



1. (a) Find the binomial expansion of

$$\frac{1}{\sqrt{9 - 10x}}, \quad |x| < \frac{9}{10}$$

in ascending powers of  $x$  up to and including the term in  $x^2$ .  
Give each coefficient as a simplified fraction.

(5)

(b) Hence, or otherwise, find the expansion of

$$\frac{3 + x}{\sqrt{9 - 10x}}, \quad |x| < \frac{9}{10}$$

in ascending powers of  $x$ , up to and including the term in  $x^2$ .  
Give each coefficient as a simplified fraction.

(3)

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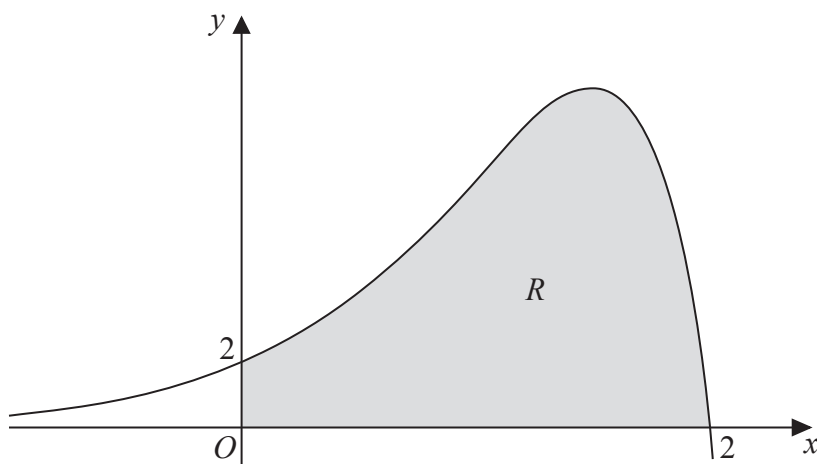


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = (2 - x)e^{2x}, \quad x \in \mathbb{R}$$

The finite region  $R$ , shown shaded in Figure 1, is bounded by the curve, the  $x$ -axis and the  $y$ -axis.

The table below shows corresponding values of  $x$  and  $y$  for  $y = (2 - x)e^{2x}$

$x$	0	0.5	1	1.5	2
$y$	2	4.077	7.389	10.043	0

- (a) Use the trapezium rule with all the values of  $y$  in the table, to obtain an approximation for the area of  $R$ , giving your answer to 2 decimal places. (3)
- (b) Explain how the trapezium rule can be used to give a more accurate approximation for the area of  $R$ . (1)
- (c) Use calculus, showing each step in your working, to obtain an exact value for the area of  $R$ . Give your answer in its simplest form. (5)

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3.  $x^2 + y^2 + 10x + 2y - 4xy = 10$

(a) Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ , fully simplifying your answer. (5)

(b) Find the values of  $y$  for which  $\frac{dy}{dx} = 0$  (5)

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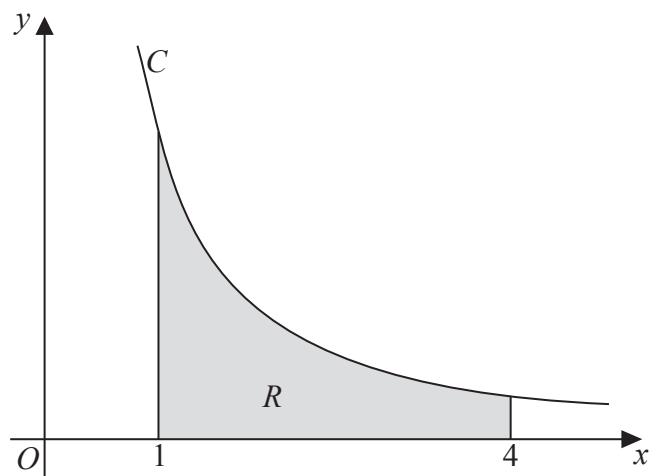
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4. (a) Express  $\frac{25}{x^2(2x+1)}$  in partial fractions. (4)



**Figure 2**

Figure 2 shows a sketch of part of the curve  $C$  with equation  $y = \frac{5}{x\sqrt{2x+1}}$ ,  $x > 0$

The finite region  $R$  is bounded by the curve  $C$ , the  $x$ -axis, the line with equation  $x = 1$  and the line with equation  $x = 4$

This region is shown shaded in Figure 2

The region  $R$  is rotated through  $360^\circ$  about the  $x$ -axis.

- (b) Use calculus to find the exact volume of the solid of revolution generated, giving your answer in the form  $a + b \ln c$ , where  $a$ ,  $b$  and  $c$  are constants. (6)

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6. With respect to a fixed origin, the point  $A$  with position vector  $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$  lies on the line  $l_1$  with equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \quad \text{where } \lambda \text{ is a scalar parameter,}$$

and the point  $B$  with position vector  $4\mathbf{i} + p\mathbf{j} + 3\mathbf{k}$ , where  $p$  is a constant, lies on the line  $l_2$  with equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}, \quad \text{where } \mu \text{ is a scalar parameter.}$$

- (a) Find the value of the constant  $p$ . (1)
  
- (b) Show that  $l_1$  and  $l_2$  intersect and find the position vector of their point of intersection,  $C$ . (4)
  
- (c) Find the size of the angle  $ACB$ , giving your answer in degrees to 3 significant figures. (3)
  
- (d) Find the area of the triangle  $ABC$ , giving your answer to 3 significant figures. (2)

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7. The rate of increase of the number,  $N$ , of fish in a lake is modelled by the differential equation

$$\frac{dN}{dt} = \frac{(kt - 1)(5000 - N)}{t} \quad t > 0, \quad 0 < N < 5000$$

In the given equation, the time  $t$  is measured in years from the start of January 2000 and  $k$  is a positive constant.

- (a) By solving the differential equation, show that

$$N = 5000 - Ate^{-kt}$$

where  $A$  is a positive constant.

**(5)**

After one year, at the start of January 2001, there are 1200 fish in the lake.

After two years, at the start of January 2002, there are 1800 fish in the lake.

- (b) Find the exact value of the constant  $A$  and the exact value of the constant  $k$ .

**(4)**

- (c) Hence find the number of fish in the lake after five years. Give your answer to the nearest hundred fish.

**(1)**

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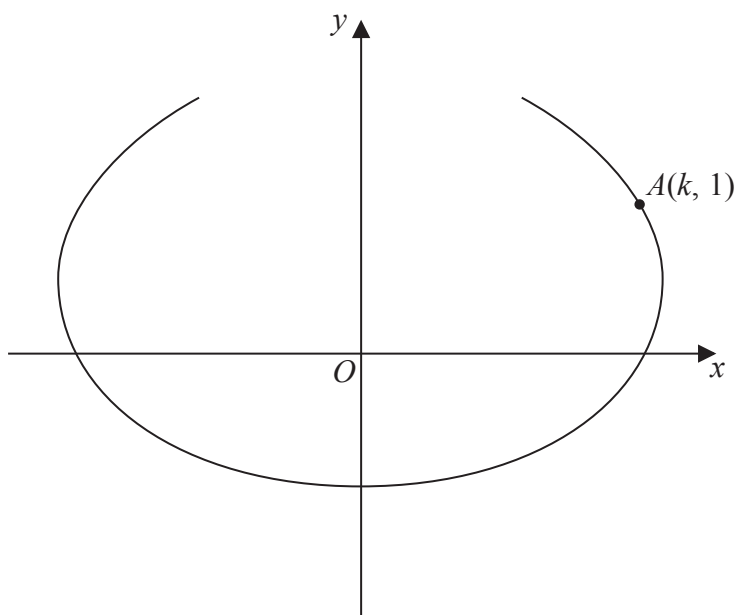


Figure 3

The curve shown in Figure 3 has parametric equations

$$x = t - 4 \sin t, \quad y = 1 - 2 \cos t, \quad -\frac{2\pi}{3} \leq t \leq \frac{2\pi}{3}$$

The point  $A$ , with coordinates  $(k, 1)$ , lies on the curve.

Given that  $k > 0$

(a) find the exact value of  $k$ , (2)

(b) find the gradient of the curve at the point  $A$ . (4)

There is one point on the curve where the gradient is equal to  $-\frac{1}{2}$

(c) Find the value of  $t$  at this point, showing each step in your working and giving your answer to 4 decimal places.

[Solutions based entirely on graphical or numerical methods are not acceptable.] (6)

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Question 8 continued

*(This area contains horizontal lines for writing an answer.)*

**Q8**

(Total 12 marks)

**TOTAL FOR PAPER: 75 MARKS**

**END**

