Examiners' Report

Summer 2014

Pearson Edexcel GCE in Core Mathematics C4 (6666/01)

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Mathematics Unit Core Mathematics 4

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General Introduction

This paper proved to be a good test of Core 4 material and discriminated well across students of all abilities.

Access to solving Q8(e) and Q8(f) was gained by those students who drew a diagram and used it to help them in assimilating the information given.

The standard of algebra seen by examiners was generally good, although a number of students made basic sign or manipulation errors.

Report on Individual Questions

Question 1

This question was well answered.

In Q01(a), many students were able to differentiate correctly, factorise out $\frac{dy}{dx}$, and rearrange their equation to arrive at a correct expression for the gradient function.

A minority did not apply the product rule correctly when differentiating $2xy$, whilst a small number left the constant term of -20 on the left hand side of their differentiated equation and a few differentiated $-x$ to give 0. Sometimes errors were seen in students' differentiation such as x^3 becoming $2x^2$, or similarly y^3 becoming $2y^2\frac{dy}{dx}$.

In Q01(b) the majority of students were able to apply a full method for finding an equation for the tangent, although some were unsuccessful in finding the correct answer due to errors in their manipulation or because of their incorrect answer to Q01(a). Common errors included making arithmetic errors in evaluating $\frac{dy}{dx}$ at $(3,-2)$, finding the equation of the normal instead of the equation of the tangent, and not leaving the equation of the tangent in the form $ax+by+c=0$, where a, b and c are integers.

Question 2

Students improved their chance of success in this question by writing out the expansion of $(1+kx)^{-4}$ as $1+(-4)(kx)+\frac{(-4)(-5)}{2!}(kx)^2+\ldots$ before attempting to answer Q02(a) and Q02(b). This strategy reduced the risk of making a sign error when equating their x coefficients in Q02(a).

In Q02(a), the majority of students wrote down $-4k=-6$ which led to the correct answer of $k=\frac{3}{2}$, although some deduced that $k=\frac{3}{2}$ with no intermediate working. The most common error was to find $k=-\frac{3}{2}$ as a result of writing down $4k=-6$ (or equivalent).

In Q02(b), the majority of students substituted their k from Q02(a) into either $\frac{(-4)(-5)}{2!}k^2$ or a simplified $10k^2$ in order to find the value of A. Students only achieved full marks in Q02(b) if they found $A=\frac{45}{2}$ from a correct $k=\frac{3}{2}$. Common errors in this part included using k instead of k^2, using a binomial coefficient of $\frac{(-4)(-3)}{2!}$ or stating $A=\frac{45}{2}x^2$ instead of $\frac{45}{2}$.

Question 3

In Q03(a) most students were able to find the y value corresponding to $x = 3$. The most common error was to truncate 0.682116... to 0.68211 rather than rounding it to 0.68212

In applying the trapezium rule in Q03(b), a small minority of students multiplied $\frac{1}{2}$ by $\frac{3}{4}$ instead of $\frac{1}{2}$ by 1. Whilst the table of values shows an interval width of 1, the application of a formula $h = \frac{b-a}{n}$ with $n = 4$ instead of $n = 3$ sometimes caused this error. Other errors included a bracketing mistake or rounding their answer incorrectly to give 2.5775.

Whilst many students in Q03(c) identified their estimate of R as an overestimate, many found some difficulty in articulating a reason for this. Those who used a diagram were most successful in showing clearly the extra area. Whilst some students gave no reason, others believed it was an underestimate. Some referred to a negative gradient but this alone was not a sufficient explanation.

In Q03(d), the majority of students differentiated the substitution correctly. The most common errors on substitution included ignoring dx to obtain $\int \frac{10}{2u^2 + 5u}$, partially substituting for x and thus writing an integral involving both x and u, or writing an integral of the form $\int \frac{\pm k}{u(2u^2 + 5u)}\, \mathrm{d}u$. A number of students attempted to integrate $\int \frac{20u}{2u^2 + 5u}\, du$ directly to give $k\ln(u^2 + 5u)$ instead of cancelling the u's to obtain $\int \frac{20}{2u + 5}\, du$. Some students made erroneous algebraic assumptions such as $\frac{20}{(2u + 5)} = \frac{20}{2u} + 4$ prior to their integration. Although a number of students integrated $\frac{20}{(2u + 5)}$ incorrectly to give $20\ln(2u + 5)$, most correctly obtained $10\ln(2u + 5)$ with a few giving $10\ln(u + 2.5)$.

The majority of students applied the changed limits of 2 and 1 correctly to an 'integrated' function in u and gave an exact final answer. A return to x limits would have been acceptable but was seldom seen and only occasionally x limits were used erroneously in a function in u.

Question 4

A number of cases were seen where 80π was misread as 80.

The majority of students multiplied out $V = 4\pi h(h + 4)$ and then found $\frac{dV}{dh}$, although on occasion the second term in their $V = 4\pi h^2 + 16\pi h$ was sometimes incorrect. A significant number of students used the product rule with $u = 4\pi h$ and $v = h + 4$, to find $\frac{dV}{dh}$. The differentiation was sometimes incorrect and it tended to be as a result of using the product rule. A significant number of students stopped after either finding their $\frac{dV}{dh}$ in terms of h or after evaluating $\frac{dV}{dh}$ as 64π when $h = 6$.

At this stage the majority of students applied the chain rule to correctly write down an equation for $\frac{dh}{dt}$. They divided 80π by their $\frac{dV}{dh}$ and substituted $h = 6$ to find a value for $\frac{dh}{dt}$. Common errors at this stage included applying $80\pi \times$ their $\frac{dV}{dh}$, applying their $\frac{dV}{dh} \div 80\pi$ and leaving their final answer as $\frac{dh}{dt} = \frac{80\pi}{64\pi}$.

Question 5

It was interesting to observe that a minority of students could not apply or were unaware of the identity $\cos\left(t + \frac{\pi}{6}\right) \equiv \cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right)$.

In Q05(a), the majority of students were able to prove $x + y = 2\sqrt{3}\cos t$ in one of two ways. About half of them expanded $4\cos\left(t + \frac{\pi}{6}\right)$ and added this to $2\sin t$ and achieved the correct result. The other half expanded $x = 4\cos\left(t + \frac{\pi}{6}\right)$ and deduced that $x = 2\sqrt{3}\cos t - y$ which also led to the correct result. The most common error was for students to expand $\cos\left(t + \frac{\pi}{6}\right)$ to give $\cos t + \cos\left(\frac{\pi}{6}\right)$.

Q05(b) was found to be much more discriminating. Some students used the result from Q05(a) to write down $(x + y)^2 = 12\cos^2 t$ (with a some believing $(2\sqrt{3})^2 = 6$) but could not progress further, although a number tried to find a strategy to eliminate the parameter t. Those students who applied $\cos^2 t \equiv 1 - \sin^2 t$ to obtain $(x + y)^2 = 12(1 - \sin^2 t)$ usually went on to achieve the correct Cartesian equation. A significant number converted $-12\sin^2 t$ to $-6y^2$ which then led to an incorrect answer of $(x + y)^2 + 6y^2 = 12$. A few students who deduced that $a = 3$ and $b = 12$ in $(x + y)^2 + ay^2 = b$, without deriving the Cartesian equation were penalised the final mark in this part.

Question 6

In Q06(i), most students recognised the need to use integration by parts and many fully correct solutions were seen. A few students labelled *u* and $\frac{dv}{dx}$ the wrong way round and a common error was to integrate e^{4x} to give either $4e^{4x}$ or even e^{4x}. During the second stage of the method it was not uncommon to see students integrating $\frac{1}{4}e^{4x}$ to give $\frac{1}{8}e^{4x}$.

In Q06(ii), successful students either applied the method of $\int (ax+b)^n \mathrm{d}x = \frac{(ax+b)^{n+1}}{a(n+1)}$, or applied a substitution of $u = (2x-1)$. Many achieved an answer in the form $\lambda(2x-1)^{-2}$, with some incorrectly giving λ as 4, -4, 1 or $-\frac{1}{4}$. Common erroneous integration led to logarithmic answers and also to $\lambda(2x-1)^{-4}$. There were a few students who differentiated $y = (2x-1)^{-2}$ to give $\frac{dy}{dx} = -4(2x-1)^{-3}$ and proceeded to deduce the correct answer.

Q06(iii) was found to be challenging by the majority of students. They were often able to separate the variables, but although almost all could integrate e^x correctly, many struggled with the integration of $\frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y}$. The most common method was to use the identity $\sin 2y \equiv 2\sin y \cos y$ to give $\int 2\sin^2 y \cos y \, \mathrm{d}y$. Those who integrated this to give $\frac{2}{3}\sin^3 y$ usually went on to score full marks, but some proceeded to $\int 2(\cos y - \cos^3 y)\,\mathrm{d}y$ and often made little further progress or believed that $\int \cos^3 y \,\mathrm{d}y = \frac{1}{4}\sin^4 y$. Some alternative methods of integration were seen such as rewriting the product $\sin 2y \sin y$ as a sum $-\frac{1}{2}\cos 3y + \frac{1}{2}\cos y$ which could be integrated using standard results. Those who attempted to use integration by parts for $\int \sin 2y \sin y \,\mathrm{d}y$ often proceeded no further than a first application. A significant number of students substituted $x = 0$ and $y = \frac{\pi}{6}$ into their integrated equation containing $+c$, but in most cases previous integration errors prevented them achieving the correct answer.

Those students who failed to separate the variables from the outset either attempted to integrate the given expression by parts, despite being a function of both *x* and *y*, or substituted the given values of $x = 0$ and $y = \frac{\pi}{6}$ to obtain a value for $\frac{dy}{dx}$.

Question 7

In Q07(a) the majority of students found $\frac{dy}{dx}$, used it to find the linear equation of the normal to the curve at $(3, 2)$, put $y = 0$ and solved for *x*. A significant number of students were let down by their differentiation and the quality of their algebra. Although most students differentiated $x = 3\tan\theta$ correctly, a significant minority found it challenging to differentiate $y = 4\cos^2\theta$ correctly. Some students used $\cos 2\theta \equiv 2\cos^2\theta - 1$ to give $y = 2 + 2\cos 2\theta$ before obtaining $\frac{dy}{d\theta} = -4\sin 2\theta$,

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whilst others applied the chain rule correctly to give $\frac{dy}{d\theta} = -8\cos\theta\sin\theta$. Common errors in this part included finding $\frac{dy}{d\theta}$ as either $4\sin 2\theta$ or $8\cos\theta\sin\theta$, simplifying a correct $\frac{dy}{dx} = \frac{-8\cos\theta\sin\theta}{3\sec^2\theta}$ to give $\frac{dy}{dx} = -24\cos^3\theta\sin\theta$, or finding the equation of the tangent instead of the equation of the normal. Some students substituted an incorrect value of θ into their $\frac{dy}{dx}$. This was usually $\theta = \frac{\pi}{2}$ which was found by prematurely setting $y = 0$. Few students found the Cartesian equation of *C* and used it successfully to find the *x*-coordinate of *Q*.

In Q07(b) some students were unable to find a volume of revolution by using the parametric equations. Those who adopted a Cartesian equation approach also made little progress. Some stated the volume as $\pi\int y^2 \mathrm{d}x$ but did not know how to continue or rewrote the volume formula as $\pi\int y^2 \mathrm{d}\theta$, with $\mathrm{d}x$ being replaced by $\mathrm{d}\theta$. Those who did not apply $\pi\int y^2 \frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta$ gained little access to this question, and some students attempted to apply an incorrect $\int\left(y\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2\mathrm{d}\theta$. Only a minority of students applied $\pi\int(4\cos^2\theta)^2 3\sec^2\theta\,\mathrm{d}\theta$ to give a correct $48\pi\int\cos^2\theta\,\mathrm{d}\theta$, with a significant number using incorrect manipulation to give $\frac{16}{3}\int\cos^2\theta\,\mathrm{d}\theta$. Those that reached a stage where they were integrating a multiple $\cos^2\theta$ realised the need for using $\cos 2\theta \equiv 2\cos^2\theta - 1$. Whilst the double angle formula was generally quoted correctly, this did not always lead to a correct expression for integration as a result of sign or bracketing errors. After integrating most students used the correct limits of $\frac{\pi}{4}$ and 0, and only a small minority achieved the correct answer of $6\pi^2 + 12\pi$. Occasionally, however, incorrect limits such as 3 and 0 were used.

The majority of students applied $V = \frac{1}{3}\pi r^2 h$ to find the volume of the cone, although a small number attempted to find this volume by applying $\pi\int_{\text{their } \frac{5}{3}}^{3}\left(\frac{3}{2}x - \frac{5}{2}\right)^2 \mathrm{d}x$.

Common errors for finding the volume of the cone included mixing up the values for r and h or using $h = \text{their } x_Q$ rather than $h = 3 - \text{their } x_Q$. A large number of students did not attempt to find the volume of the cone, whilst others found the area of a triangle and subtracted this from their $6\pi^2 + 12\pi$.

Question 8

Q08(a) was well answered with only a few students adding $\overrightarrow{OB}$ to $\overrightarrow{OA}$ instead of applying $\overrightarrow{OB} - \overrightarrow{OA}$. Only a few made arithmetic errors in applying $\overrightarrow{OB} - \overrightarrow{OA}$.

In Q08(b) most students were able to write down a correct expression for l_1, but some students did not form a correct equation by writing $\mathbf{r} = \ldots$

In Q08(c) most students were able to apply the scalar product formula using either $\overrightarrow{AB}$ and $\overrightarrow{PB}$ or $\overrightarrow{BA}$ and $\overrightarrow{BP}$, to obtain the correct answer of $\cos\theta = \frac{1}{3}$. The most common error was to apply the scalar product formula with either $\overrightarrow{AB}$ and $\overrightarrow{BP}$ or $\overrightarrow{BA}$ and $\overrightarrow{PB}$, which usually resulted in giving $\cos\theta = -\frac{1}{3}$, although the minus sign was dropped by a significant number of students. A small minority of students applied the cosine rule correctly to achieve the correct answer. A number of students found this part challenging and usually applied the scalar product formula with non-relevant vectors such as $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

Q08(d) was generally very well answered, particularly if the student had already gained all the marks in Q08(a) and Q08(b). Few wrote down the line for l_2 with position vector and direction vector the wrong way round.

In Q08(e) a significant number of students over-complicated the problem by forming and solving an equation in μ (or λ), to give $\mu = \pm 1$, with some solving their equation incorrectly. Those students who drew a clear diagram, quickly found the coordinates of *C* and *D* by applying $\overrightarrow{OP} \pm$ their $\overrightarrow{AB}$.

Q08(f) was found to be challenging. Those students who drew a diagram tended to be more successful in gaining marks. In order to make progress, it was necessary to find either the perpendicular height of the trapezium or the area of one of the triangles *APB*, *APD* or *BCP*. The majority of students were unable to do this, and did not score any marks for this part. Although many students knew the formula for the area of a trapezium, many erroneously assumed that *AD* was the perpendicular height.

Grade Boundaries

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