Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 7 / 0 1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 10 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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P 4 3 1 3 8 A 0 1 3 2

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PEARSON

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1.

$$\mathbf{M} = \begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$$

Given that the matrix **M** is singular, find the possible values of x.

(4)

Leave blank

Question 1 continued

Q1

(Total 4 marks)

P 4 3 1 3 8 A 0 3 3 2

Turn over

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2. $$f(x) = \cos(x^2) - x + 3, \qquad 0 < x < \pi$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [2.5, 3].

(2)

(b) Use linear interpolation once on the interval [2.5, 3] to find an approximation for α, giving your answer to 2 decimal places.

(3)

Leave blank

Question 2 continued

(Total 5 marks)

Q2

P 4 3 1 3 8 A 0 5 3 2

Turn over

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3. Given that $x = \frac{1}{2}$ is a root of the equation

$$2x^3 - 9x^2 + kx - 13 = 0, \qquad k \in \mathbb{R}$$

find

(a) the value of k,

(3)

(b) the other 2 roots of the equation.

(4)

Leave blank

Question 3 continued

Q3

(Total 7 marks)

P 4 3 1 3 8 A 0 7 3 2

Turn over

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4. The rectangular hyperbola H has Cartesian equation $xy = 4$

The point $P\left(2t, \frac{2}{t}\right)$ lies on H, where $t \neq 0$

(a) Show that an equation of the normal to H at the point P is

$$ty - t^3x = 2 - 2t^4$$

(5)

The normal to H at the point where $t = -\frac{1}{2}$ meets H again at the point Q.

(b) Find the coordinates of the point Q.

(4)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

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Question 4 continued

Q4

(Total 9 marks)

P 4 3 1 3 8 A 0 1 1 3 2

Turn over

Leave blank

5. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (r + 2)(r + 3) = \frac{1}{3}n(n^2 + 9n + 26)$$

for all positive integers n.

(6)

(b) Hence show that

$$\sum_{r=n+1}^{3n} (r + 2)(r + 3) = \frac{2}{3}n(an^2 + bn + c)$$

where a, b and c are integers to be found.

(4)

Leave blank

Question 5 continued

P 4 3 1 3 8 A 0 1 3 3 2

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Question 5 continued

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Question 5 continued

(Total 10 marks)

Q5

P 4 3 1 3 8 A 0 1 5 3 2

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6. A parabola C has equation $y^2 = 4ax, \quad a > 0$

The points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ lie on C, where $p \neq 0, q \neq 0, p \neq q$.

(a) Show that an equation of the tangent to the parabola at P is

$$py - x = ap^2$$

(4)

(b) Write down the equation of the tangent at Q.

(1)

The tangent at P meets the tangent at Q at the point R.

(c) Find, in terms of p and q, the coordinates of R, giving your answers in their simplest form.

(4)

Given that R lies on the directrix of C,

(d) find the value of pq.

(2)

Leave blank

Question 6 continued

P 4 3 1 3 8 A 0 1 7 3 2

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 11 marks)

P 4 3 1 3 8 A 0 1 9 3 2

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7. $z_1 = 2 + 3\text{i}, \quad z_2 = 3 + 2\text{i}, \quad z_3 = a + b\text{i}, \quad a, b \in \mathbb{R}$

(a) Find the exact value of $|z_1 + z_2|$.

(2)

Given that $w = \dfrac{z_1 z_3}{z_2}$,

(b) find w in terms of a and b, giving your answer in the form $x + \text{i}y, \quad x, y \in \mathbb{R}$

(4)

Given also that $w = \dfrac{17}{13} - \dfrac{7}{13}\text{i}$,

(c) find the value of a and the value of b,

(3)

(d) find arg w, giving your answer in radians to 3 decimal places.

(2)

Leave blank

Question 7 continued

P 4 3 1 3 8 A 0 2 1 3 2

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Question 7 continued

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Question 7 continued

Q7

(Total 11 marks)

P 4 3 1 3 8 A 0 2 3 3 2

Turn over

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8.

$$\mathbf{A} = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}$$

and **I** is the 2 × 2 identity matrix.

(a) Prove that

$$\mathbf{A}^2 = 7\mathbf{A} + 2\mathbf{I}$$

(2)

(b) Hence show that

$$\mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A} - 7\mathbf{I})$$

(2)

The transformation represented by **A** maps the point P onto the point Q.

Given that Q has coordinates $(2k + 8, -2k - 5)$, where k is a constant,

(c) find, in terms of k, the coordinates of P.

(4)

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Question 8 continued

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Question 8 continued

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Question 8 continued

(Total 8 marks)

Q8

P 4 3 1 3 8 A 0 2 7 3 2

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9. (a) A sequence of numbers is defined by

$$u_1 = 8$$
$$u_{n+1} = 4u_n - 9n, \quad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 4^n + 3n + 1$$

(5)

(b) Prove by induction that, for $m \in \mathbb{Z}^+$,

$$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^m = \begin{pmatrix} 2m+1 & -4m \\ m & 1-2m \end{pmatrix}$$

(5)

Leave blank

Question 9 continued

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Question 9 continued

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Question 9 continued

P 4 3 1 3 8 A 0 3 1 3 2

Turn over

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Question 9 continued

Q9

(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

END