportion did not state the limits for θ, or, more often, gave their value for *k* as the upper limit, β.

Many students offered incorrect responses to part (c), with various reasons for errors seen. A significant number failed to comprehend that the integral could be split into two terms and that integration by parts could be used on $\theta\sec^2\theta$, and standard integration (reverse chain rule or integration by substitution) on $\tan\theta\sec^2\theta$. For those students who were able to see what to do, many of these integrated $\theta\sec^2\theta$ correctly to give $\theta\tan\theta - \ln(\sec\theta)$. Integration of $\tan\theta\sec^2\theta$ caused more problems with many students integrating $\tan\theta\sec^2\theta$ to give $\frac{1}{3}\sec^3\theta$. Those who integrated $\tan\theta\sec^2\theta$ to give either $\frac{1}{2}\sec^2\theta$ or $\frac{1}{2}\tan^2\theta$ did so either by recognising $\tan\theta\sec^2\theta$ was in the form $\mathrm{f}(\theta)\mathrm{f}'(\theta)$; or differentiated either $\sec^2\theta$ or $\tan^2\theta$ to give $2\sec^2\theta\tan\theta$ and then applied a reverse process. Most students who applied integration by parts to give $\int\tan\theta\sec^2\theta\,\mathrm{d}\theta = \tan^2\theta - \int\tan\theta\sec^2\theta\,\mathrm{d}\theta$ abandoned the attempt at this stage, with only a few combining the integrals together to give $\int\tan\theta\sec^2\theta\,\mathrm{d}\theta = \frac{1}{2}\tan^2\theta$. A few students attempted to apply integration by parts to $\int(\theta + \tan\theta)\sec^2\theta\,\mathrm{d}\theta$, but the majority of them made little progress with this method. Most students who integrated $\theta\sec^2\theta + \tan\theta\sec^2\theta$ correctly used the limits of $\frac{\pi}{3}$ and 0 to achieve the correct exact answer. Errors at this stage included applying their *k* as the upper limit, not giving an exact answer, or forgetting to incorporate the multiple 3 (i.e. $\lambda = 3$) to their integrated result.

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